A level Mathematics Practice Paper – Kinematics of a particle – Mark scheme

Question	Scheme	Marks
1(a)	$\mathbf{a} = \dfrac{\mathrm{d}\mathbf{v}}{\mathrm{d}t} = 6t\mathbf{i} + (4-2t)\mathbf{j}$	M1
		A1
	When $t = 1$, $\mathbf{a} = 6\mathbf{i} + 2\mathbf{j}$	DM1
	$\lvert\mathbf{a}\rvert = \sqrt{6^2 + 2^2} = \sqrt{40} = 6.32$ (m s^{-2})	DM1
		A1
		(5)
1(b)	$\mathbf{r} = \int (3t^2 - 1)\mathbf{i} + (4t - t^2)\mathbf{j}\ \mathrm{d}t$	M1
	$= (t^3 - t + C)\mathbf{i} + (2t^2 - \frac{1}{3}t^3 + D)\mathbf{j}$	A1
	$t = 0, \mathbf{r} = \mathbf{i} \Rightarrow C = 1, D = 0$	DM1
	When $t = 3, \mathbf{r} = 25\mathbf{i} + 9\mathbf{j}$ (m)	DM1
		A1
		(5)
		(10 marks)
2(a)	Integrate: $\boldsymbol{v} = \left(t^3 - 2t^2\right)\mathbf{i} + \left(3t^2 - 5t\right)\mathbf{j} + \mathbf{C}$	M1
		A2
	$t = 3$: $\mathbf{v} = 9\mathbf{i} + 12\mathbf{j} + \mathbf{C} = 11\mathbf{i} + 10\mathbf{j}$ $\quad \mathbf{C} = 2\mathbf{i} - 2\mathbf{j}$	DM1
	$\mathbf{v} = \left(t^3 - 2t^2 + 2\right)\mathbf{i} + \left(3t^2 - 5t - 2\right)\mathbf{j}$	A1
		(5)
2(b)	Parallel to $\mathbf{i}$ $\Rightarrow 3t^2 - 5t - 2 = 0$	M1
	$(3t+1)(t-2) = 0, \quad t = 2$	A1
	$\lvert\mathbf{v}\rvert = 8 - 8 + 2 = 2$ (m s^{-1})	DM1
		A1
		(4)
		(9 marks)

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Question	Scheme	Marks
3(a)	Speed = $\sqrt{8^2 + 48^2} = \sqrt{2368} = 48.7\,(\text{ms}^{-1})$	M1 A1
		(2)
3(b)	$\mathbf{a} = 2\mathbf{i} - 6t\,\mathbf{j}$	M1 A1
	When $t = 4$, $\mathbf{a} = 2\mathbf{i} - 24\,\mathbf{j}$ (ms^{-2})	A1
		(3)
3(b)	$\mathbf{r} = t^2\mathbf{i} - t^3\mathbf{j} + \mathbf{C}$	M1 A1
	t = 1, $-4\mathbf{i} + \mathbf{j} = \mathbf{i} - \mathbf{j} + \mathbf{C}$, $\mathbf{C} = -5\mathbf{i} + 2\mathbf{j}$	M1
	$\mathbf{r} = (t^2 - 5)\mathbf{i} + \left(-t^3 + 2\right)\mathbf{j}$	
	When $t = 4$, $\mathbf{r} = (16-5)\mathbf{i} + (-64 + 2)\mathbf{j} = 11\mathbf{i} - 62\mathbf{j}$	M1 A1
		(5)
		(10 marks)
4(a)	$2mu = 2mv_P + mv_Q$	M1
		A1
	$\frac{3}{4}mu^2 = \frac{1}{2}2mv_P{}^2 + \frac{1}{2}mv_Q{}^2$	M1
		A1
	$3v_Q{}^2 - 4uv_Q + u^2 = 0$ or $12v_P{}^2 - 16uv_P + 5u^2 = 0$	M1
		A1
	$v_Q = \frac{u}{3}$, $v_P = \frac{5u}{6}$ or $v_Q = u, v_P = \frac{u}{2}$	DM1
		A1
	$v_Q = u$	DM1
	..... since $v_Q > v_P$	A1
		(10)
4(b)	$e = \dfrac{u - \frac{u}{2}}{u}$ $\left(\dfrac{v_Q - v_P}{u}\right)$	M1
		A1 ft
	$= \dfrac{1}{2}$	A1
		(3)
		(13 marks)

