FM1 – Mechanics Chapter 2
Exam Questions: Work and Energy

Q1.

Unless otherwise indicated, whenever a numerical value of g is required, take $g = 9.8$ m s^{-2} and give your answer to either 2 significant figures or 3 significant figures.

A small stone of mass 0.5 kg is thrown vertically upwards from a point A with an initial speed of 25 m s^{-1}. The stone first comes to instantaneous rest at the point B which is 20 m vertically above the point A. As the stone moves it is subject to air resistance. The stone is modelled as a particle.

(a) Find the energy lost due to air resistance by the stone, as it moves from A to B.

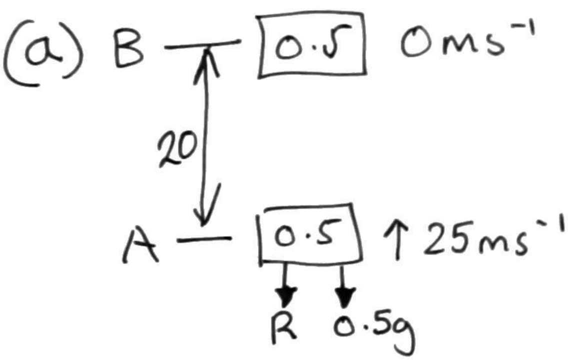
The air resistance is modelled as a constant force of magnitude R newtons.

(b) Find the value of R.

(c) State how the model for air resistance could be refined to make it more realistic.

(Total for question = 6 marks)

(a)



A PE = 0

$KE = \frac{1}{2}mu^2$

$= \frac{1}{2}(0.5)(25)^2 = 156.25$

B PE = 0

KE = mgh

$= 0.5(9.8)(20)$

$= 98$

$156.25 - 98 = \underline{58.25 \text{ J}}$

(b) Initial Energy = Final Energy

Work In + KE + PE = KE + PE + work out

0 + 156.25 + 0 = 0 + 98 + 20R

58.25 = 20R

R = 2.9125

(c) variable resistance

Q2.

A small ball of mass 0.3 kg is released from rest from a point 3.6 m above horizontal ground. The ball falls freely under gravity, hits the ground and rebounds vertically upwards.

In the first impact with the ground, the ball receives an impulse of magnitude 4.2 N s. The ball is modelled as a particle.

(a) Find the speed of the ball immediately after it first hits the ground.

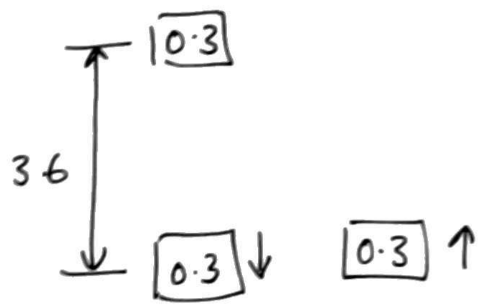
(5)

(b) Find the kinetic energy lost by the ball as a result of the impact with the ground.

(3)

(Total for question = 8 marks)

(a)



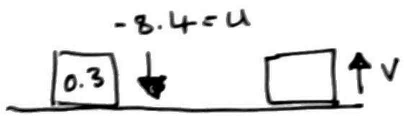
$s = -3.6$
$u = 0$
$v = ?$
$a = -9.8$
$t =$

$v^2 = u^2 + 2as$

$v^2 = 0^2 + 2(-9.8)(-3.6)$

$v^2 = 70.56$

$v = 8.4$



$I = mv - mu$

$4.2 = 0.3(v) - 0.3(-8.4)$

$v = \underline{\underline{5.6}}\ ms^{-1}$

(b)

$KE = \frac{1}{2}mu^2$

$= \frac{1}{2}(0.3)(-8.4)^2$

$= 10.584$

$KE = \frac{1}{2}mv^2$

$= \frac{1}{2}(0.3)(5.6)^2$

$= 4.704$

$$10.584 - 4.704 = \underline{\underline{5.88\ J}}$$

Q3.

