

Name: _____

Level 2 Further Maths

Equation of a Circle



Corbettmaths

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/



1. The equation of a circle C, with centre O, is:

$$x^2 + y^2 = 289$$

- (a) Find the coordinates of the centre O.

(.....,)
(1)

- (b) Find the radius of C.

$$\sqrt{289} = 17$$

17
.....
(1)

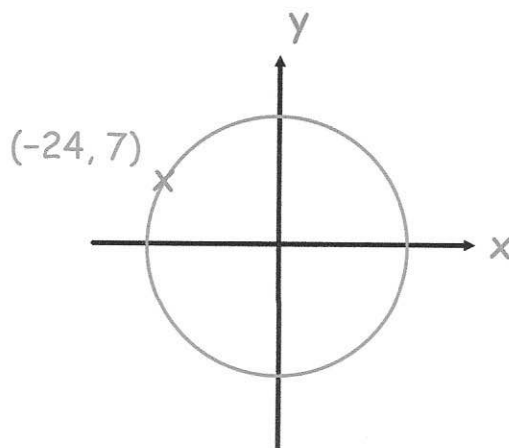
- (c) Show the point (8, 15) lies on C.

$$8^2 + 15^2 = 289$$

(2)

2. The circle below has centre $(0, 0)$.
The point $(-24, 7)$ is a point on the circle.

Find the equation of the circle.



$$24^2 + 7^2 = 625$$

$$\sqrt{625}$$

$$x^2 + y^2 = 25^2$$

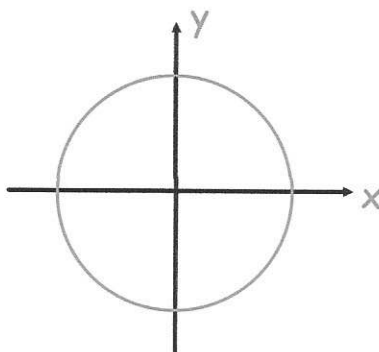
(2)

3. The circle shown has $x^2 + y^2 = 42.25$

Find the circumference of the circle.
Give your answer in terms of π

$$\sqrt{42.25} = 6.5$$

$$6.5 \times 2 = 13$$



$$13\pi$$

(2)

4. A circle has the equation $x^2 + y^2 = 400$

Find the area of the circle.

Give your answer in terms of π

$$r = 20$$

$$A = \pi r^2$$

$$= \pi \times 20^2$$

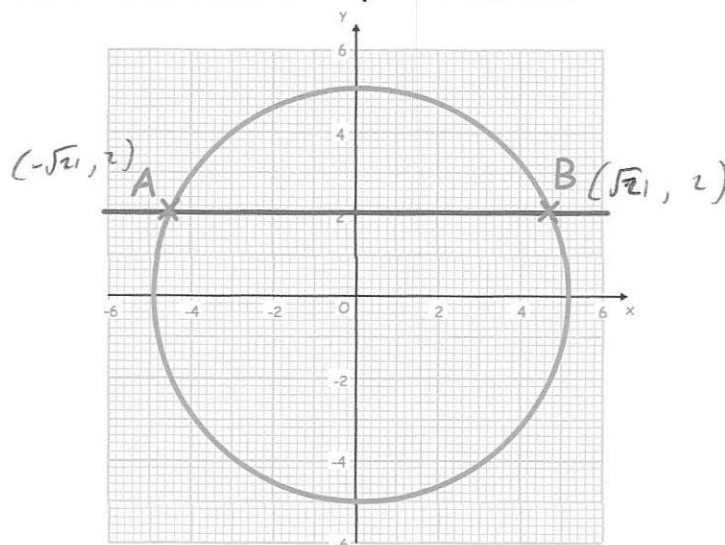
$$= 400\pi$$

$$400\pi$$

(2)

5. A circle has equation $x^2 + y^2 = 25$

A straight line meets the circle at the points A and B.



