

# FM1 – Mechanics Chapter 4

## Exam Questions: Collisions with a Plane

**Q1.**

A particle of mass  $m$  kg lies on a smooth horizontal surface.

Initially the particle is at rest at a point  $O$  midway between a pair of fixed parallel vertical walls.

The walls are 2 m apart.

At time  $t = 0$  the particle is projected from  $O$  with speed  $u$  m s<sup>-1</sup> in a direction perpendicular to the walls.

The coefficient of restitution between the particle and each wall is  $\frac{2}{3}$ .

The magnitude of the impulse on the particle due to the first impact with a wall is  $\lambda mu$  N s.

(a) Find the value of  $\lambda$ .

(3)

The particle returns to  $O$ , having bounced off each wall once, at time  $t = 3$  seconds.

(b) Find the value of  $u$ .

(6)

**(Total 9 marks)**

**Q2.**

Two particles  $A$  and  $B$ , of mass  $2m$  and  $3m$  respectively, are initially at rest on a smooth horizontal surface.

Particle  $A$  is projected with speed  $3u$  towards  $B$ . Particle  $A$  collides directly with particle  $B$ .

The coefficient of restitution between  $A$  and  $B$  is  $\frac{3}{4}$

(a) Find

(i) the speed of  $A$  immediately after the collision,

(ii) the speed of  $B$  immediately after the collision.

(7)

After the collision  $B$  hits a fixed smooth vertical wall and rebounds.

The wall is perpendicular to the direction of motion of  $B$ .

The coefficient of restitution between  $B$  and the wall is  $e$ .

The magnitude of the impulse received by  $B$  when it hits the wall is  $\frac{27}{4} mu$ .

(b) Find the value of  $e$ .

(3)

(c) Determine whether there is a further collision between  $A$  and  $B$  after  $B$  rebounds from the wall.

(2)

**(Total for question = 12 marks)**

**Q3.**

A particle  $A$  of mass  $2m$  is moving in a straight line with speed  $3u$  on a smooth horizontal plane.

Particle  $A$  collides directly with a particle  $B$  of mass  $m$  which is at rest on the plane.

The coefficient of restitution between  $A$  and  $B$  is  $e$ , where  $e > 0$

(a) Show that the speed of  $B$  immediately after the collision is  $2u(1 + e)$  .

(6)

After the collision,  $B$  hits a smooth fixed vertical wall which is perpendicular to the direction of motion of  $B$ .

(b) Show that there will be a second collision between  $A$  and  $B$ .

(3)

The coefficient of restitution between  $B$  and the wall is  $\frac{1}{2}$

Find, **in simplified form**, in terms of  $m$ ,  $u$  and  $e$ ,

(c) the magnitude of the impulse received by  $B$  in its collision with the wall,

(3)

**(Total for question = 12 marks)**

**Q4.**

A particle  $P$  of mass  $3m$  is moving in a straight line on a smooth horizontal table.

A particle  $Q$  of mass  $m$  is moving in the opposite direction to  $P$  along the same straight line.

The particles collide directly. Immediately before the collision the speed of  $P$  is  $u$  and the speed of  $Q$  is  $2u$ .

The velocities of  $P$  and  $Q$  immediately after the collision, measured in the direction of motion of  $P$  before the collision, are  $v$  and  $w$  respectively.

The coefficient of restitution between  $P$  and  $Q$  is  $e$ .

(a) Find an expression for  $v$  in terms of  $u$  and  $e$ .

(6)

Given that the direction of motion of  $P$  is changed by the collision,

(b) find the range of possible values of  $e$ .

(2)

(c) Show that  $w = \frac{u}{4}(1 + 9e)$ .

(2)

Following the collision with  $P$ , the particle  $Q$  then collides with and rebounds from a fixed vertical wall which is perpendicular to the direction of motion of  $Q$ .

The coefficient of restitution between  $Q$  and the wall is  $f$ .

Given that  $e = \frac{5}{9}$ , and that  $P$  and  $Q$  collide again in the subsequent motion,

(d) find the range of possible values of  $f$ .

(6)

**(Total for question = 16 marks)**

**Q5.**

A small ball  $A$  of mass  $3m$  is moving with speed  $u$  in a straight line on a smooth horizontal table.

The ball collides directly with another small ball  $B$  of mass  $m$  moving with speed  $u$  towards  $A$  along the same straight line.

The coefficient of restitution between  $A$  and  $B$  is  $\frac{1}{2}$ .

The balls have the same radius and can be modelled as particles.

(a) Find

(i) the speed of  $A$  immediately after the collision,

(ii) the speed of  $B$  immediately after the collision.

(7)

After the collision  $B$  hits a smooth vertical wall which is perpendicular to the direction of motion of  $B$ . The coefficient of restitution between  $B$  and the wall is  $\frac{2}{5}$ .

(b) Find the speed of  $B$  immediately after hitting the wall.

(2)

The first collision between  $A$  and  $B$  occurred at a distance  $4a$  from the wall.

The balls collide again  $T$  seconds after the first collision.

(c) Show that  $T = \frac{112a}{15u}$ .

(6)

**(Total 15 marks)**

**Q6.**

A particle  $P$  of mass  $2m$  is moving in a straight line with speed  $3u$  on a smooth horizontal plane.

It collides directly with a particle  $Q$  of mass  $m$  that is moving on the plane with speed  $2u$  in the opposite direction to  $P$ .

The coefficient of restitution between  $P$  and  $Q$  is  $e$ , where  $e > \frac{4}{5}$

(a) Show that the speed of  $Q$  immediately after the collision is  $\frac{(4 + 10e)u}{3}$

(6)

After the collision  $Q$  hits a smooth fixed vertical wall that is perpendicular to the direction of motion of  $Q$ .

The coefficient of restitution between  $Q$  and the wall is  $f$ .

(b) Find, **in terms of  $e$** , the set of values of  $f$  for which there will be a second collision between  $P$  and  $Q$ .

(4)

**(Total for question = 10 marks)**