A block of mass 10 kg is pulled along a straight horizontal road by a constant horizontal force of magnitude 70 N in the direction of the road. The block moves in a straight line passing through two points *A* and *B* on the road, where *AB* = 50 m. The block is modelled as a particle and the road is modelled as a rough plane. The coefficient of friction between the block and the road is $\frac{4}{7}$.

(a) Calculate the work done against friction in moving the block from *A* to *B*.

(4)

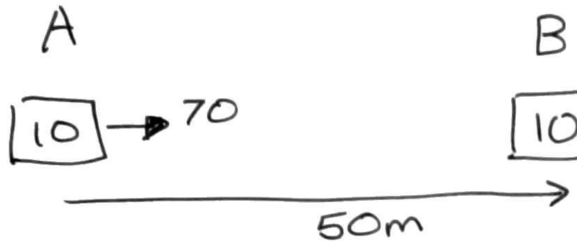
The block passes through *A* with a speed of 2 m s^{-1}.

(b) Find the speed of the block at *B*.

(4)

(Total 8 marks)

(a)



$\mu = \frac{4}{7}$

WD against Friction = Friction × Distance

= μR × Distance

= $\frac{4}{7}(10g) \times 50$

= $\underline{\underline{2800}}$ J

(b)

Initial Energy = Final Energy

Work In + PE + KE = Work Out + PE + KE

70 x50

$3500 + 0 + \frac{1}{2}(10)(2)^2 = 2800 + 0 + \frac{1}{2}(10)(v)^2$

$3520 = 2800 + 5v^2$

$v = \underline{\underline{12}}$

Q4.

The points *A* and *B* are 10 m apart on a line of greatest slope of a fixed rough inclined plane, with *A* above *B*. The plane is inclined at 25° to the horizontal. A particle *P* of mass 5 kg is released from rest at *A* and slides down the slope. As *P* passes *B*, it is moving with speed 7 m s^{-1}.

(a) Find, using the work-energy principle, the work done against friction as *P* moves from *A* to *B*.

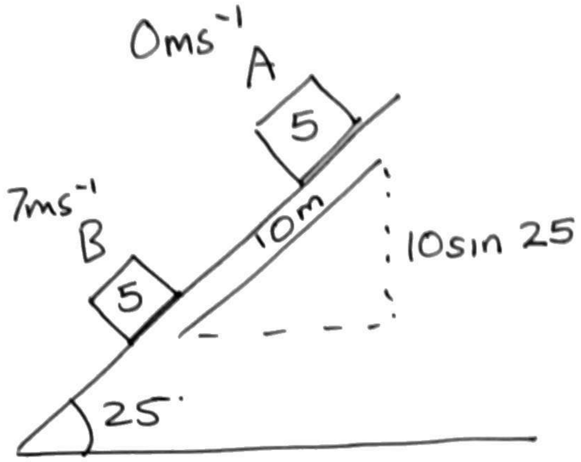
(4)

(b) Find the coefficient of friction between the particle and the plane.

(5)

(Total 9 marks)

(a)



Initial Energy = Final Energy

Work ~~In~~ + KE + PE = Work Out + KE + PE

$0 + 0 + mgh = W + \frac{1}{2}mv^2 + 0$

$5(9.8)(10\sin 25) = W + \frac{1}{2}(5)(7)^2$

$W = \underline{\underline{84.6\text{ J}}}$

(b) Friction x Distance = 84.5829....

$\mu R \times 10 = 84.5829....$

$\mu(5g\cos 25) \times 10 = 84.5829....$

$\therefore \mu = \underline{\underline{0.190}}$

Q5.