- (a) Write down the equation of the straight line.

$$y = 2$$

(1)

- (b) Find the distance AB

$$x^2 + 2^2 = 25$$

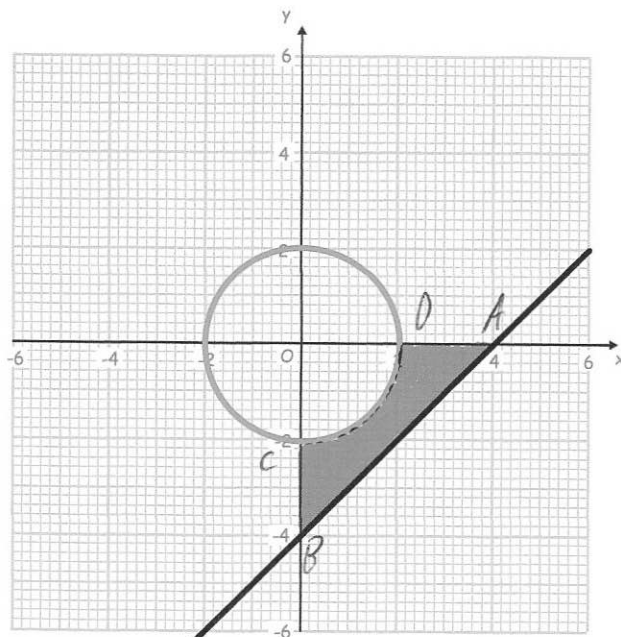
$$x^2 = 21$$

$$x = \pm \sqrt{21}$$

$$2\sqrt{21}$$

(4)

6. The circle below has equation $x^2 + y^2 = 4$
The line has equation $y = x - 4$



- (a) Find the area of the shaded region.

Area of triangle OAB
 $\frac{1}{2} \times 4 \times 4 = 8$

Area of sector ODC
 $\frac{1}{4} (\pi \times 2^2)$
 $= \pi$

$8 - \pi$

(4)

- (b) Find the perimeter of the shaded region.

$AD = 2$ Arc DC = $\frac{1}{4} (\pi \times 4) = \pi$
 $BC = 2$ Line AB = $\sqrt{32}$ or $4\sqrt{2}$

$4 + \pi + 4\sqrt{2}$

(4)

or 12.798

7. The equation of a circle C, with centre A, is:

$$(x - 3)^2 + (y + 2)^2 = 25$$

- (a) Find the coordinates of the centre A.

$$(\dots 3, \dots -2) \\ (1)$$

- (b) Find the radius of C.

$$\frac{5}{\dots} \\ (1)$$

- (c) Show the point (6, 2) lies on C.

$$(6-3)^2 + (2+2)^2 \\ 3^2 + 4^2 = 25$$

$\therefore$ it is on C

(2)

8. A circle has centre (5, 2) and radius 4.

- (a) Write down the equation of the circle.

$$(x-5)^2 + (y-2)^2 = 4^2 \\ \text{or } (x-5)^2 + (y-2)^2 = 16 \quad (2)$$

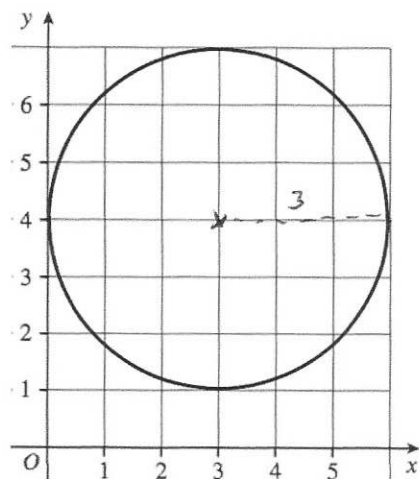
- (b) Does the point (7, 4) lie on the circle?

$$(7-5)^2 + (4-2)^2 = \\ 2^2 + 2^2 = 8 \\ 8 \neq 16$$

$\therefore$ it is not on C

(2)

9.

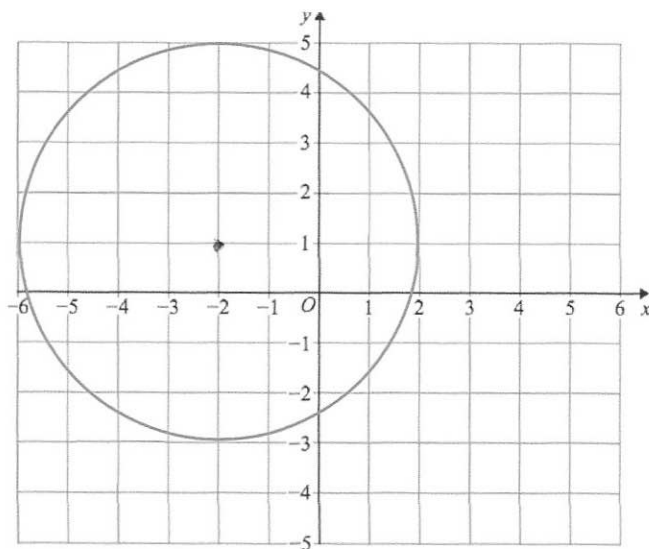


Find the equation of the circle.

$$\underline{(x-3)^2 + (y-4)^2 = 3^2}$$

(2)

10.



