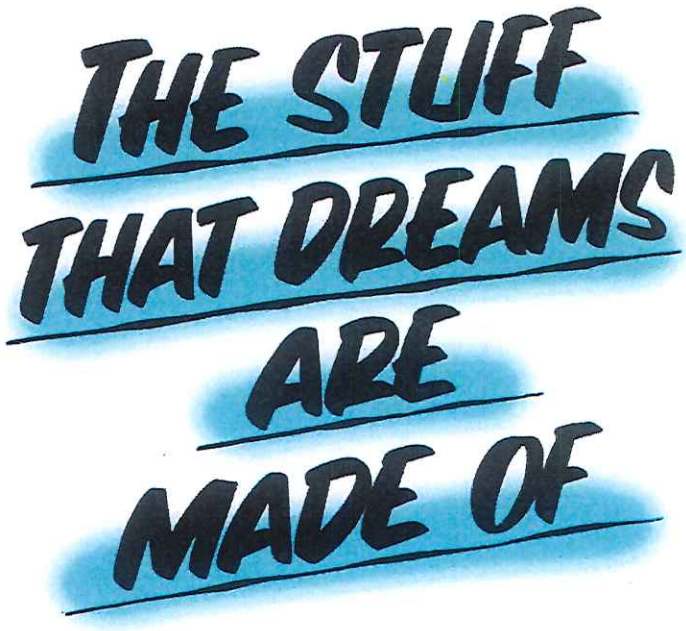
Dreams Pack

Maths Crib Sheets

Applied Year 1



Year 1 Statistics

Standard deviation = $\sqrt{\frac{S_{xx}}{n}}$ or $\sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$

Statistical tables

The following statistical tables are required for A Level Mathematics:

Binomial Cumulative Distribution Function (see page 29)

Random Numbers (see page 38)

Year 1 Mechanics

Kinematics

For motion in a straight line with constant acceleration:

$v = u + at$

$s = ut + \frac{1}{2}at^2$

$s = vt - \frac{1}{2}at^2$

$v^2 = u^2 + 2as$

$s = \frac{1}{2}(u + v)t$

Year 1 Applied Chapter 1 – Data Collection

Key Definitions

Population – all data

Sample – a selection of the data

Sampling Unit – a piece of data

Sampling Frame – a list of data

Surveys

Census – survey of the entire population

Advantage	Disadvantage
- Accurate results	- Time consuming - Expensive

Sampling – selection of the population

SAMPLING – selection of the population

Advantage	Disadvantage
- Quicker - cheaper	- may not be accurate - sub group may not be represented

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Random Sampling

Every sampling unit has an equal chance of being chosen.

Simple Sampling

Names out of a hat.
Random number generator.

Advantages	Disadvantages
- Easy + cheap - Bias Free - Data has equal chance of being selected	- Time consuming - Sample frame needed

Systemic Sampling

Sampling unit chosen at equal intervals. (starting position chosen at random).

Advantages	Disadvantages
- Suitable for large sample - simple + quick to use	- Introduces bias if sample frame not bias - Sample frame needed

Stratified Sampling

Sample groups (strata) in proportion to the population groups (strata).

Advantages	Disadvantages
- Simple + quick to use - Each Strata gets represented	- Population must be in distinct strata - Can introduce bias

Non-Random Sampling

You decide which sampling units you choose.

Quota Sampling

Collect sampling units until you have your quota for each strata.

Advantages	Disadvantages
- Quick + Easy - Inexpensive	- Bias can be introduced - Non responces are not recorded

Opportunity Sampling

Collect sampling units until you have your quota for each strata.

Advantages	Disadvantages
- Easy to carry out - Cheap	- Sample not likely to represent the population

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Types of Data

QUALITATIVE – Non-numerical

QUANTITATIVE – Numerical values

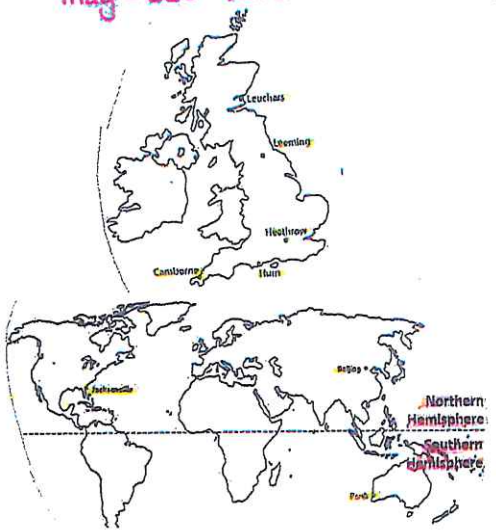
- Discrete – Not all values possible.
- Continuous – Any value possible.

