Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

AS and A level Further Mathematics
Core Pure Mathematics

Practice Paper
Series

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 80.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1. Show, using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, that

$$\sum_{r=1}^{n} 3(2r-1)^2 = n(2n+1)(2n-1), \text{ for all positive integers } n.$$

(Total 5 marks)

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2. (*a*) Using the formula for $\sum_{r=1}^{n} r^2$ write down, in terms of n only, an expression for

$$\sum_{r=1}^{3n} r^2$$

(1)

(*b*) Show that, for all integers n, where $n > 0$,

$$\sum_{r=2n+1}^{3n} r^2 = \frac{n}{6}(an^2 + bn + c)$$

where the values of the constants a, b and c are to be found.

(4)

(Total 5 marks)

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3. (*a*) Using the formulae for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$, show that

$$\sum_{r=1}^{n} (r+1)(r+4) = \frac{n}{3}(n+4)(n+5)$$

for all positive integers n.

(5)

(*b*) Hence show that

$$\sum_{r=n+1}^{2n} (r+1)(r+4) = \frac{n}{3}(n+1)(an+b)$$

where a and b are integers to be found.

(3)

(Total 8 marks)

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4. (*a*) Use the standard results for $\sum_{r=1}^{n} r^3$ and $\sum_{r=1}^{n} r$ to show that

$$\sum_{r=1}^{n}(r^3 + 6r - 3) = \frac{1}{4}n^2(n + 2n + 13)$$

for all positive integers n.

(5)

(*b*) Hence find the exact value of

$$\sum_{r=16}^{30}(r^3 + 6r - 3).$$

(2)

(Total 7 marks)

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5. (*a*) Use the results for $\sum_{r=1}^{n} r$, $\sum_{r=1}^{n} r^2$ and $\sum_{r=1}^{n} r^3$, to prove that

$$\sum_{r=1}^{n} r(r+1)(r+5) = \frac{1}{4}n(n + 1)(n + 2)(n + 7)$$

for all positive integers n.

(5)

(*b*) Hence, or otherwise, find the value of

$$\sum_{r=20}^{50} r(r+1)(r+5)$$

(2)

(Total 7 marks)

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6. (*a*) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^3$ to show that

$$\sum_{r=1}^{n} r\left(r^2-3\right)=\frac{1}{4}n\left(n+1\right)\left(n+3\right)\left(n-2\right)$$

(5)

(*b*) Calculate the value of $\sum_{r=10}^{50} r\left(r^2-3\right)$.

(3)

(Total 8 marks)

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7. (*a*) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(2r-1)^2=\frac{1}{3}n(4n^2-1)$$

(6)

(*b*) Hence show that

$$\sum_{r=2n+1}^{4n}(2r-1)^2=an(bn^2-1)$$

where *a* and *b* are constants to be found.

(3)

(Total 9 marks)

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8. (*a*) Use the standard results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(r+2)(r+3) = \frac{1}{3}n(n^2+9n+26)$$

for all positive integers n.

(6)

(*b*) Hence show that

$$\sum_{r=n+1}^{3n}(r+2)(r+3) = \frac{2}{3}n(an^2+bn+c)$$

where a, b and c are integers to be found.

(4)

(Total 10 marks)

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9. (*a*) Use the results for $\sum_{r=1}^{n} r$ and $\sum_{r=1}^{n} r^2$ to show that

$$\sum_{r=1}^{n}(2r-1)^2 = \frac{1}{3}n(2n+1)(2n-1)$$

for all positive integers n.

(6)

(*b*) Hence show that

$$\sum_{r=n+1}^{3n}(2r-1)^2 = \frac{2}{3}n(an^2+b)$$

where a and b are integers to be found.

(4)

(Total 10 marks)

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10. (*a*) Prove by induction

$$\sum_{r=1}^{n} r^3 = \frac{1}{4}n^2(n+1)^2.$$

(5)

(*b*) Using the result in part (*a*), show that

$$\sum_{r=1}^{n} (r^3 - 2) = \frac{1}{4}n(n^3 + 2n^2 + n - 8).$$

(3)

(*c*) Calculate the exact value of $\sum_{r=20}^{50} (r^3 - 2)$.

(3)

(Total 11 marks)

TOTAL FOR PAPER: 80 MARKS