1. Prove that the sum of three consecutive integers is divisible by 3.

three consecutive integers; n
$n+1$
$n+2$

sum of;

$n+(n+1)+(n+2)$

$= 3n+3$

$= 3(n+1)$

$\therefore$ divisible by 3.

(3)

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2. Prove $(n + 6)^2 – (n + 2)^2$ is always a multiple of 8

$(n+6)(n+6)$

$n^2+12n+36$

$(n+2)(n+2)$

n^2+4n+4

$= n^2+12n+36-n^2-4n-4$

$= 8n+32$

$= 8(n+4)$

$\therefore$ divisible by 8 and is therefore a multiple of 8

(4)

3. Prove $(n + 10)^2 - (n + 5)^2$ is always a multiple of 5

$(n+10)(n+10) - (n+5)(n+5)$

$= n^2+20n+100 - (n^2+10n+25)$

$= n^2+20n+100-n^2-10n-25$

$= 10n+75$

$= 5(2n+15)$

$\therefore$ a multiple of 5

(4)

4. Prove the sum of two consecutive odd numbers is even.

two consecutive odd numbers; $2n+1$

$2n+3$

Sum of:

$= 2n+1+2n+3$

$= 4n+4$

$= 4(n+1)$

$\therefore$ an even number.

(3)

5. $(2n + 1)(3n - 2) - (6n - 1)(n - 2)$ is always even

$6n^2 + 3n - 4n - 2 - (6n^2 - n - 12n + 2)$

$= 6n^2 - n - 2 - 6n^2 + 13n - 2$

$= 12n - 4$

$= 4(3n - 1)$

$\therefore$ an even outcome.

(3)

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6. Prove that the sum of three consecutive even numbers is always a multiple of 6

three consecutive even numbers; $2n$

$2n+2$

$2n+4$

Sum of:

$= 2n + 2n + 2 + 2n + 4$

$= 6n + 6$

$= 6(n+1)$

$\therefore$ a multiple of 6.

(3)

7. Prove the sum of four consecutive odd numbers is always a multiple of 8

four consecutive odd numbers; $2n+1$
$2n+3$
$2n+5$
$2n+7$

Sum of:

$2n+1+2n+3+2n+5+2n+7$

$= 8n+16$

$= 8(n+2)$

$\therefore$ a multiple of 8.

(4)

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8. Prove $(2n + 9)^2 - (2n + 5)^2$ is always a multiple of 4

$(2n+9)(2n+9) - (2n+5)(2n+5)$

$= 4n^2 + 36n + 81 - (4n^2 + 20n + 25)$

$= 4n^2 + 36n + 81 - 4n^2 - 20n - 25$

$= 16n + 56$

$= 4(4n+14)$

$\therefore$ a multiple of 4

(4)

9. Prove $(n + 1)^2 + (n + 3)^2 - (n + 5)^2 = (n + 3)(n - 5)$

$(n+1)(n+1) + (n+3)(n+3) - (n+5)(n+5)$

$= n^2 + 2n + 1 + (n^2 + 6n + 9) - (n^2 + 10n + 25)$

$= n^2 + 2n + 1 + n^2 + 6n + 9 - n^2 - 10n - 25$

$= n^2 - 2n - 15$

$= (n+3)(n-5)$

(4)

10. Prove the product of two even numbers is always even

Even numbers: $2n$

$2m$

product: $2n \times 2m = 4mn$

$2(2mn)$

$\therefore$ always even.

(3)

11. Prove the product of three consecutive odd numbers is odd

Three consecutive odd numbers: $2n+1$

$2n+3$

$2n+5$

product: $(2n+1)(2n+3)(2n+5)$

$8n^3 + 36n^2 + 46n + 15$

$\underbrace{2(4n^3 + 18n^2 + 23n)}_{\text{Even}} + 15$

$+ \text{ odd} = \text{odd}$

(3)

12. Prove algebraically that the sum of the squares of two odd integers is always even.

$2n + 1$ and $2k + 1$

$(2n + 1)^2 + (2k + 1)^2$

$= 4n^2 + 4n + 1 + 4k^2 + 4k + 1$

$= 4n^2 + 4n + 4k^2 + 4k + 2$

$= 2(2n^2 + 2n + 2k^2 + 2k + 1)$

$\therefore$ always even.

(4)

13. Prove that when two consecutive integers are squared, that the difference is equal to the sum of the two consecutive integers.

two consecutive integers: n
$n+1$

Sum of: $n + n+1$
$= 2n+1$

Difference of squares:
$= (n+1)^2 - n^2$
$= (n+1)(n+1) - n^2$
$= n^2 + 2n + 1 - n^2$
$= 2n+1$

$\therefore$ equal to the sum of.

(4)

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14. Prove algebraically that

$(4n + 1)^2 - (2n - 1)$ is an even number

for all positive integer values of n.

$(4n+1)(4n+1) - (2n-1)$
$= 16n^2 + 8n + 1 - 2n + 1$
$= 16n^2 + 6n + 2$
$= 2(8n^2 + 3n + 1)$

$\therefore$ even outcome

(4)

15. Prove that $3n(3n + 4) + (n - 6)^2$ is positive for all values of ~~x~~ n

$9n^2 + 12n + (n^2 - 6n - 6n + 36)$

$10n^2 + 36$

as $n > 0$

$n^2 > 0$

$10n^2 > 0$

$10n^2 + 36 > 0$

$\therefore$ $10n^2 + 36$ is always positive.

(4)

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16. The first five terms of a linear sequence are 5, 11, 17, 23, 29 …

(a) Find the nth term of the sequence

$6n - 1$

(2)

A new sequence is generated by squaring each term of the linear sequence and then adding 5.

(b) Prove that all terms in the new sequence are divisible by 6.

$(6n - 1)^2 + 5$

$(36n^2 - 6n - 6n + 1) + 5$

$36n^2 - 12n + 6$

$6(6n^2 - 2n + 1)$ $\therefore$ divisible by 6.

(4)

17. Prove that the product of two consecutive even numbers is a multiple of 4.

Two consecutive even numbers: $2n$

$2n+2$

product: $2n(2n+2)$

$4n^2+4n$

$4n(n+1)$

$\therefore$ a multiple of 4

(3)

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18. Prove that when any odd integer is squared, the result is always one more than a multiple of 8.

odd integer: $2n+1$

odd integer squared: $(2n+1)(2n+1)$

$= 4n^2+4n+1$

$= 4n(n+1)+1$

Since either n or $n+1$ is even, $n(n+1)$ is even.

4 times even is always a multiple of 8.

$\therefore$ $4n(n+1)$ is a multiple of 8

(4)

$\therefore$ $4n(n+1)+1$ is one more than a multiple of 8

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19. Prove that the product of two odd numbers is always odd.

odd numbers: $2k+1$

$2m+1$

product: $(2k+1)(2m+1)$

$4km+2k+2m+1$

$\underbrace{2(2km+k+m)}_{\text{Even}} + 1$

Even + odd = odd.

(3)

20. Given $2^{89} - 1$ is prime.

Show that $2^{89} + 1$ is a multiple of 3

Let n, $n+1$, $n+2$ be 3 consecutive numbers.

one number must be a multiple of 3.

$2^{89} - 1$ is prime $\therefore$ not a multiple of 3.

2^{89} is not a multiple of 3 (considering prime factors)

$2^{89} + 1$ $\therefore$ must be a multiple of 3.

(3)