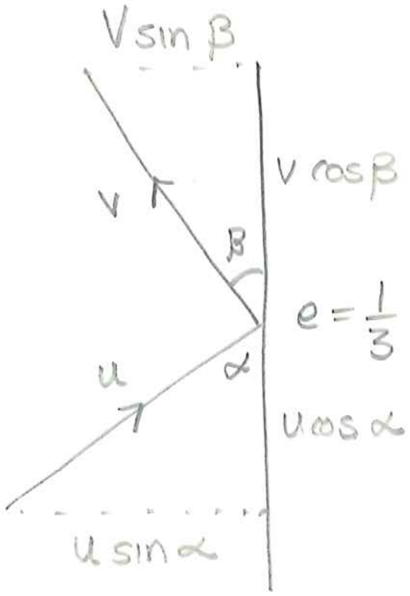
5A

1) a)



$\tan\alpha = \frac{3}{4}$

$\sin\alpha = \frac{3}{5}$

$\cos\alpha = \frac{4}{5}$

(11) $V\cos\beta = u\cos\alpha$

$V\cos\beta = \frac{4u}{5}$

$(=)$

$\frac{1}{3}u\sin\alpha = V\sin\beta$

$\frac{u}{3}\,\frac{3}{5} = V\sin\beta$

$\frac{3u}{15} = V\sin\beta$

① $V\cos\beta = \frac{4u}{5}$

② $V\sin\beta = \frac{3u}{15}$

①^2 $V^2\cos^2\beta = \frac{16u^2}{25}$

②^2 $V^2\sin^2\beta = \frac{9u^2}{225}$

①^2 + ②^2

$V^2\cos^2\beta + V^2\sin^2\beta = \frac{16u^2}{25} + \frac{u^2}{25}$

$V^2(\cos^2\beta + \sin^2\beta) = \frac{17u^2}{25}$

$\boxed{\cos x^2 + \sin^2 x = 1}$

$V^2 = \frac{17u^2}{25}$

$V = \frac{\sqrt{17}\,u}{5}$

(5A)

1) b)

(1) $V\cos\beta = \frac{4u}{5}$

(2) $V\sin\beta = \frac{3u}{15}$

(2) ÷ (1)

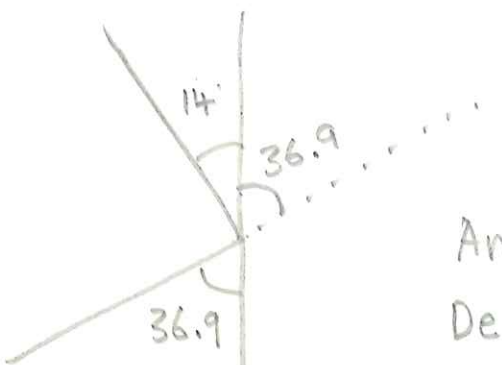
$$\frac{\sin\beta}{\cos\beta} = \frac{1}{4}$$

$$\tan\beta = \frac{1}{4}$$

$$\beta = 14.0^\circ \text{ (3sf)}$$

$$\tan\alpha = \frac{3}{4}$$

$$\alpha = 36.9^\circ \text{ (3sf)}$$

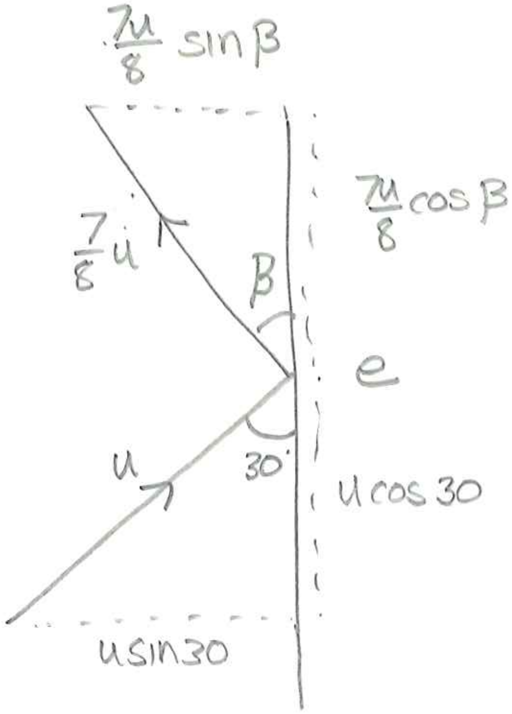


Angle of Deflection $= 14^\circ + 36.9^\circ$

$= \underline{\underline{50.9^\circ}}$

(5A)

2)



(11)

$$\frac{7u}{8}\cos\beta = u\cos 30$$

$$\frac{7u}{8}\cos\beta = \frac{u\sqrt{3}}{2}$$

$$\frac{7u}{4}\cos\beta = u\sqrt{3}$$

(=)

$$\frac{7u}{8}\sin\beta = eu\sin 30$$

$$\frac{7u}{8}\sin\beta = \frac{eu}{2}$$

$$\frac{7u}{4}\sin\beta = eu$$

① $\frac{7u}{4}\cos\beta = u\sqrt{3}$

② $\frac{7u}{4}\sin\beta = eu$

①^2 $\frac{49u^2}{16}\cos^2\beta = 3u^2$

②^2 $\frac{49u^2}{16}\sin^2\beta = e^2u^2$

①^2 + ②^2

$$\frac{49u^2}{16}\cos^2\beta + \frac{49u^2}{16}\sin^2\beta = 3u^2 + e^2u^2$$

$$\frac{49u^2}{16}(\cos^2\beta + \sin^2\beta) = 3u^2 + e^2u^2$$

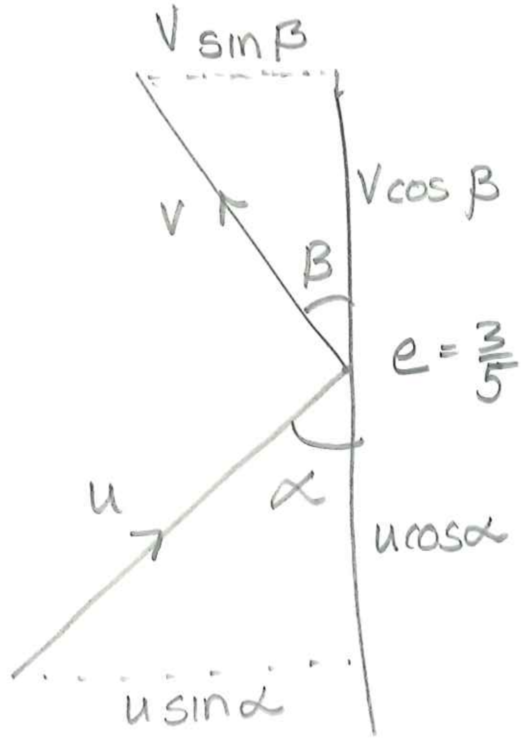
$$\frac{49u^2}{16} = 3u^2 + e^2u^2$$

$$\frac{49}{16} = 3 + e^2$$

$$e = \frac{1}{4}$$

5A

3)