Large Data Set

Weather Stations

may - Oct 1987

may - Oct 2015



Measurements made

Total Rainfall - lots of zeros + tr

Wind Speed

Humidity

Visibility

mean temp

Total sunshine

Cloud cover

Pressure

Max gust speed

Wind Speed - Beaufort Scale qualitative

n/a no data available.

tr trace (Not zero but very small)

Key Notation

x data value

$\bar{x}$ mean

f frequency

$\sum x$ sum of the data

n number of pieces of data

Mean

$$\bar{x} = \frac{\sum x}{n}$$

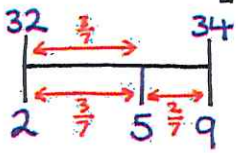
if group frequency table, then x is the midpoints.

Estimate Median

(continuous data)

$30 < t \leq 32$	$32 < t \leq 34$	$34 < t \leq 35$
2	7	1

$n = 10$ median = $10 \div 2 = 5^{th}$ number (no need to do $\frac{n+1}{2}$)



Median = $32 + (2 \times \frac{3}{7}) = 32.9$

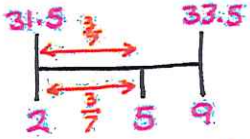
Estimate Median

(continuous data rounded)

Watch out for gaps between the intervals.

Interval $30 - 31$ should be rewritten as $29.5 - 31.5$ etc.

30 – 31	32 – 33	34 – 35
2	7	1



median = $31.5 + (2 \times \frac{3}{7}) = 32.4$

Standard Deviation

$$\sigma = \sqrt{\frac{\sum x^2}{n} - (\bar{x})^2}$$

Standard Deviation = σ

Variance = σ^2

For continuous data the x is the mid-point

Short cut if you are given S_{xx}

$$\sigma = \sqrt{\frac{S_{xx}}{n}}$$

Formula Booklet

Standard deviation = √(Variance)

Interquartile range = $IQR = Q_3 - Q_1$

For a set of n values $x_1, x_2, \ldots x_i, \ldots x_n$

$$S_{xx} = \Sigma(x_i - \bar{x})^2 = \Sigma x_i^2 - \frac{(\Sigma x_i)^2}{n}$$

Standard deviation = $\sqrt{\frac{S_{xx}}{n}}$ or $\sqrt{\frac{\sum x^2}{n} - \bar{x}^2}$

Coding

ORIGINAL DATA ⟷ USER FRIENDLY DATA

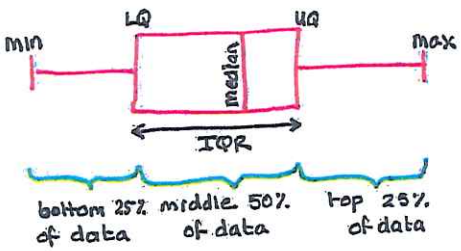
Mean
is affected by all calculations.

Standard Deviation
is affected by multiplication and division only.

Original Mean	coding	New Mean ($\bar{y}$)
$\bar{x}$	$\times 3$	$3\bar{x}$
$\bar{x}$	$\div 3$	$\frac{\bar{x}}{3}$
$\bar{x}$	$\times 3 + 2$	$3\bar{x} + 2$
$\bar{x}$	$\div 2 - 4$	$\frac{\bar{x}}{2} - 4$

Original Standard Deviation	coding	Original Mean (σ_y)
σ_x	$\times 3$	$3\sigma_x$
σ_x	$+ 3$	σ_x
σ_x	$\times 3 - 2$	$3\sigma_x$
σ_x	$\div 3 + 2$	$\frac{\sigma_x}{3}$

Box Plots



Locating Median, LQ and UQ

3 4 4 6 8 9 9 10 12
↓ median 8

3 4 4 6 9 9 10 12
↓ LQ 4 ↓ UQ 9.5

3 4 4 6 8 9 9 10 12 13
↓ median 8.5

3 4 4 6 8 9 9 10 12 13
↓ LQ 4 ↓ UQ 10

Outliers

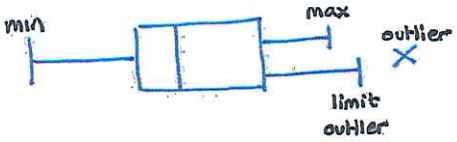
Outliers are extreme values.