Find the equation of the circle.

$$\underline{(x+2)^2 + (y-1)^2 = 4^2}$$

(2)

11. A circle C has centre P

The points A (0, 6) and B (8, 6) lie on the diameter of C.

(a) Find the coordinates of the centre P.

$$(\dots\dots\dots 4 \dots\dots\dots, \dots\dots\dots 6 \dots\dots\dots)$$

(1)

(b) Write down the equation of the circle.

$$\dots\dots\dots (x-4)^2 + (y-6)^2 = 4^2 \dots\dots\dots$$

(2)

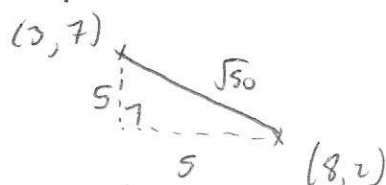
12. AB is a diameter of a circle C.
Q is the centre of the circle
A has coordinates $(-2, 12)$ and B has coordinates $(8, 2)$.

(a) Find the centre of the circle, Q.

$$(\underline{3}, \underline{7})$$

(1)

(b) Find the equation of C



$$(\underline{x-3})^2 + (\underline{y-7})^2$$

(2)

(c) Show the point D, $(10, 8)$ lies on C.

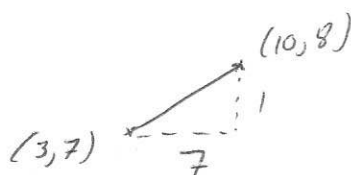
$$(10-3)^2 + (8-7)^2$$

$$7^2 + 1^2 = 50$$

$\therefore D$ is on the circle

(2)

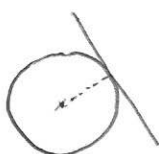
(d) Find the gradient of OD.



$$\underline{\frac{1}{7}}$$

(2)

(e) Find the equation of the tangent to C at the point D.



$$y = -7x + c$$

$$8 = -7(10) + c$$

$$c = 78$$

$$\underline{y = -7x + 78}$$

(3)

13. A circle has equation $(x - 5)^2 + (y + 2)^2 = 20$

(a) Find the centre of the circle.

$$(\underline{5}, \underline{-2})$$

(1)

(b) State, with a reason, whether this circle intersects the y-axis.

no, as centre is $(5, -2)$ but radius is $\sqrt{20}$

(2)

(c) Find the equation of a line parallel to $y = 2x$ that passes through the centre of the circle.

$$y = 2x + c$$
$$-2 = 10 + c$$
$$c = -12$$

$$\underline{y = 2x - 12}$$

(3)

(d) Show $y = 2x - 2$ is a tangent to the circle

$$(x - 5)^2 + (2x - 2 + 2)^2 = 20$$

$$(x - 5)^2 + (2x)^2 = 20$$

$$x^2 - 10x + 25 + 4x^2 = 20$$

$$5x^2 - 10x + 5 = 0$$

$$x^2 - 2x + 1 = 0$$

$$(x - 1)(x - 1) = 0$$

$$x = 1$$

$$y = 0 \quad (1, 0)$$

(3)

(e) Find the coordinates of the point of contact.

Since only 1 point of intersection, it must be a tangent

$$\underline{(1, 0)}$$

(2)

14. A circle has centre C and equation $x^2 + y^2 - 6x + 14y + 49 = 0$

(a) Find the centre of the circle.

$$x^2 - 6x + y^2 + 14y + 49 = 0$$

$$(x-3)^2 - 9 + (y+7)^2 - 49 + 49 = 0$$

$$(x-3)^2 + (y+7)^2 = 9$$

(.....3,-7.....)
(2)

(b) Find the radius of the circle

.....3.....
(2)