$\tan\alpha = \frac{5}{12}$

$\sin\alpha = \frac{5}{13}$

$\cos\alpha = \frac{12}{13}$

(‖)

$$u\cos\alpha = V\cos\beta$$

$$\frac{12u}{13} = V\cos\beta$$

(=)

$$\frac{3}{5} u\sin\alpha = V\sin\beta$$

$$\frac{3u}{5}\,\frac{5}{13} = V\sin\beta$$

$$\frac{3u}{13} = V\sin\beta$$

---

① $\dfrac{12u}{13} = V\cos\beta$

② $\dfrac{3u}{13} = V\sin\beta$

①^2 $\dfrac{144u^2}{169} = V^2\cos^2\beta$

②^2 $\dfrac{9u^2}{169} = V^2\sin^2\beta$

①^2 + ②^2

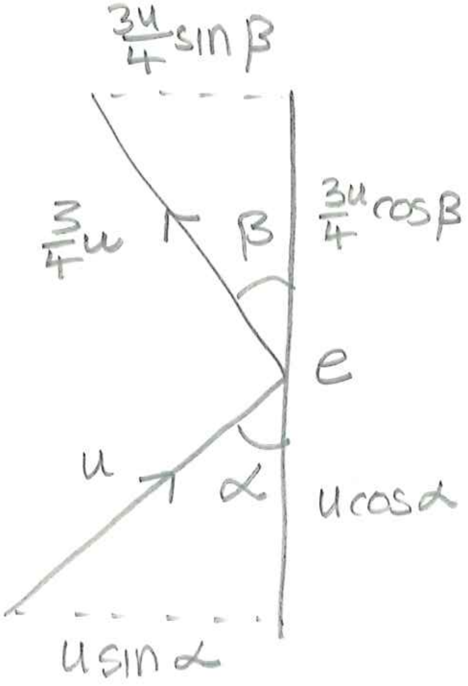
$$\frac{144u^2}{169} + \frac{9u^2}{169} = V^2(\cos^2\beta + \sin^2\beta)$$

$$\frac{153u^2}{169} = V^2$$

$$V = \frac{3\sqrt{17}\,u}{13}$$

5A

4)



$\tan\alpha = \frac{2}{1}$

$\sin\alpha = \frac{2}{\sqrt{5}}$

$\cos\alpha = \frac{1}{\sqrt{5}}$

(ll)

$$\frac{3u}{4}\cos\beta = u\cos\alpha$$

$$\frac{3u}{4}\cos\beta = u\frac{1}{\sqrt{5}}$$

(=)

$$eu\sin\alpha = \frac{3u}{4}\sin\beta$$

$$\frac{2ue}{\sqrt{5}} = \frac{3u}{4}\sin\beta$$

$①^2$ $$\frac{9u^2}{16}\cos^2\beta = \frac{u^2}{5}$$

$②^2$ $$\frac{4u^2e^2}{5} = \frac{9u^2}{16}\sin^2\beta$$

$①^2 + ②^2$

$$\frac{9u^2}{16}(\cos^2\beta + \sin^2\beta) = \frac{u^2}{5} + \frac{4u^2e^2}{5}$$

$$\frac{9u^2}{16} = \frac{u^2}{5} + \frac{4u^2e^2}{5}$$

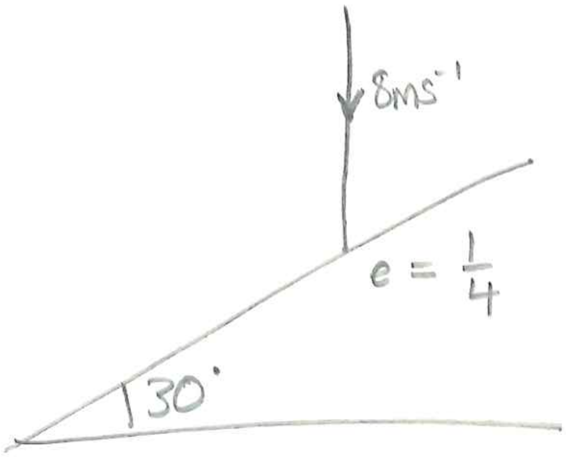
$$\frac{9}{16} = \frac{1}{5} + \frac{4e^2}{5}$$

