

Matrix Transformations – Section Test

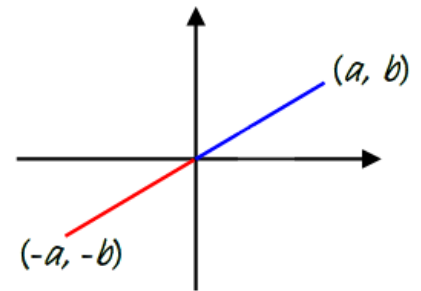
(Answers)

1.

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -a \\ -b \end{pmatrix}$$

so, the image of (a, b) is $(-a, -b)$.

The matrix represents a rotation of 180° about the origin.

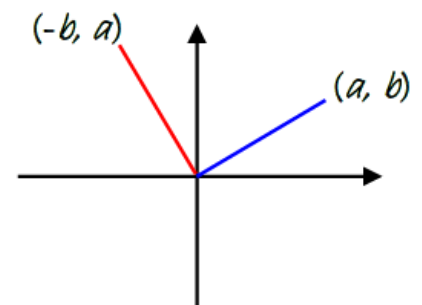


2.

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix} \begin{pmatrix} a \\ b \end{pmatrix} = \begin{pmatrix} -b \\ a \end{pmatrix}$$

so, the image of (a, b) is $(-b, a)$

The matrix represents a rotation through 90° anticlockwise about the origin.



3.

Under an enlargement scale factor 4, the point $(1, 0)$ is mapped to $(4, 0)$ and the point $(0, 1)$ is mapped to $(0, 4)$.

So the matrix representing this transformation is $\begin{pmatrix} 4 & 0 \\ 0 & 4 \end{pmatrix}$.

4.

Under a reflection in the x-axis, the point $(1, 0)$ is mapped to itself, and the point $(0, 1)$ is mapped to $(0, -1)$.

So the matrix representing this transformation is $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

5.

$$\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 4 \\ 6 \end{pmatrix}$$

The image of A is $(4, 6)$.

6.

$$\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} 3 \\ 1 \end{pmatrix} = \begin{pmatrix} 7 \\ 13 \end{pmatrix}$$

The image of B is $(7, 13)$.

7.

$$\begin{pmatrix} 2 & 1 \\ 3 & 4 \end{pmatrix} \begin{pmatrix} -1 \\ 3 \end{pmatrix} = \begin{pmatrix} 1 \\ 9 \end{pmatrix}$$

The image of C is (1, 9).

8.

The composite transformation "A followed by B" is represented by the matrix

$$BA = \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} = \begin{pmatrix} 0 & 6 \\ -1 & -9 \end{pmatrix}$$

9.

The composite transformation "B followed by A" is represented by the matrix

$$AB = \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix} = \begin{pmatrix} 2 & 4 \\ -7 & -11 \end{pmatrix}$$

10.

The transformation "A followed by B followed by A again" is represented by the matrix

$$\begin{aligned} ABA &= \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ -2 & -3 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 2 & 4 \\ -7 & -11 \end{pmatrix} \begin{pmatrix} 2 & 0 \\ -1 & 3 \end{pmatrix} \\ &= \begin{pmatrix} 0 & 12 \\ -3 & -33 \end{pmatrix} \end{aligned}$$

The image of the point (3, 2) under this transformation is

$$\begin{pmatrix} 0 & 12 \\ -3 & -33 \end{pmatrix} \begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} 24 \\ -75 \end{pmatrix}$$