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Question	Scheme	Marks
5(a)	Considering energy: $\frac{1}{2}m\times 14^2=\frac{1}{2}m\times 10^2+mgh$	M1
		A2
	$h=\frac{48}{g}=4.90$	A1
		(4)
5(b)	Vertical distance: $h=14\sin\alpha t-\frac{1}{2}\times 9.8t^2$	M1
	$4.9t^2-11.9t+h=0$	A2
	$t=\frac{11.9\pm\sqrt{11.9^2-4\times 4.9^2}}{9.8}$	DM1
	$t=1.903.....$	A1
	Horizontal distance: $x=14\cos\alpha\times t$	M1
		A1
	=14.0 (m)	A1
		(8)
		(12 marks)

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Question	Scheme	Marks
6(a)	$2 = -2u\sin\theta + \frac{1}{2}g \times 4$	M1
	$\left(-2 = u\sin\theta t - \frac{1}{2}gt^2\right)$	A1
	$u\sin\theta = g - 1$	
	$2u\cos\theta = 8 \quad (u\cos\theta = 4)$ $(u\cos\theta t = 8)$	B1
	$\tan\theta = \frac{g-1}{4} = 2.2$ *	M1 A1
		(5)
6(b)	$u\cos\theta = 4$	M1
	$u = \frac{4}{\cos\theta} = 9.66... = 9.7$	A1
	OR use components from (a) and Pythagoras.	
		(2)
6(c)	$6 = (1-g)T + \frac{1}{2} \times 9.8T^2$	M1
	$4.9T^2 - 8.8T - 6 = 0$	
	$T = \frac{8.8 \pm \sqrt{\left[(-)8.8\right]^2 + 24 \times 4.9}}{9.8}$	M1
	$T = 2.323... = 2.32$ or 2.3	A1
		(3)
6(d)	$v^2 = 8.8^2 + 2g \times 6$ or $v = -8.8 + gT$	M1
	$v =$ 13.96	A1
	Horiz speed = 4	
	$\tan\alpha = \frac{v}{4}$	M1
		A1
	$\alpha = 74.01... = 74°$	A1
		(5)
		(15 marks)

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Question	Scheme	Marks
7(a)	Using $s = ut + \frac{1}{2}at^2$ Method must be clear	M1
	$\mathbf{r} = (3t)\mathbf{i} + (10 + 5t - 4.9t^2)\mathbf{j}$ Answer given	A1 A1
		(3)
7(b)	**j** component $= 0$: $10 + 5t - 4.9t^2$	M1
	quadratic formula: $t = \frac{5 \pm \sqrt{25 + 196}}{9.8} = \frac{5 \pm \sqrt{221}}{9.8}$	DM1
	$T = 2.03$(s), 2.0 (s) positive solution only.	A1
		(3)
7(c)	Differentiating the position vector (or working from first principles)	M1
	$\mathbf{v} = 3\mathbf{i} + (5 - 9.8t)\mathbf{j}$ (ms^{-1})	A1
		(2)
7(d)	At B the **j** component of the velocity is the negative of the **i** component: 5	M1
	$-9.8t = -3$, $8 = 9.8t$,	A1
		(2)
7(e)	$t = 0.82$	
	$\mathbf{v} = 3\mathbf{i} - 3\mathbf{j}$, speed $= \sqrt{3^2 + 3^2} = \sqrt{18} = 4.24\,(\text{m s}^{-1})$	M1A1
		(2)
		(12 marks)