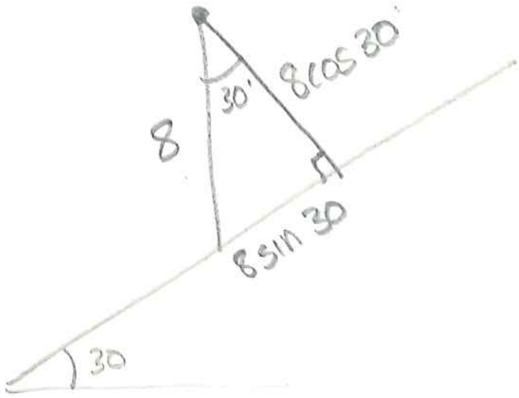
$$\frac{29}{64} = e^2$$

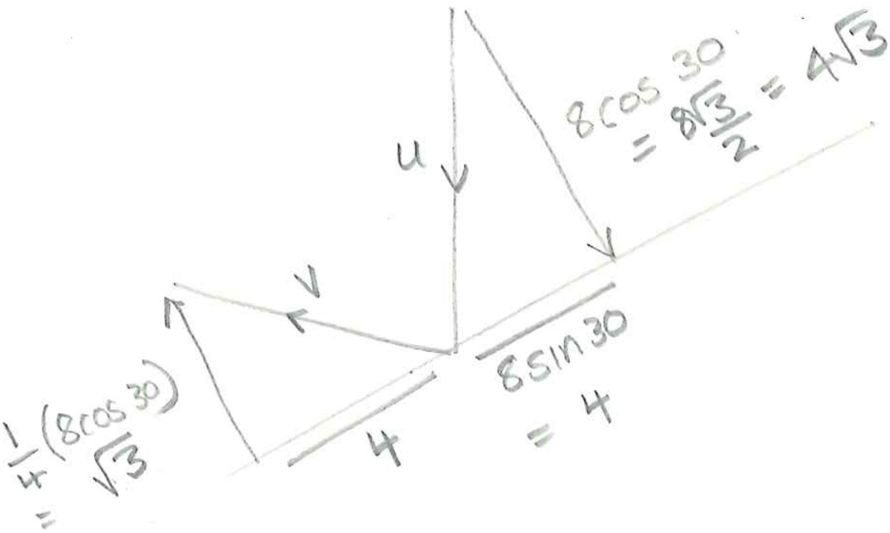
$$e = \frac{\sqrt{29}}{8}$$

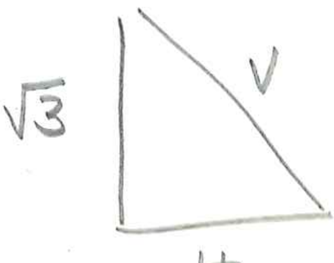
5A

5)









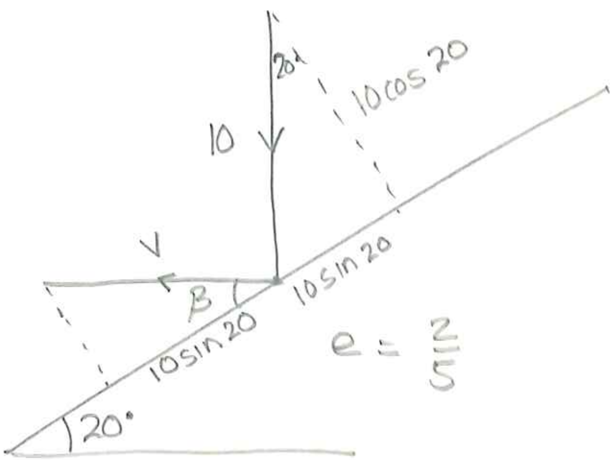
$$\sqrt{4^2 + (\sqrt{3})^2} = v$$

$$\sqrt{16 + 3} = v$$

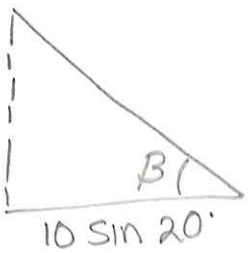
$$V = \sqrt{19}$$

(5A)

6)



$e = \frac{2}{5}$



$\frac{2}{5}(10\cos 20)$

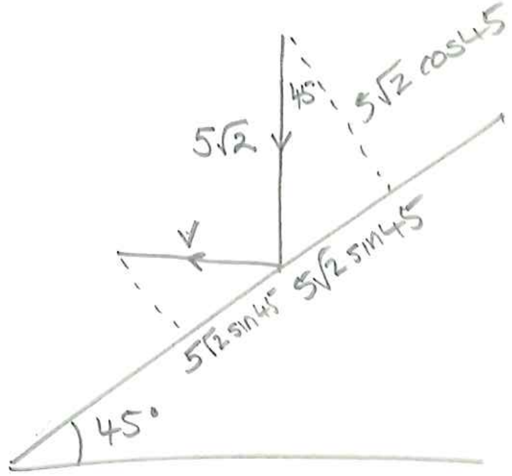
$= 4\cos 20$

$$v = \sqrt{(4\cos 20)^2 + (10\sin 20)^2} = \underline{\underline{5.08\ ms^{-1}}}\ (3sf)$$

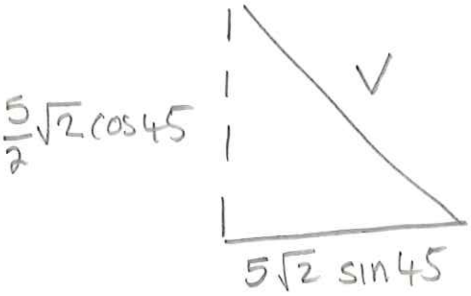
5A

7)

a)



$e = \frac{1}{2}$



$$V = \sqrt{\left(\frac{5\sqrt{2}}{2}\cos 45\right)^2 + \left(5\sqrt{2}\sin 45\right)^2}$$

$$V = \frac{5\sqrt{5}}{2} = \underline{\underline{5.59\,ms^{-1}}}$$

b) momentum before $(0.75)(\overset{\text{down}}{-}5\sqrt{2}\cos 45) = -\frac{15}{4}$

momentum after $(0.75)\left(\frac{5\sqrt{2}}{2}\cos 45\right) = \frac{15}{8}$

$$\text{Impulse} = \frac{15}{8} - -\frac{15}{4} = \frac{45}{8} = \underline{\underline{5.625\,ms^{-1}}}$$

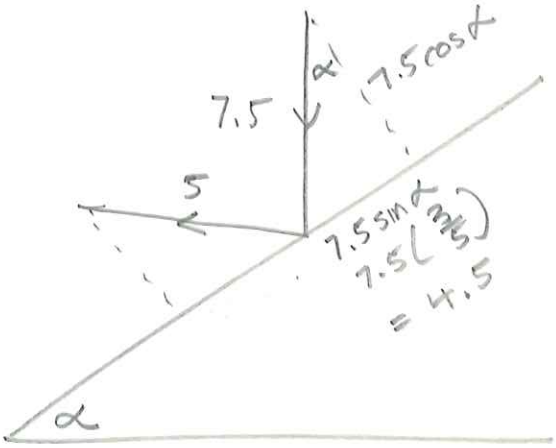
5A

8)