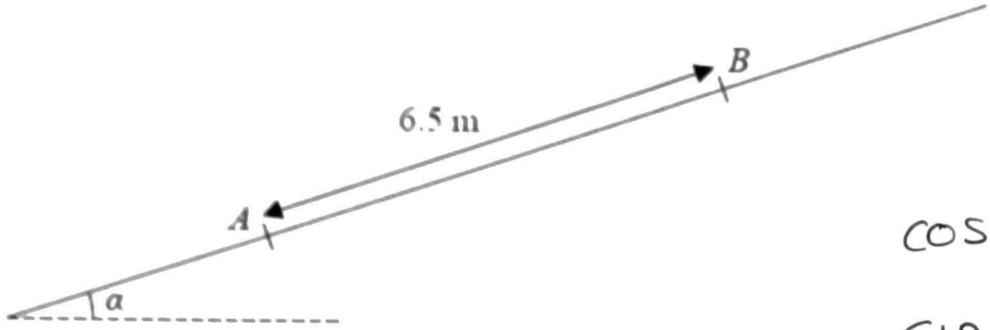


Figure 2

$\cos\alpha = \frac{12}{13}$

$\sin\alpha = \frac{5}{13}$

A particle P of mass 10 kg is projected from a point A up a line of greatest slope AB of a fixed rough plane. The plane is inclined at angle α to the horizontal, where $\tan\alpha = \frac{5}{12}$ and AB = 6.5 m, as shown in Figure 2. The coefficient of friction between P and the plane is μ. The work done against friction as P moves from A to B is 245 J.

(a) Find the value of μ.

(5)

The particle is projected from A with speed 11.5 m s^{-1}. By using the work–energy principle,

(b) find the speed of the particle as it passes through B.

(4)

(Total for question = 9 marks)

(a) WDFriction = 245 J

$\mu R \times$ Distance $= 245$

$\mu(10g\cos\alpha) \times 6.5 = 245$

$\mu\left(10g\,\frac{12}{13}\right) \times 6.5 = 245$

$\mu = 0.417$

(b) work In + KE + PE = Work out + KE + PE

$0 + \frac{1}{2}mu^2 + 0 = 245 + \frac{1}{2}mv^2 + mgh$

$\frac{1}{2}(10)(11.5)^2 = 245 + \frac{1}{2}(10)v^2 + 10(9.8)(6.5\sin\alpha)$

$661.25 = 245 + 5v^2 + 245$

$V = \underline{\underline{5.85}}$ ms^{-1}

Q6.

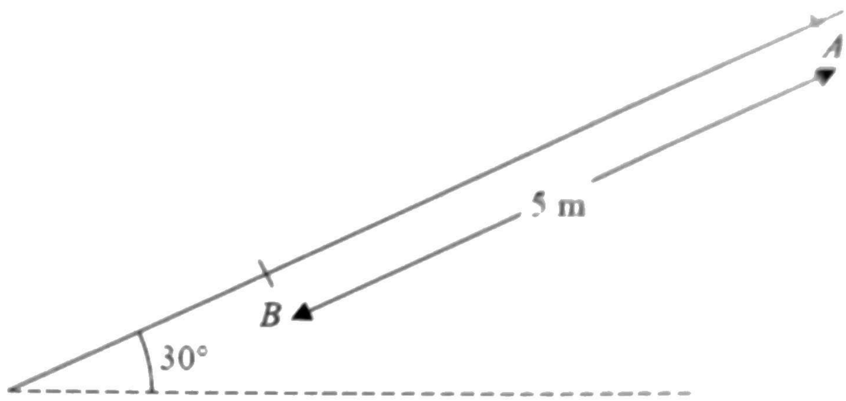


Figure 3

A particle P of mass 2 kg is released from rest at a point A on a rough inclined plane and slides down a line of greatest slope. The plane is inclined at 30° to the horizontal. The point B is 5 m from A on the line of greatest slope through A, as shown in Figure 3.

(a) Find the potential energy lost by P as it moves from A to B.

The speed of P as it reaches B is 4 m s^{-1}.

(b) (i) Use the work-energy principle to find the magnitude of the constant frictional force acting on P as it moves from A to B.

(ii) Find the coefficient of friction between P and the plane.

The particle P is now placed at A and projected down the plane towards B with speed 3 m s^{-1}. Given that the frictional force remains constant,

(c) find the speed of P as it reaches B.