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Question	Scheme	Marks
8(a)	**i** → distance = $6t$	B1
	j ↑ distance = $12t - \frac{1}{2}gt^2$	M1 A1
	At B, $2\left(12t - \frac{1}{2}gt^2\right) = 6t$	M1 A1
	$(24-6)t = gt^2$	M1
	$18 = gt$, $t = \frac{18}{g} (= 1.84\text{s})$	A1
		(7)
8(b)	**i** → speed = 6	B1
	j ↑ velocity = $12 - gt = -6$	M1 A1
	speed at A	
	$= \sqrt{6^2 + 6^2} = \sqrt{72} = 6\sqrt{2} (= 8.49) (\text{ms}^{-1})$	M1 A1
		(5)
8(c)	↑ speed = $12 - gt = +6$	M1 A1 ft
	$t = \frac{6}{g} (= 0.61\text{s})$	A1
		(3)
		(15 marks)

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Question	Scheme	Marks
9(a)	After 4 seconds from O, horizontal speed $= u\cos\theta$	B1
	Vertical component of speed at $A = u + at$	M1
	$= u\sin\theta - 4g$	A1
	At A, components are $15\cos 20$ (horizontal) and $15\sin 20$ (vertical)	B1
	$u\cos\theta = 15\cos 20$ $\qquad$ $u\sin\theta = 15\sin 20 + 4g$	dM1
	$\theta = 72.4 \quad (72)$	A1
	$u = 46.5 \quad (47)$	A1
		(7)
9(b)	$-15\sin 20 = 15\sin 20 - gt$ $\quad$ or $\quad$ $0 = 15\sin 20t - \frac{1}{2}gt^2$	M1
	$t = 1.05$ (s) or 1.0 (s)	A1
		(2)
9(c)	Total time $= 4 + (1.05) + 4$	B1ft
	Range $= 46.5 \times \cos 72.4 \times (8 + 1.05)$ $\quad$ (or $15\cos 20 \times 9.05$)	M1
	$= 128$ (m) or 127 (m) (130)	A1
		(3)
		(12 marks)

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Question	Scheme	Marks
10(a)	$(\rightarrow)\sqrt{27ag}\cos\theta.\ t = 9a$	M1
		A1
	$(\uparrow)\sqrt{27ag}\sin\theta.\ t - \frac{1}{2}gt^2 = 6a$	M1
		A1
	$(\uparrow)\sqrt{27ag}\sin\theta.\ \frac{9a}{\sqrt{27ag}\cos\theta} - \frac{1}{2}g\left(\frac{9a}{\sqrt{27ag}\cos\theta}\right)^2 = 6a$	DM1
	$9a\tan\theta - \frac{1}{2}g.81a^2\frac{(1+\tan^2\theta)}{27ag} = 6a$	DM1
	$\tan^2\theta - 6\tan\theta + 5 = 0$	A1
		(7)
10(b)	$\tan^2\theta - 6\tan\theta + 5 = 0$	
	$(\tan\theta - 1)(\tan\theta - 5) = 0$	M1
	$\tan\theta_2 = 1$ or $\tan\theta_1 = 5$	A1 A1
		(3)
10(c)	$t = \frac{9a}{\sqrt{27ag}\cos\theta} = \frac{9a}{\sqrt{27ag}} \times \frac{\sqrt{26}}{1}$	M1 A1ft
	$= \sqrt{\frac{81a^2.26}{27a}} = \sqrt{\frac{78a}{g}}$ ***Answer given***	A1
		(3)
10(d)	$\frac{1}{2}m(27ag - v^2) = mg6a$	M1 A1
	$v = \sqrt{15ag}$	A1
		(3)
		(15 marks)