Possible ways you may be asked to calculate outliers:

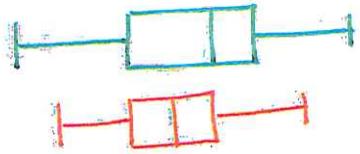
(1.5 x IQR) ± UQ and LQ

(2 σ) ± UQ and LQ

represent on a box plots



Comparing Box Plots



Make 2 comparision

- SPREAD (IQR or Range)
- AVERAGE (median)

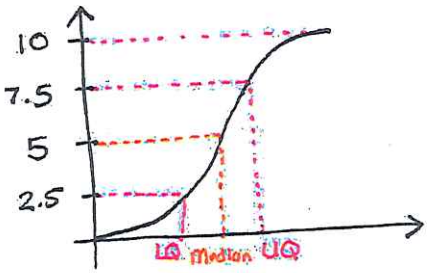
Cumulative Frequency Graphs

Time	Freq	Cumulative Frequency
$9 < t \leq 10$	1	1
$10 < t \leq 11$	4	1+4 = 5
$11 < t \leq 12$	3	1+4+3 = 8

(upper value, Cumulative Frequency)

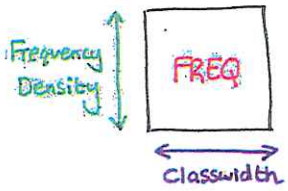
(10, 1)
(11, 5)
(12, 8)

Locating Median, LQ and UQ



Histograms

- Continuous Data
- Uneven Intervals
- Frequency = Area
- No gaps between bars



Weight	Freq	Freq Density
$10 < w \leq 15$ (5)	15	15 ÷ 5 = 4

Scalers

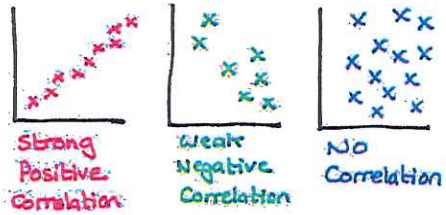
Always make sure

Total Area = Total Frequency

If not you need to find a SCALER

$$\text{SCALER} = \frac{\text{Total Frequency}}{\text{Total Area}}$$

Correlation



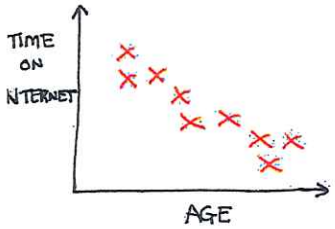
Describe Correlation

Positive Correlation + Strength

Negative Correlation + Strength

No correlation

Describe Relationship

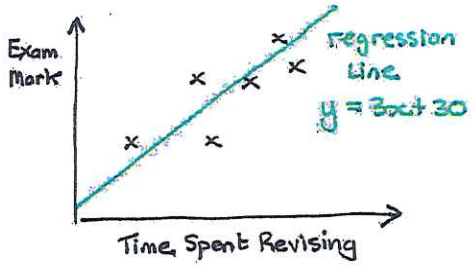


Correlation : Weak Negative Correlation

Describe Relationship : As you get older you use the internet less.

Regression Line

A regression line is a line of best fit (calculated).



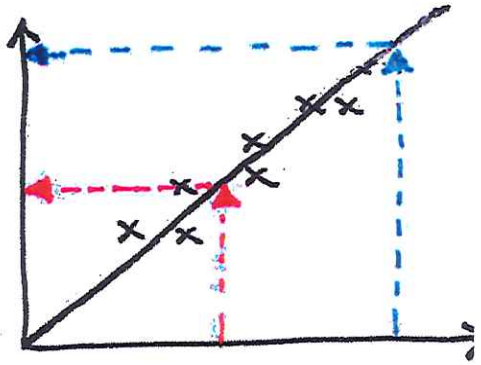
Gradient : 3 marks for every hour of revision

Y Intercept : 30 marks would be obtained with no revision

Interpolation and Extrapolation

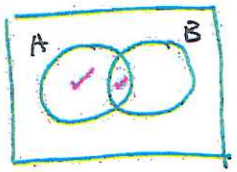
Interpolation – estimating values inside the data range.

Extroplating – estimating values outside the data range.

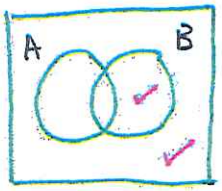


Year 1 Applied Chapter 5 - Probability

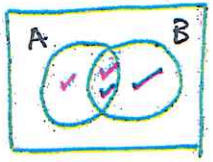
Venn Diagrams



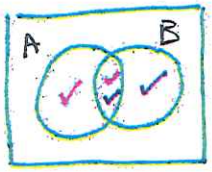
$P(A)$



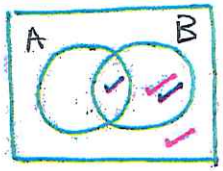
$P(A')$



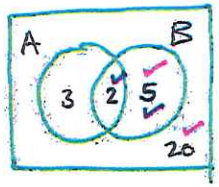
$P(A \cap B)$
and
double tick



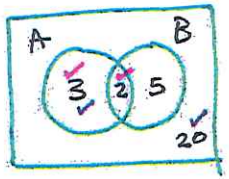
$P(A \cup B)$
or
anything ticked



$P(A' \cap B)$
double tick



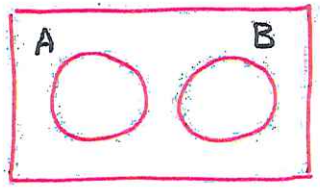
$P(A' \cup B)$
anything ticked
$\frac{2+5+20}{30} = \frac{27}{30}$



$P(A \cap B')$
double tick
$\frac{3}{30}$

Mutually Exclusive

Cannot happen at the same time.