(Total 13 marks)

$PE = mgh$

(a) $= 2(9.8)(5\sin 30) = \underline{\underline{49\text{J}}}$

(b) Initial Energy = Final Energy

Work In + KE + PE = Work Out + KE + PE

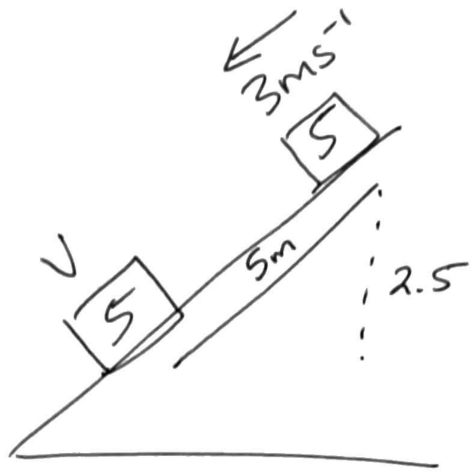
$0 + 0 + 49 = \mu R \times \text{Distance} + \frac{1}{2}(2)(4)^2 + 0$

$49 = \mu(2g\cos 30) \times 5 + 16$

$\mu = \dfrac{33}{5(2g\cos 30)}$

$\mu = 0.389$

(c)



WD against friction $= \frac{33}{10g\cos 30} \times 5 \times 2g\cos 30 = 33$

Initial Energy = Final Energy

Work In + KE + PE = Work Out + KE + PE

$0 + \frac{1}{2}(2)(3)^2 + 49 = 33 + \frac{1}{2}(2)v^2 + 0$

$9 + 49 = 33 + v^2$

$v^2 = 25$

$v = \underline{\underline{5 \text{ ms}^{-1}}}$

Q7.

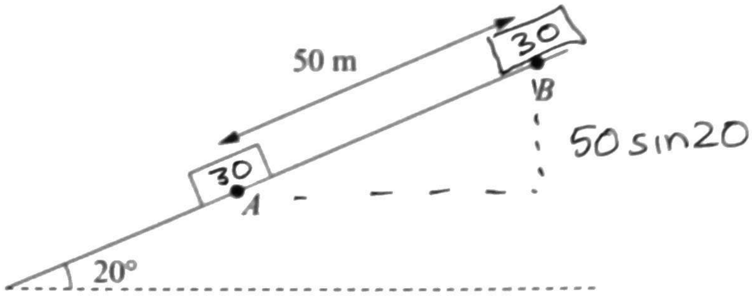


Figure 1

A box of mass 30 kg is held at rest at point A on a rough inclined plane. The plane is inclined at 20° to the horizontal. Point B is 50 m from A up a line of greatest slope of the plane, as shown in Figure 1. The box is dragged from A to B by a force acting parallel to AB and then held at rest at B. The coefficient of friction between the box and the plane is $\frac{1}{4}$. Friction is the only non-gravitational resistive force acting on the box. Modelling the box as a particle,

(a) find the work done in dragging the box from A to B.

(6)

The box is released from rest at the point B and slides down the slope. Using the work-energy principle, or otherwise,

(b) find the speed of the box as it reaches A.

(5)

(Total 11 marks)

(a) Total WD $= \underset{\text{Gain}}{\text{Total KE}} + \underset{\text{Gain}}{\text{Total PE}} + \underset{\mu R \times \text{Distance}}{\text{WD friction}}$

$= 0 + 30(9.8)(50\sin 20) + \frac{1}{4}(30g\cos 20)50$

$= \underline{\underline{8480 \text{ J}}}$

(b) Initial Energy = Final Energy

Work In + KE + PE = Work out + KE + PE

$0 + 0 + 30(9.8)(50\sin 20) = \frac{50}{4}(30g\cos 20) + \frac{1}{2}(30)v^2 + 0$

$14700\sin 20 = 3675\cos 20 + 15v^2$

$v = \underline{\underline{10.2 \text{ ms}^{-1}}}$