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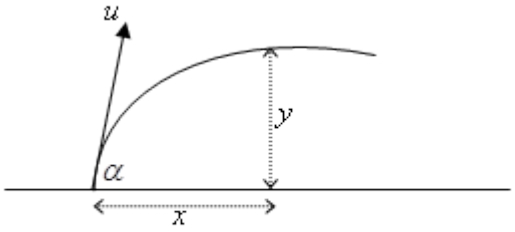
Question	Scheme		Marks
11(a)	$0^2 = u_V^2 - 2\times 9.8\times 10$		M1
			A1
	$u_V = 14$ *		A1
			(3)
OR	conservation of energy:		M1
	$\frac{1}{2}m\left(u_h{}^2 + u_v{}^2\right) = mg\times 10 + \frac{1}{2}mu_h{}^2$, $\frac{1}{2}u_v{}^2 = 98$		A1
	$u_V = 14$ *		A1
			(3)
11(b)	$(\uparrow),\ -52.5 = 14t - \frac{1}{2}gt^2$		M1
			A1
			A1
	$49t^2 - 140t - 525 = 0$		DM1
	$(t-5)(49t+105) = 0 \quad t = 5$		A1
	$(\rightarrow)\ 50 = 5u_H$		M1
	$u_H = 10$		A1
	$u_H = \sqrt{10^2 + 14^2}$		M1
	$\sqrt{296} = 17.2$ m s^{-1}		A1
			(9)
11(c)	$\tan OBA = \frac{52.5}{50} = 1.05$	Correct direction o.e. (accept reciprocal)	B1
	$v_r = 1.05 \times 10 = 10.5$	Use trig with their u_H and correct interpretation of direction to find the vertical component of the speed	M1
	$(\uparrow) - 10.5 = 14 - gt$	Use suvat to form an equation in t. Dependent on the preceding M.	DM1
		Correct equation for their u_H For incorrect direction give A0 here	A1
	$t = 2.5$	only	A1
			(5)
			(17 marks)

Question	Scheme	Marks
12(a)		
	Horiz: $x = u\cos\alpha t$	B1
	Vert: $y = u\sin\alpha t - \frac{1}{2}gt^2$	M1
	$y = u\sin\alpha \times \frac{x}{u\cos\alpha} - \frac{1}{2}g \times \frac{x^2}{u^2\cos^2\alpha}$	DM1
	$y = x\tan\alpha - \frac{gx^2}{2u^2\cos^2\alpha}$ **	A1
		(4)
12(b)	$y = -7$: $-7 = \tan 45x - \frac{gx^2}{2\times 7^2\cos^2 45}$	M1 A1
	$-7 = x - \frac{9.8x^2}{7^2}$	
	$-7 = x - \frac{x^2}{5}$	M1
	$x^2 - 5x - 35 = 0$	
	$x = \frac{5 \pm \sqrt{25 + 4\times 35}}{2}$	M1
	$x = 8.92$ or 8.9	A1
		(5)
12(c)	Time to travel 8.922 m horizontally $= \frac{8.922}{7\cos 45} = 1.802...$s	M1
	$v = \frac{8.922}{1.402}$	M1 A1 ft
	$= 6.36$ or 6.4 $\left(\text{m s}^{-1}\right)$	A1
		(4)
		(13 marks)

A level Mathematics Practice Paper – Kinematics of a particle – Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	M2 2012	1		Kinematics of a particle moving in a straight line or plane	1.1b, 3.1a
2	M2 2014	2		Kinematics of a particle moving in a straight line or plane	1.1b, 3.1a
3	M2 Jan 2012	2		Kinematics of a particle moving in a straight line or plane	1.1a, 1.1b, 3.1a
4	M1 2013R	5		Kinematics of a particle moving in a straight line, Problems with friction	1.1b, 1.2, 3.1, 2.2a, 3.1b, 3.2
5	M2 2014	6		Kinematics of a particle moving in a straight line or plane, Work and energy	1.1b, 2.1, 3.1b
6	M2 Jan 2013	6		Kinematics of a particle moving in a straight line or plane	1.1b, 2,1, 2,2a, 3.1b
7	M2 Jan 2011	6		Kinematics of a particle moving in a straight line or plane	1.1b, 2.1, 2.2a, 3.1b
8	M2 Jan 2012	7		Kinematics of a particle moving in a straight line or plane	1.1b, 2.2a, 3.1b
9	M2 2015	7		Kinematics of a particle moving in a straight line or plane	1.1b, 2.1, 3.1b
10	M2 2013R	7		Kinematics of a particle moving in a straight line or plane	1.1b, 2.1, 2.2a, 3.1b
11	M2 2012	7		Kinematics of a particle moving in a straight line or plane	1.1b, 2.2a, 3.1b
12	M2 2011	8		Kinematics of a particle moving in a straight line or plane	1.1b, 2.1, 2.2a, 3.1b, 3.3, 3.4