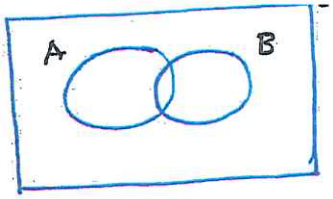


mutually exclusive if

$P(A \text{ or } B) = P(A) + P(B)$

Independent Events

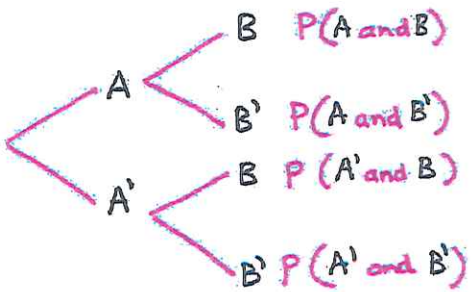
One event is not affected by another.



Independent if

$P(A \text{ and } B) = P(A) \times P(B)$

Tree Diagrams



Always double check if the question is replacement or not replacement.

Year 1 Applied Chapter 6 – Statistical Distribution

Probability Distributions

Tables

X	1	2	3	4	
$P(X=x)$	0.2	0.2	0.5	0.5	$=1$

Functions

$$P(X=x) = \begin{cases} 0.2 & x=1,2 \\ 0.5 & x=3 \\ 0.1 & x=4 \end{cases}$$

Binomial Distribution

$$X \sim B(n, p)$$

n = number trials
p = probability of success
r = number of success required

$$P(X=r) = \binom{n}{r} p^r (1-p)^{n-r}$$

where

$$\binom{n}{r} = {}^nC_r = \frac{n!}{r!(n-r)!}$$

Binomial PD and CD

Classwiz Calculator

menu
STAT
DIST
Bpd
or
Bcd Data : Variable

$P(X = x)$ Binomial PD

menu
7
4 : Binomial PD
2 : Variable

$P(X \leq x)$ Binomial CD

menu
7
Page down ↓
1 : Binomial CD
2 : Variable

Watch out for ⩾

$$P(X \geqslant 5) = 1 - P(X \leq 4)$$

1 2 3 4 5 6 7 8

$P(X \leq 4)$

N = 8 P = 0.55
$P(X = 0) = 0.00168$
$P(X = 1) = 0.00164$
$P(X = 2) = 0.0703$
$P(X = 3) = 0.172$
$P(X = 4) = 0.262$
} $P(X \leq 4) = 0.523$

$P(X = 5) = 0.257$
$P(X = 6) = 0.157$
$P(X = 7) = 0.0548$
$P(X = 8) = 0.00837$
} $P(X \geq 5) = 1 - P(X \leq 4)$
$= \mathbf{0.477}$

Example (PD)

Total outcomes (N) = 8
Probability of success (P) = 0.55
What is the probability of
0 success out of 8?

$$P(X = 0) = 0.00168$$

Example (CD)

Total outcomes (N) = 8
Probability of success (P) = 0.55
What is the probability of
3 success or less out of 8?

$$P(X \leq 3) = 0.260$$

Year 1 Applied Chapter 7 – Hypothesis Testing

Hypothesis

$H_0: p =$ $H_0: p =$ $H_0: p =$

$H_1: p <$ $H_1: p >$ $H_1: p \neq$

Probability of Sample

$P(X \leq x)$ $P(X \geq x)$ $P(X \leq x)$ or $P(X \geq x)$

Significant Level

$P(\text{SAMPLE}) >$ Significant Level

Accept H_0

$P(\text{SAMPLE}) <$ Significant Level

Accept H_1

Don't forget if it is a two tail to half the significant level.

Conclusion

Always make the conclusion in context of the original problem.

Critical Regions

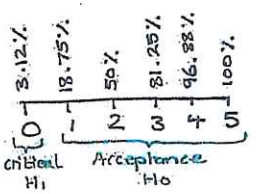
$H_0: p = 0.5$

$H_1: p < 0.5$

$p = 0.5$ $n = 5$

X	$P(X \leq x)$
0	0.0312
1	0.1875
2	0.5000
3	0.8125
4	0.9688
5	1.0000

5% significant level



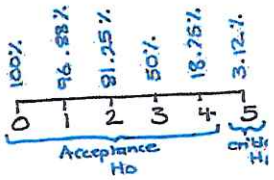
$H_0: p = 0.5$

$H_1: p > 0.5$

$p = 0.5$ $n = 5$

X	$P(X \geq x)$
0	1.0000
1	0.9688
2	0.8125
3	0.5000
4	0.1875
5	0.0312

5% significant level



$H_0: p = 0.5$

$H_1: p \neq 0.5$

$p = 0.5$ $n = 10$

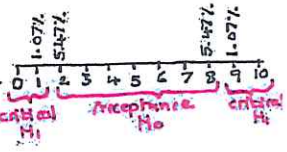
X	$P(X \leq x)$	$P(X \geq x)$
0	0.001	1.000
1	0.0107	0.999
2	0.0547	0.9893
⋮		
8	0.9893	0.0547
9	0.999	0.0107
10	1.000	0.001