$\tan\alpha = \frac{3}{4}$

$\cos\alpha = \frac{4}{5}$

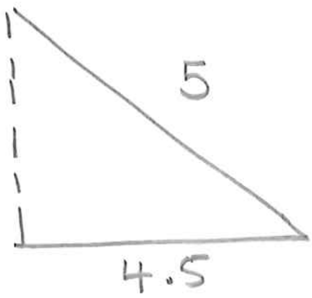
$\sin\alpha = \frac{3}{5}$



$7.5e\cos\alpha$

$= 7.5e\,\frac{4}{5}$

$= 6e$



$$(6e)^2 + (4.5)^2 = 5^2$$

$$36e^2 + 20.25 = 25$$

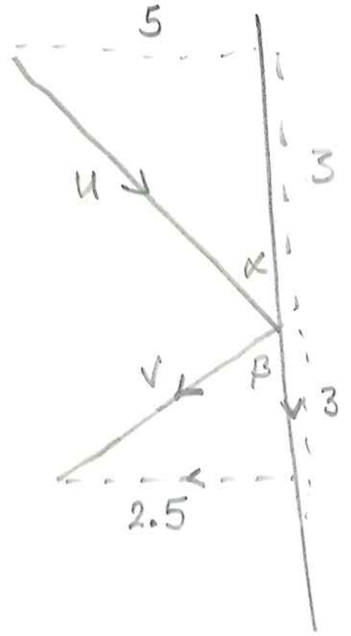
$$36e^2 = 4.75$$

$$e^2 = \frac{19}{144}$$

$$e = 0.363 \text{ (3sf)}$$

5A

9)



$e = \frac{1}{2}$

a) $v = -2.5i - 3j$

b) $KE_{\text{before}} = \frac{1}{2}mu^2 = \frac{1}{2}(0.8)(\sqrt{34})^2 = 13.6$

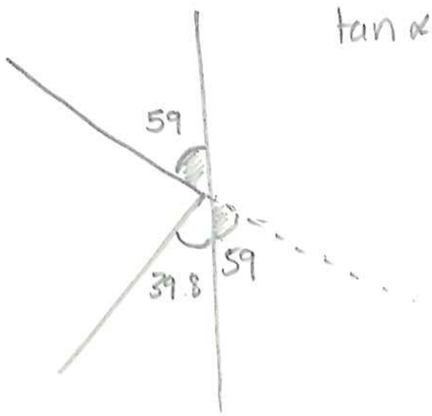
$\sqrt{5^2+3^2}$

$KE_{\text{after}} = \frac{1}{2}mv^2 = \frac{1}{2}(0.8)\left(\frac{\sqrt{61}}{2}\right)^2 = \frac{61}{10}$

$\sqrt{2.5^2+3^2}$

KE LOSS $13.6 - 6.1 = \underline{\underline{7.5\,J}}$

c)



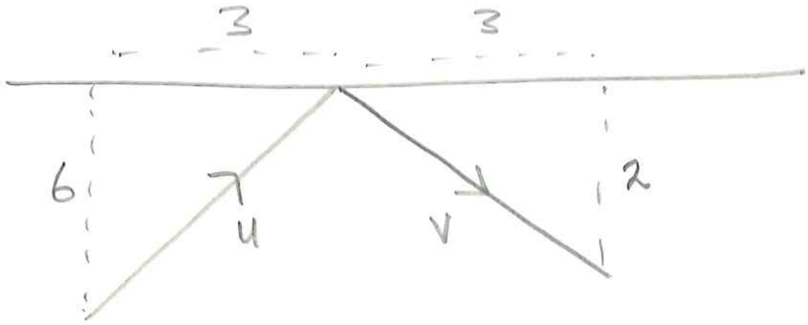
$\tan\alpha = \frac{5}{3} = 59$ $\tan\beta = \frac{2.5}{3} = 39.8$

Angle of deflection $= \underline{\underline{98.8°}}$

5A

$e = \frac{1}{3}$

10)



a) $v^2 = 3^2 + 2^2$

$v = \sqrt{13}\ \text{ms}^{-1}$

b) KE before $= \frac{1}{2}(1)(\sqrt{45})^2 = \frac{45}{2}$

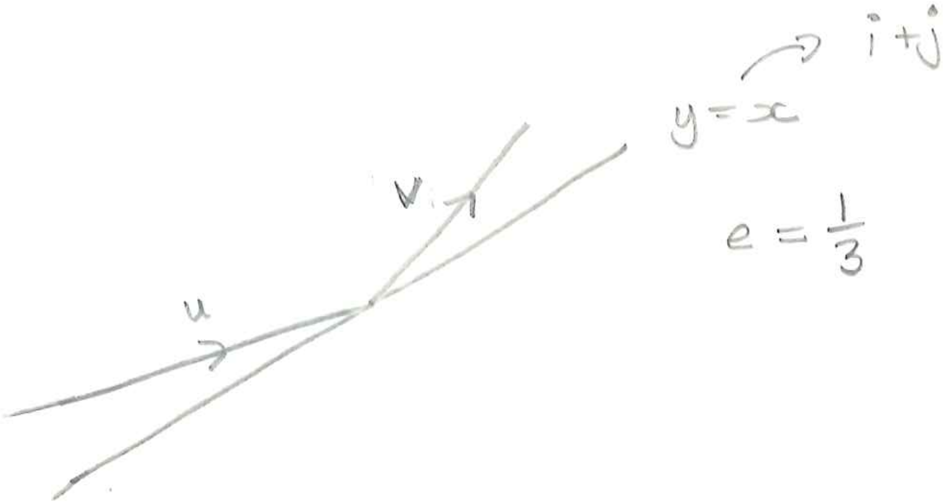
KE after $= \frac{1}{2}(1)(\sqrt{13})^2 = \frac{13}{2}$

KE LOSS $= \frac{45}{2} - \frac{13}{2} = \underline{\underline{16\text{ J}}}$

5A

11)

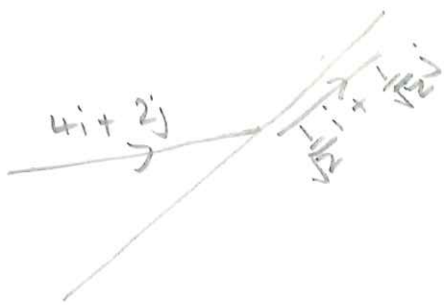
a)