15. Circle 1 has an equation of $(x - 4)^2 + (y - 1)^2 = 36$

Circle 2 has an equation of $(x + 7)^2 + (y - 8)^2 = 100$

Calculate the distance between the centres of Circle 1 and Circle 2

Circle 1 $(4, 1)$

Circle 2 $(-7, 8)$



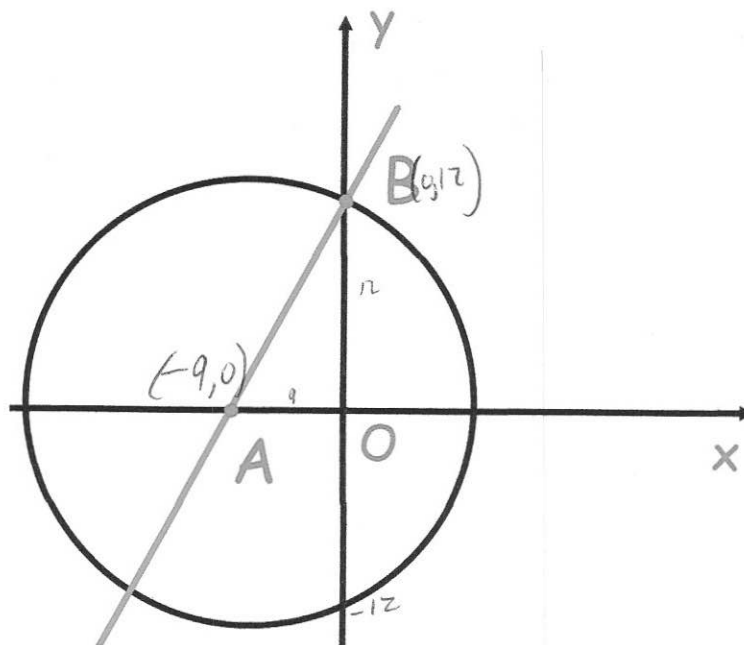
$$7^2 + 11^2 = 170$$

$$\sqrt{170}$$

$$\sqrt{170}$$

(4)

16. Shown below is the circle, centre A, with equation $(x + 9)^2 + y^2 = 225$



Find the equation of the line passing through A and B.

$$\begin{aligned} \text{B} \quad x &= 0 \\ q^2 + y^2 &= 225 \\ y^2 &= 144 \\ y &= \pm 12 \end{aligned}$$

$$y = \frac{12}{9}x + c$$

$$y = \frac{4}{3}x + c$$

$$0 = \frac{4}{3}(-9) + c$$

$$0 = -12 + c$$

$$c = 12$$

(wait that was obvious!)

$$y = \frac{4}{3}x + 12$$

(4)

17. The line $y = 2x - 1$ intersects the circle $(x - 2)^2 + (y + 5)^2 = 144$ at the points A and B.

Find the coordinates of A and B.

$$(x-2)^2 + (2x-1+5)^2 = 144$$

$$x^2 - 4x + 4 + (2x+4)^2 = 144$$

$$x^2 - 4x + 4 + 4x^2 + 16x + 16 = 144$$

$$5x^2 + 12x - 124 = 0$$

$$a=5 \quad b=12 \quad c=-124$$

$$x = \frac{-6 + 4\sqrt{41}}{5} \quad \text{or} \quad x = \frac{-6 - 4\sqrt{41}}{5}$$

$$y = \frac{-17 + 8\sqrt{41}}{5} \quad y = \frac{-17 - 8\sqrt{41}}{5}$$

$$\left(\frac{-6 + 4\sqrt{41}}{5}, \frac{-17 + 8\sqrt{41}}{5} \right)$$

$$\left(\frac{-6 - 4\sqrt{41}}{5}, \frac{-17 - 8\sqrt{41}}{5} \right)$$

$\swarrow$ $8\sqrt{41}$

.....
(6)

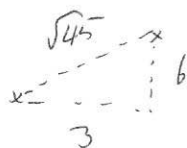
18. A circle has equation $(x + 7)^2 + (y - 6)^2 = 49$

Is the point $(-4, 12)$ inside or outside the circle?
You must show your workings.

$$r = 7$$

Centre $(-7, 6)$

point $(-4, 12)$



$$3^2 + 6^2 = 45$$

$$\sqrt{45} < 7$$

As the distance between the centre $(-7, 6)$ and the point $(-4, 12)$ is less than the radius of 7 then the point is inside the circle.

.....
(6)