5% significant level

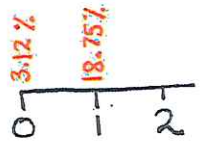
2tail

Bottom tail 2.5%

Top tail 2.5%



Actual Significant Level



Significant level 5%

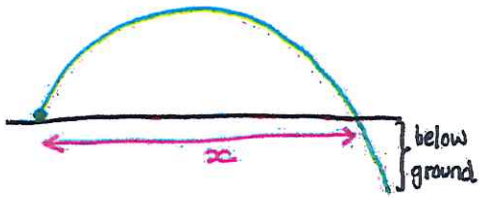
Actual significant level 3.12%

Year 1 Applied Chapter 8 – Modelling in Mechanics

Modelling

You need to think how the maths applies to the context of the problem.

$h = 2 + 1.1t - 0.3t^2$



Equation only valid if in the range of x.
Use common sense!

Definitions

Particle – Dimensions are negligible. Mass acts at single p

Rod – Thinness is negligible. Mass acts at single poi

Inextensible String – string does not strech

Light Object – Mass is negliabible. Compare to other objects.

Uniform Body – mass is distributed evenly. Mass acts at the centre.

Assumptions

Make life easier for yourself!

- Ignore air resistance.
- Ignore rational rotation.
- No friction.
- Mass acts at a single point.
- Object has no dimensions.

Units

Always check the units in the question are as follows.
In not them convert.

mass	kg
Length	metres
Time	seconds
Speed	ms^{-1}
Velocity	ms^{-1}
Acceleration	ms^{-2}
Force	Newtons
Weight	Newtons

$6\ kmh^{-1} = 6\ \frac{km}{h} = 6\ \frac{1000}{3600} = \frac{5}{3} ms^{-1}$

Vectors

i components

right and left

j components

up and down

Velocity and Vectors

Velocity

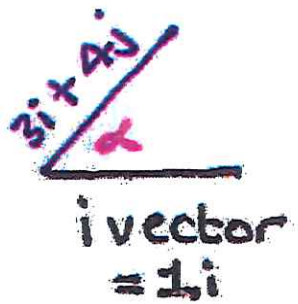
$3i + 4j\ ms^{-1}$

Speed

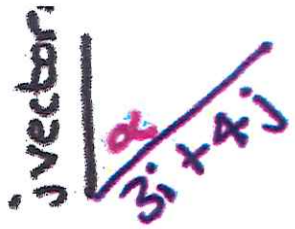
$\sqrt{3^2 + 4^2} = 5\ ms^{-1}$

Unit Vectors

i unit vectors = $1i$



j unit vectors = $1j$



Distance Time Graphs

Average Velocity = $\frac{\text{displacement}}{\text{time}}$

Average Speed = $\frac{\text{total distance}}{\text{time}}$

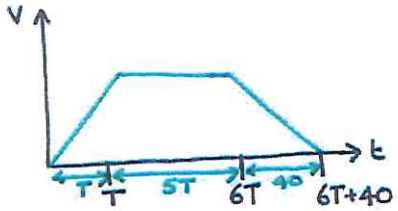
Displacement = distance from origin

Velocity Time Graphs

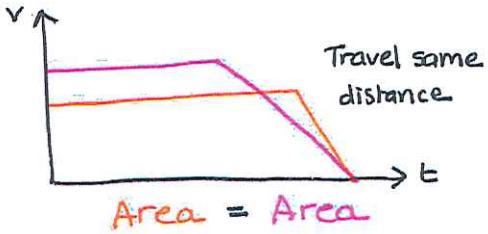
Gradient = Acceleration +
or
Deceleration −

Distance Travelled = Area under Graph

watch out for how you label the axis. (Gaps or points)



Velocity Time Graphs (Two Vehicles)



If they have travelled the same distance, then area = area.

Make sure not to draw the one graph inside the other graph, as they could not be equal in area.

SUVAT Formula

Only use when acceleration is constant.

$$V = u + at$$

$$V^2 = u^2 + 2as$$

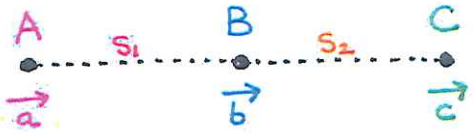
$$S = \left(\frac{u+v}{2}\right)t$$

$$S = ut + \frac{1}{2}at^2$$

$$S = vt - \frac{1}{2}at^2$$

Given in the formula booklet.