Slope $= i + j$

$|s| = \sqrt{1^2 + 1^2} = \sqrt{2}$

Unit vector Slope $= \hat{s} = \frac{1}{\sqrt{2}} i + \frac{1}{\sqrt{2}} j$



$k = u \cdot \hat{s}$

$= \begin{pmatrix} 4 \\ 2 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix}$

$= \frac{4}{\sqrt{2}} + \frac{2}{\sqrt{2}} = \frac{6}{\sqrt{2}}$

parallel component $= k \cdot \hat{s}$

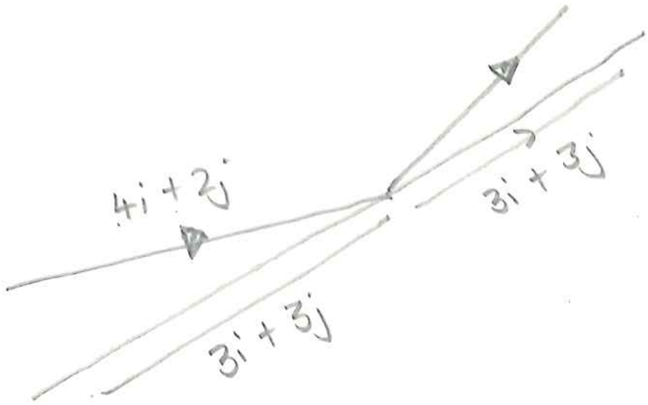
$= \frac{6}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \end{pmatrix}$

$= 3i + 3j$

(5A)

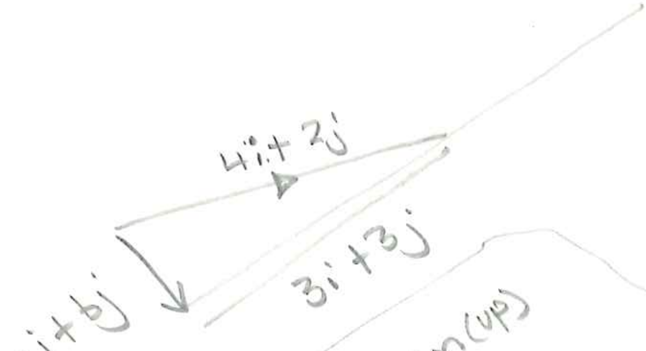
11)

a) conti



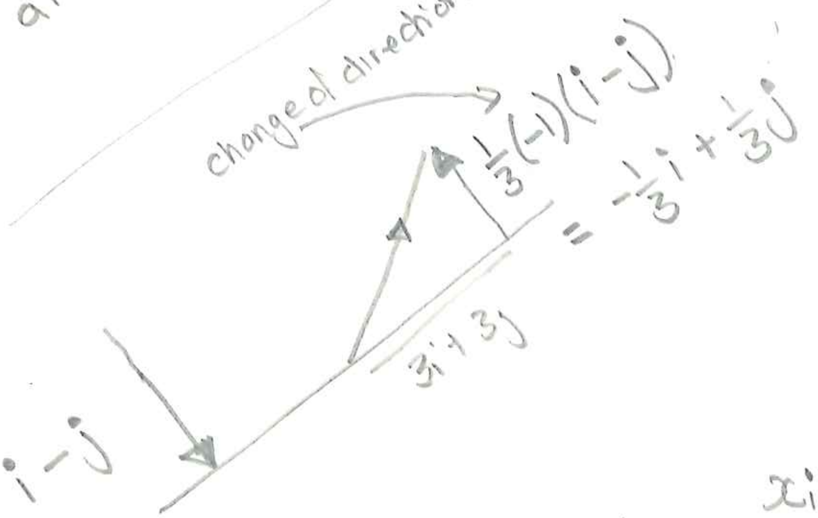
$e = \frac{1}{3}$

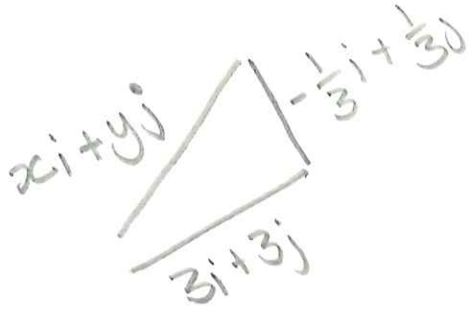
Perpendicular Component



$4i + 2j = ai + bj + 3i + 3j$

$ai + bj = 1i - 1j$





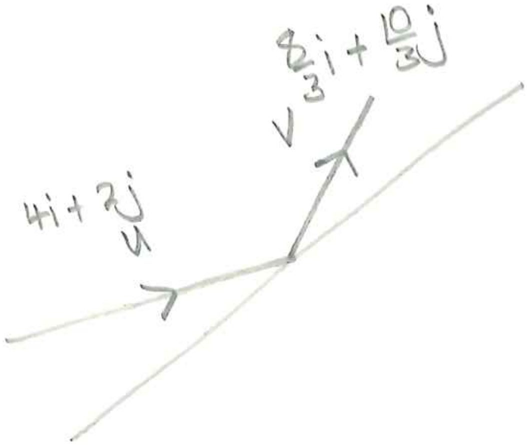
$xi + yj = 3i + 3j - \frac{1}{3}i + \frac{1}{3}j$

$xi + yj = \underline{\underline{\frac{8}{3}i + \frac{10}{3}j}}$

5A

11

b)



$|u| = \sqrt{4^2 + 2^2} = \sqrt{20}$

$|v| = \sqrt{\left(\frac{8}{3}\right)^2 + \left(\frac{10}{3}\right)^2} = \frac{2\sqrt{41}}{3}$

