Year 2 Pure Chapter 9 - Differentiation

Trigonometric Functions

$$y = \sin f(x)$$

$$\frac{dy}{dx} = f'(x) \cos f(x)$$

$$y = \cos f(x)$$

$$\frac{dy}{dx} = -f'(x) \sin f(x)$$

More Trig Functions

(in the formula booklet)

$\tan kx \rightarrow k \sec^2 kx$

$\sec kx \rightarrow k \sec kx \tan kx$

$\cot kx \rightarrow -k \operatorname{cosec}^2 kx$

$\operatorname{cosec} kx \rightarrow -k \operatorname{cosec} kx \cot kx$

Exponents

$$y = e^{f(x)}$$

$$\frac{dy}{dx} = f'(x)\, e^{f(x)}$$

$$y = a^{f(x)}$$

$$\frac{dy}{dx} = f'(x)\, a^{f(x)} \ln a$$

Logs

$$y = \ln f(x)$$

$$\frac{dy}{dx} = \frac{f'(x)}{f(x)}$$

Chain Rule

(Function within a function)

Examples of chain rule functions

$y = (1 + 2x)^4$

$y = (7 - x)^{\frac{1}{2}}$

$y = e^{\cos x}$

$y = \cos(2x - 1)$

Example

$$y = (5x^2 + 2)^4$$

$5x^2 + 2$	u^4
$10x$	$4u^3$

$$\frac{dy}{dy} = 40xu^3$$

$$\frac{dy}{dy} = 40x(5x^2 + 2)^3$$

Product Rule

(Function multiply a function)

$$y = x(1+3x)^5$$

$$y = \sin 2x \cos 3x$$

$$y = e^x \sin x$$

Example

$$y = x^2 \sin x$$

x^2	$\sin x$
$2x$	$\cos x$

Multiply opposite corners and add.

$$\frac{dy}{dy} = x^2 \cos x + 2x \sin x$$

Quotient Rule

(Function divided a function)

$$y = \frac{5x}{x+1}$$

$$y = \frac{\ln x}{x+1}$$

$$y = \frac{\sin x}{\ln x}$$

Example

$$y = \frac{2x}{x^2-3}$$

$2x$	$x^2 - 3$
2	$2x$

$$\frac{f(x)}{g(x)} = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

given in the formula booklet.

$$\frac{dy}{dx} = \frac{2(x^2-3) - 2x(2x)}{[x^2-3]^2}$$

Parametric Equations

Take the x equations and get

$$\frac{dx}{dt}$$

Then take the reciprocal to get

$$\frac{dt}{dx}$$

Take the y equations and get

$$\frac{dy}{dt}$$

$$\frac{dy}{dx} = \frac{dy}{dt} \times \frac{dt}{dx}$$

Implicit Differentiation

How to deal y terms:
Differentiate with respect to y and multiply by $\frac{dy}{dx}$

$$y^2 \rightarrow 2y\frac{dy}{dx}$$

Example

$$y^2x$$

y^2	x
$2y\frac{dy}{dx}$	1

$$\frac{dy}{dx} = 1y^2 + 2yx\frac{dy}{dx}$$

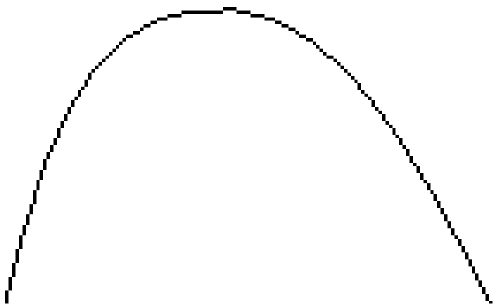
Now make $\frac{dy}{dx}$ the subject of the equation

Second Derivatives

Concave

when f(x) is concave

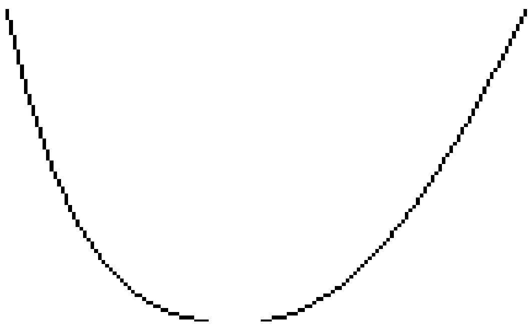
f''(x) ≤ 0



Convex

when f(x) is concave

f''(x) ≥ 0



Point of inflection

when f(x) is a point of inflection

f''(x) = 0

Rates of Change

Use chain rule to connect rates of change.

Area changing with time

$$= \frac{dA}{dt}$$

$$\frac{dA}{dt} = \frac{dA}{dx} \text{ x } \frac{dx}{dt}$$

Volume changing with time

$$= \frac{dA}{dt}$$

$$\frac{dV}{dt} = \frac{dV}{dx} \text{ x } \frac{dx}{dt}$$

Watch out for decreasing rates of change.

$$\frac{dV}{dt} = -\frac{2v}{15}$$

Negative because decreasing