SUVAT – Horizontal (multi journey)



constant acceleration x

AB

$s = s_1$
$u = a$
$v = b$
$a = x$
$t = t_1$

BC

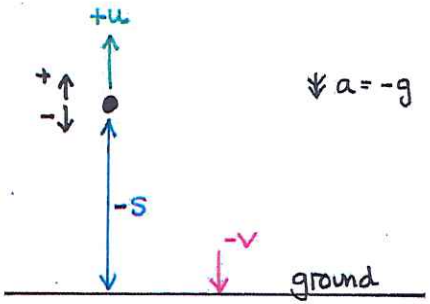
$s = s_2$
$u = b$
$v = c$
$a = x$
$t = t_2$

AC

$s = s_1 + s_2$
$u = a$
$v = c$
$a = x$
$t = t_1 + t_2$

SUVAT - Vertical

Direction Matters!



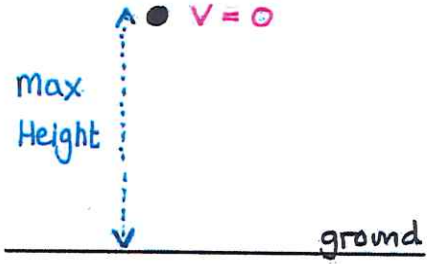
SUVAT - Vertical (Time above a given height)

$$0 = ut + \frac{1}{2}at^2$$

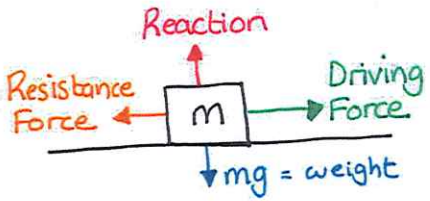
Quadratic (Two solutions)

Time Above $S = t_1 - t_2$

SUVAT - Vertical (Maximum Height)



Forces (Horizontal)



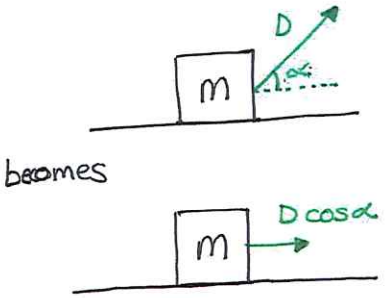
$F = ma$

F = Resultant Force

= Driving Forces − Resistant Forces

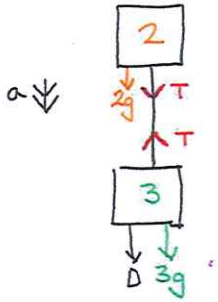
Forces at an Angle

Forces should be parallel to the ground.

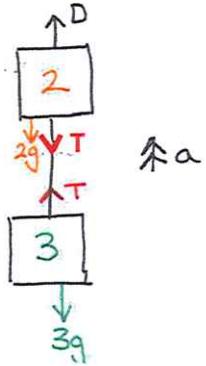


Tension

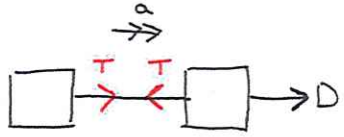
Descending



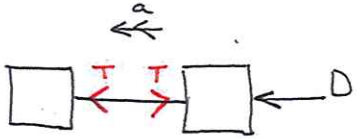
Ascending



Tension

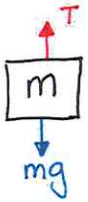


Thrust (Reversing)



Forces (Vertical)

Direction Matters!



$g = 9.8\ ms^{-2}$

Direction of motion matters

a ↑ $F = ma$

$T - mg = ma$

a ↓ $F = ma$

$mg - T = ma$

Vectors and Forces

Adding vector forces

$F_1 = 2i + 3j$

$F_2 = 4i - j$

$F_1 + F_2 = 6i + 2j$

Magnitude

$a = 4i + 3j$

$|a| = \sqrt{4^2+3^2} = 5$

Bearings

Northline Clockwise

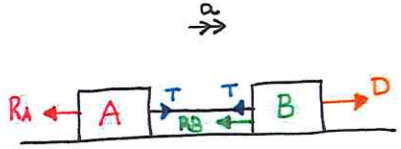
Parallel

$ai + bj$

$xi + yj$

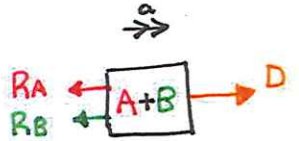
$\frac{b}{a} = \frac{y}{x}$

Connected Particles (Horizontal Only)



Overall System

Good for calculating acceleration.