KE Before $= \frac{1}{2}mu^2 = \frac{1}{2}(2)(\sqrt{20})^2 = 20$

KE After $= \frac{1}{2}mu^2 = \frac{1}{2}(2)\left(\frac{2\sqrt{41}}{3}\right)^2 = \frac{164}{9}$

$$\frac{20 - \frac{164}{9}}{20} \times 100 = 8.89\% \ (3sf)$$

5A

11 c) Angle of deflection

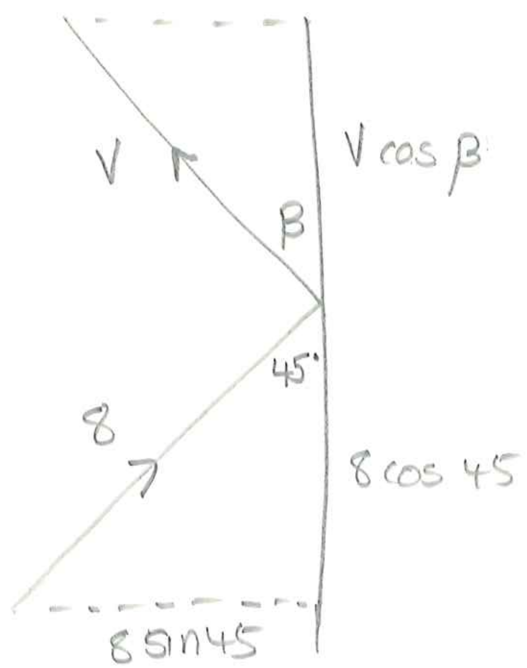
$$\cos\theta = \frac{u \cdot v}{|u||v|}$$

$$\cos\theta = \frac{\begin{pmatrix}4\\2\end{pmatrix}\begin{pmatrix}8/3\\10/3\end{pmatrix}}{\left(\sqrt{20}\right)\left(\frac{2\sqrt{41}}{3}\right)}$$

$$\theta = \underline{\underline{24.8^\circ}} \text{ (3sf)}$$

5A

12)



$e = \frac{2}{5}$

① (‖) $V \cos \beta = 8 \cos 45$

$V \cos \beta = 4\sqrt{2}$

② (⊥)

$V \sin \beta = \left(\frac{2}{5}\right)(8 \sin 45)$

$V \sin \beta = \frac{8\sqrt{2}}{5}$

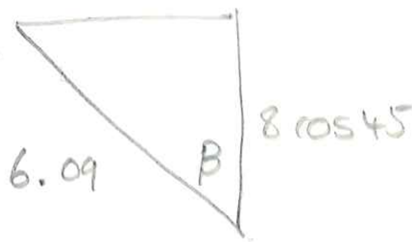
$①^2 \quad V^2 \cos^2 \beta = 32$

$②^2 \quad V^2 \sin^2 \beta = \frac{128}{25}$

$①^2 + ②^2$

$V^2 (\cos^2 \beta + \sin^2 \beta) = \frac{928}{25}$

$V = 6.09 \text{ ms}^{-1}$ (3sf)



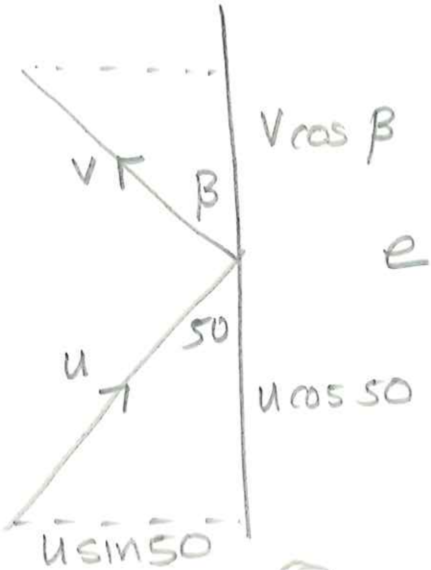
$\cos \beta = \frac{8 \cos 45}{6.09}$

$\beta = 21.7°$

5A

13)

a)



(1)

(ii)

$V \cos \beta = u \cos 50$

(2)

(=)

$eu \sin 50 = V \sin \beta$

(2) ÷ (1)

$$\frac{V \sin \beta}{V \cos \beta} = \frac{eu \sin 50}{u \cos 50}$$

$$\tan \beta = e \tan 50$$

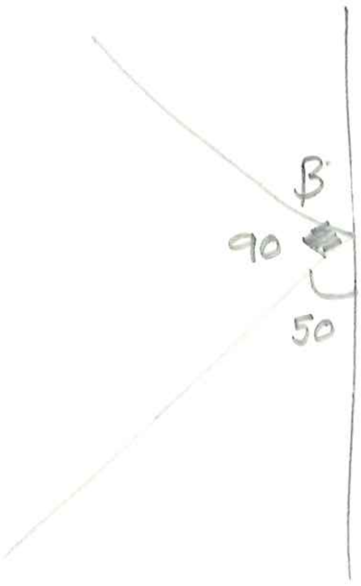
hence

β is not dependent on u

5A

13

b)



$\beta = 40°$

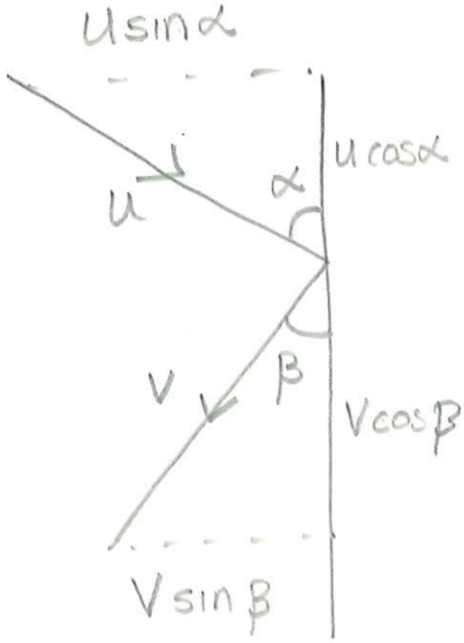
$$\tan \beta = e \tan 50$$

$$e = \frac{\tan 40}{\tan 50}$$

$$e = 0.704 \text{ (3sf)}$$

5A

14)