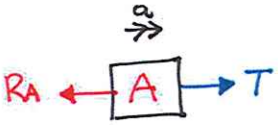


overall $F = ma$

$D - R_A - R_B = (A+B)a$

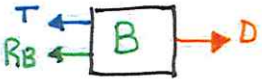
Individual Particles

Good for calculating tension.



partical A $F = ma$

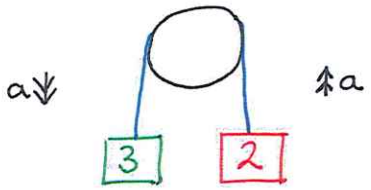
$T - R_A = Aa$



partical B $F = ma$

$D - T - R_B = Ba$

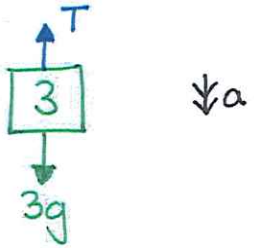
Pulleys



Overall System

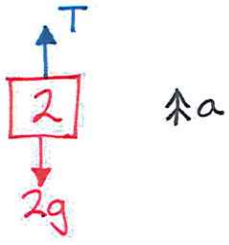
Cannot look at the an overall system as the particles are moving in direction directions.

Individually



$F = ma$

$3g - T = 3a$

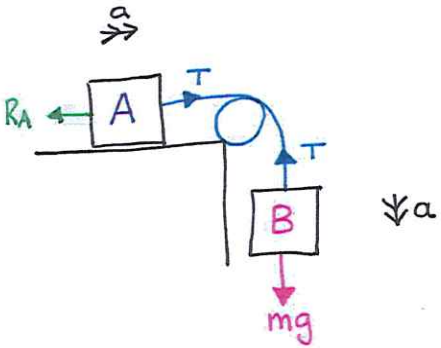


$F = ma$

$T - 2g = 2a$

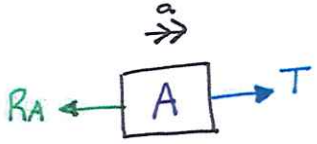
Connected Particles

(Horizontal and vertical)



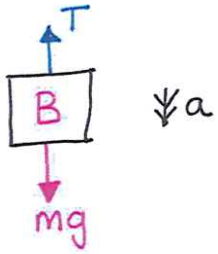
Individual Particles

Good for calculating tension.



$F = ma$

$T - R_A = Aa$



$F = ma$

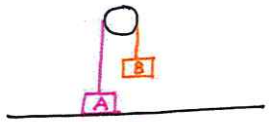
$mg - T = Ba$

Forces and SUVAT

Journey Part A

system is released ($u = 0$)

Journey Part B



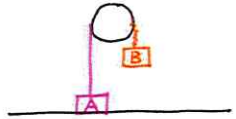
A hits ground

B keeps traveling up

For B driving force is now zero

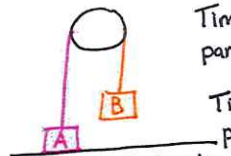
Recalculate acceleration ($F = ma$)

Journey Part C



B reaches its greatest height where $v = 0$

Journey Part D

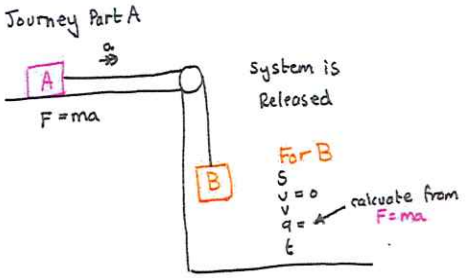


Time from part B to part C = Time from part C to part D

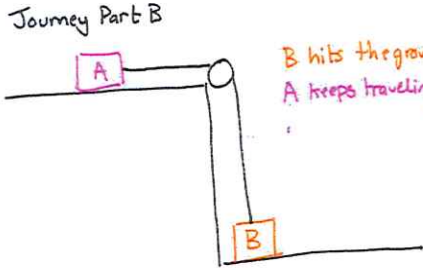
String Taut Again

Forces and SUVAT

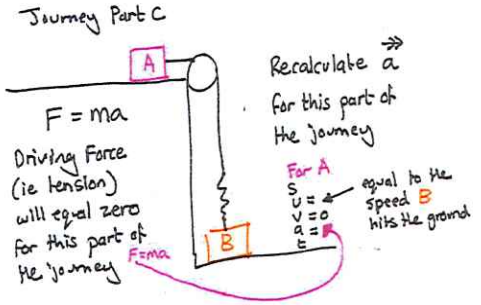
Journey Part A



Journey Part B

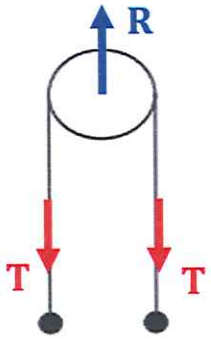


Journey Part C



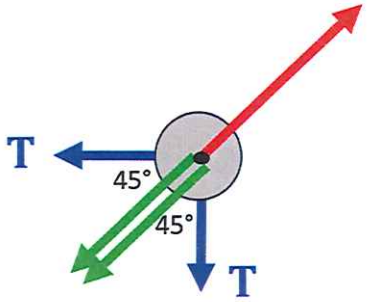
Forces Exerted on a Pulley

Both vertical



The force exerted on the pulley is $2T$ in the downwards direction.