$\tan \alpha = \frac{3}{4}$

$\sin \alpha = \frac{3}{5}$

$\cos \alpha = \frac{4}{5}$

$\tan \beta = \frac{5}{12}$

$\sin \beta = \frac{5}{13}$

$\cos \beta = \frac{12}{13}$

a) (ii) $V \cos \beta = u \cos \alpha$

$$\frac{12V}{13} = \frac{4u}{5}$$

$$V = \frac{13u}{15}$$

$$\text{KE After} = \frac{1}{2}(m)\left(\frac{169u^2}{225}\right)$$

$$\text{KE Before} = \frac{1}{2}(m)(u)^2$$

$$\text{KE Loss} = \frac{1}{2}(m)\left(u^2 - \frac{169u^2}{225}\right)$$

$$= \frac{1}{2}m\left(\frac{56u^2}{225}\right) = \frac{28mu^2}{225}$$

5A

14)

a) conti

$$\frac{KE \text{ Loss}}{KE \text{ Before}}$$

$$= \frac{28mu^2}{225} \div \frac{mu^2}{2}$$

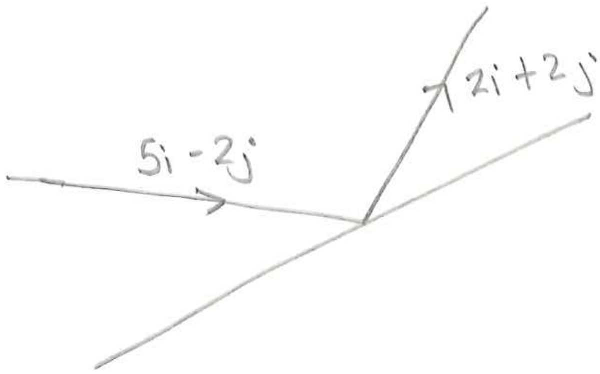
$$= \frac{28mu^2}{225} \times \frac{2}{mu^2}$$

$$= \frac{56}{225}$$

(5A)

15)

a)



mass = m

$$\text{Impulse} = m(2i+2j) - m(5i-2j)$$

$$= 2mi + 2mj - 5mi + 2mj$$

$$= -3mi + 4mj$$

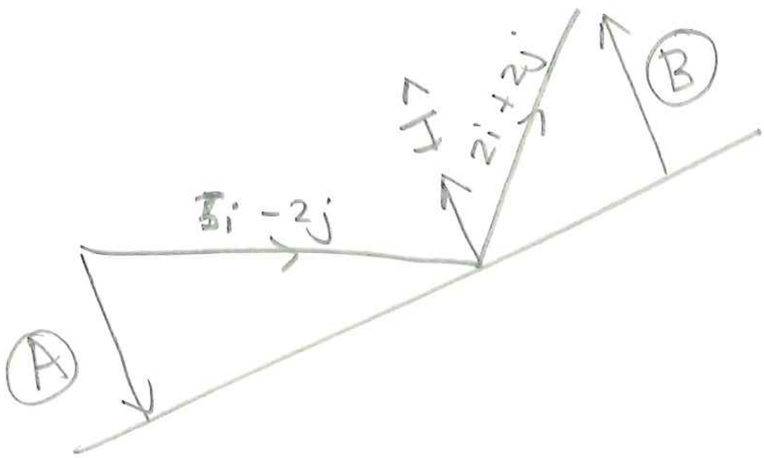
$$|I| = \sqrt{3^2 + 4^2} = \sqrt{25} = 5$$

$$\hat{I} = \frac{-3}{5}mi + \frac{4}{5}mj$$

5A

15

b)



$$\hat{I} = -\frac{3m}{5}i + \frac{4m}{5}j$$

A Scalar $= u \cdot \hat{I}$

$$= \begin{pmatrix} 5 \\ -2 \end{pmatrix} \begin{pmatrix} -3m/5 \\ 4m/5 \end{pmatrix} = -\frac{15m}{5} - \frac{8m}{5}$$

$$= \frac{-23m}{5}$$

B Scalar $= v \cdot \hat{I}$

$$= \begin{pmatrix} 2 \\ 2 \end{pmatrix} \begin{pmatrix} -3m/5 \\ 2m/5 \end{pmatrix} = -\frac{6m}{5} + \frac{8m}{5}$$

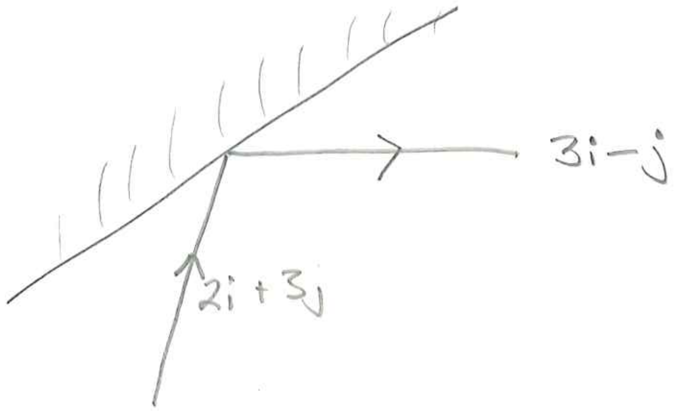
$$= \frac{2m}{5}$$

$$e = \frac{2m/5}{23m/5} = \frac{2}{23}$$

5A

16)

a)

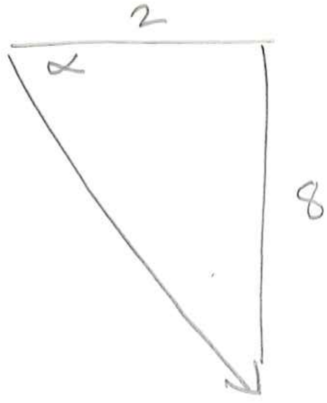


mass = 2kg

$$\text{Impulse} = mv - mu$$
$$= 2(3i-j) - 2(2i+3j)$$
$$= 6i - 2j - 4i - 6j$$
$$= 2i - 8j$$

$$\sqrt{2^2+8^2} = 2\sqrt{17}$$

Direction



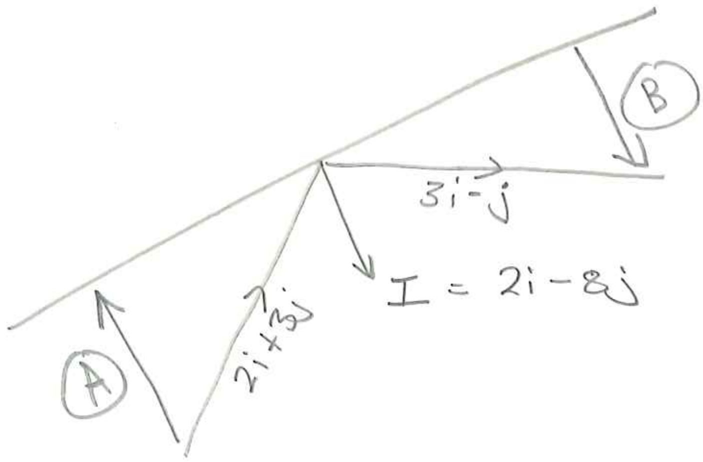
$\tan\alpha = \frac{8}{2} = 76.0°$

to the slope.