Horizontal and Vertical

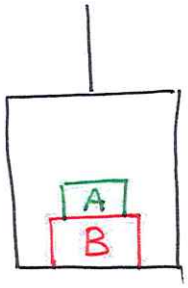


The force exerted on the pulley is $2T\cos 45$ in the south-west direction.

Lifts and Scale Pans

Newtons 3rd Law
Everything has an equal and opposite reaction.

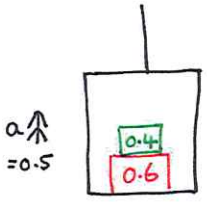
Stacked Particles
Always easier to work out the forces exerted using the particle on top.



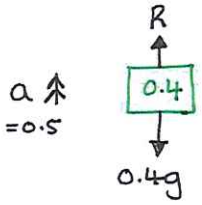
Force exerted on B by A = Force exerted on A by B

Ascending Example

Find the force exerted on A by B.
(In other words, find the driving force of A)



Partical on Top



$F = ma$

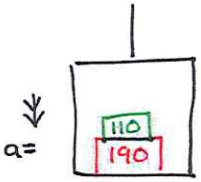
$R - 0.4g = 0.4(0.5)$

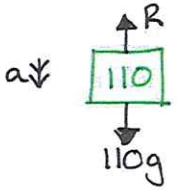
$R = 4.12$

Force excerted on A by B is 4.12

Descending Example

Find the force exerted on A by B.
(In other words, find the reactional force of A)





$F = ma$

$100g - R = 110(1.8)$

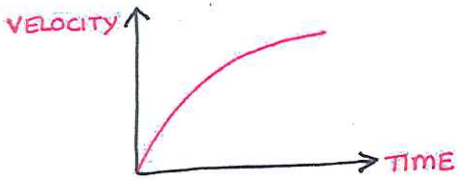
$R = 880$

Force excerted on A by B is 880

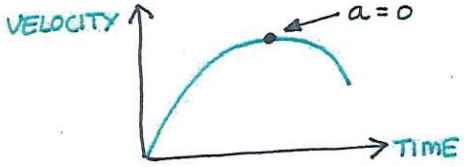
Year 1 Applied Chapter 11 – Variable Acceleration

Increasing Acceleration

Decreasing Acceleration

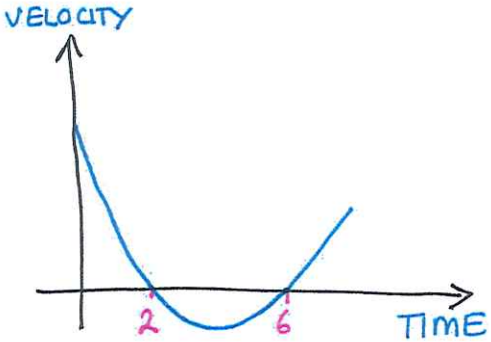


Turning Point



Changing Direction

When a particle reaches zero it could indicate a change of direction.



From 0 to 2
slowing down but moving →

From 2 to 6
Change of direction ←

From 6 and beyond
Change of direction →

Variable Acceleration (Do not use SUVAT)

S = displacement

↓ differentiate

V = velocity

↓ differentiate

a = acceleration

S = displacement

↑ integration

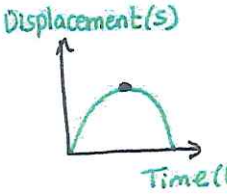
V = velocity

↑ integration

a = acceleration

Don't forget the constant value +c

Maximum and Minimum

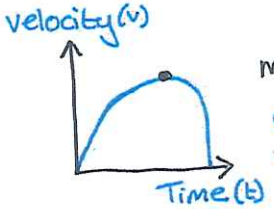


max displacement

$\frac{ds}{dt} = V = 0$

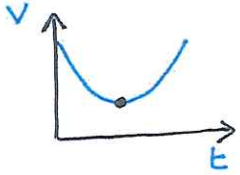
min displacement

$\frac{ds}{dt} = V = 0$



max velocity

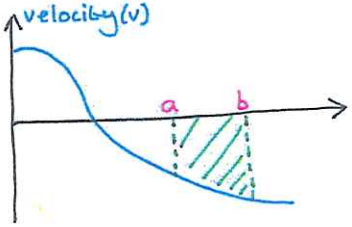
$\frac{dv}{dt} = a = 0$



min velocity

$\frac{dv}{dt} = a = 0$

Area Under a Graph



$S = \int_a^b V \, dt$

Area = Distance Travelled