5A

16)

b)



$|I| = \sqrt{2^2 + 8^2}$

$= 2\sqrt{17}$

$$\hat{I} = \frac{2}{2\sqrt{17}} i - \frac{8}{2\sqrt{17}} j$$

$$= \frac{1}{\sqrt{17}} i - \frac{4}{\sqrt{17}} j$$

A Scalar $\begin{pmatrix} 2 \\ 3 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{17}} \\ \frac{-4}{\sqrt{17}} \end{pmatrix} = \frac{2}{\sqrt{17}} - \frac{12}{\sqrt{17}}$

$$= -\frac{10}{\sqrt{17}}$$

Perpendicular Component $= k\,\hat{I}$

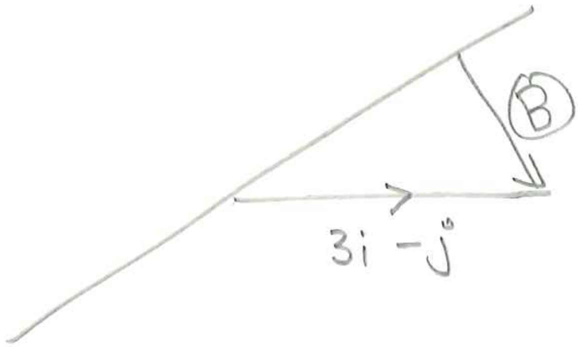
$$= \frac{-10}{\sqrt{17}} \begin{pmatrix} \frac{1}{\sqrt{17}} \\ \frac{-4}{\sqrt{17}} \end{pmatrix}$$

$$= \frac{-10}{17} i + \frac{40}{17} j$$

5A

16)

b) conti



B

Scalar $\begin{pmatrix} 3 \\ -1 \end{pmatrix} \cdot \begin{pmatrix} \frac{1}{\sqrt{17}} \\ \frac{-4}{\sqrt{17}} \end{pmatrix} = \frac{3}{\sqrt{17}} + \frac{4}{\sqrt{17}}$

Perpendicular Component $= p \cdot \hat{I}$

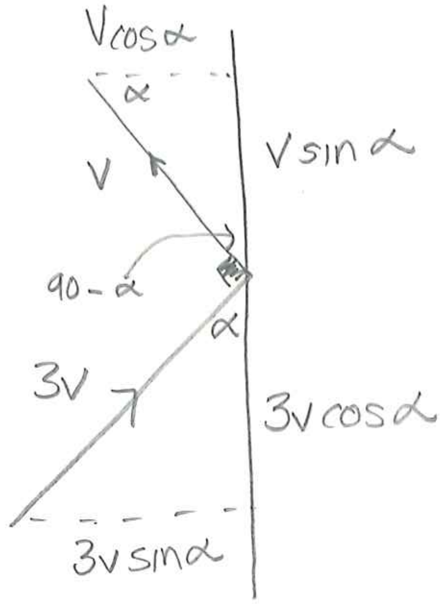
$$= \left(\frac{7}{\sqrt{17}}\right) \begin{pmatrix} \frac{1}{\sqrt{17}} \\ \frac{-4}{\sqrt{17}} \end{pmatrix} = \frac{7}{17}i - \frac{28}{17}j$$

$\frac{10}{17}(-i + 4j)$ $\quad$ $\frac{7}{17}(i - 4j)$

$$e = \frac{7/17}{10/17} = 7/10$$

(5A)

17)



(ii)

$$3V\cos\alpha = V\sin\alpha$$

$$3 = \frac{\sin\alpha}{\cos\alpha}$$

$$\tan\alpha = 3$$

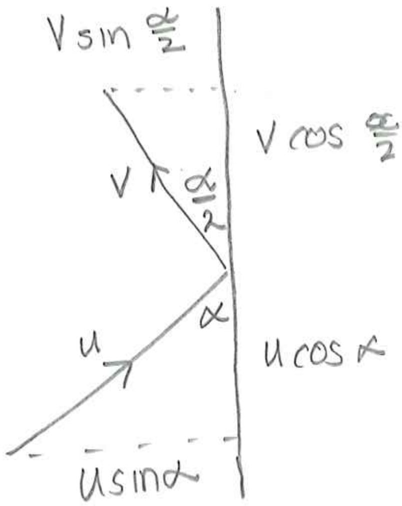
b)

$$e = \frac{V\cos\alpha}{3V\sin\alpha}$$

$$e = \frac{1}{3\tan\alpha} = \frac{1}{3(3)} = \frac{1}{9}$$

(5A)

18)



$e = 0.4$

(1)

(II) $V\cos\frac{\alpha}{2} = u\cos\alpha$

(2)

$(=)$

$(0.4)\,u\sin\alpha = V\sin\frac{\alpha}{2}$

(2) ÷ (1)

$$\frac{V\sin\frac{\alpha}{2}}{V\cos\frac{\alpha}{2}} = \frac{0.4\,u\sin\alpha}{u\cos\alpha}$$

$$\tan\frac{\alpha}{2} = 0.4\tan\alpha$$

Double Angle

$$\tan(2A) = \frac{2\tan A}{1-\tan^2 A}$$

let $A = \frac{\alpha}{2}$

$$\tan\left(2\frac{\alpha}{2}\right) = \frac{2\tan\frac{\alpha}{2}}{1-\tan^2\left(\frac{\alpha}{2}\right)}$$

let $\tan\frac{\alpha}{2} = t$

$$\tan\alpha = \frac{2t}{1-t^2}$$

sub

5A

18) conti $t(1-t^2) = 0.8t$

$1 - t^2 = 0.8$

$0.2 = t^2$

$t = \sqrt{0.2}$

$\tan\frac{\alpha}{2} = \sqrt{0.2}$

$\frac{\alpha}{2} = 24.09°$

$\alpha = 48.2°$ (3sf)