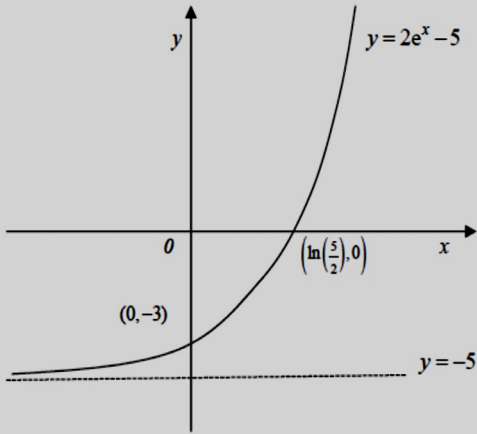
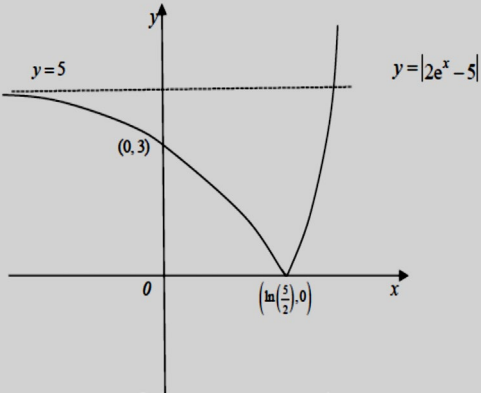


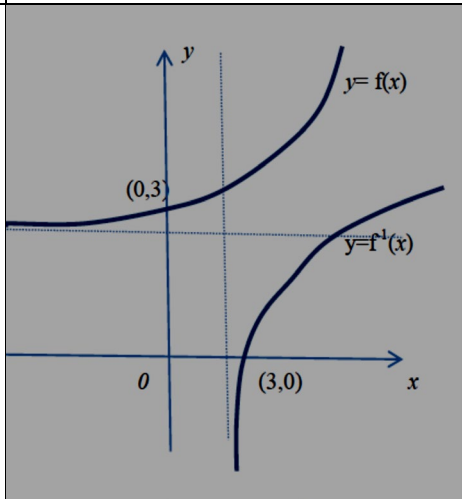
A level Mathematics Practice Paper – Trigonometry (part 3) – Mark scheme

Question	Scheme	Marks
1(a)	$2\ln(2x+1)-10=0 \Rightarrow \ln(2x+1)=5 \Rightarrow 2x+1=e^5 \Rightarrow x=..$	M1
	$\Rightarrow x = \frac{e^5-1}{2}$	A1
		(2)
1(b)	$3^x e^{4x} = e^7 \Rightarrow \ln(3^x e^{4x}) = \ln e^7$	
	$\ln 3^x + \ln e^{4x} = \ln e^7 \Rightarrow x \ln 3 + 4x \ln e = 7 \ln e$	M1 M1
	$x(\ln 3 + 4) = 7 \Rightarrow x = ...$	dM1
	$x = \frac{7}{(\ln 3 + 4)}$ oe	A1
		(4)
		(6 marks)
2(a)	$\ln(4-2x)(9-3x), = \ln(x+1)^2$	M1 M1
	So $36-30x+6x^2 = x^2+2x+1$ and $5x^2-32x+35=0$	A1
	Solve $5x^2-32x+35=0$ to give $x = \frac{7}{5}$ oe (Ignore the solution $x=5$)	M1 A1
		(5)
2(b)	Take log _e 's to give $\ln 2^x + \ln e^{3x+1} = \ln 10$	M1
	$x \ln 2 + (3x+1) \ln e = \ln 10$	M1
	$x(\ln 2 + 3 \ln e) = \ln 10 - \ln e \Rightarrow x = ..$	dM1
	and uses $\ln e = 1$	M1
	$x = \frac{-1 + \ln 10}{3 + \ln 2}$	A1
		(5)
		(10 marks)

A level Mathematics Practice Paper – Trigonometry (part 3) – Mark scheme

Question	Scheme		Marks
3(a)(i)		Shape	B1
		$\left(\ln\left(\frac{5}{2}\right), 0\right)$ and $(0, -3)$	B1
		$y = -5$	B1
			(3)
3(a)(ii)		Shape inc cusp	B1ft
		$\left(\ln\left(\frac{5}{2}\right), 0\right)$ and $(0, 3)$	B1ft
		$y = 5$	B1ft
			(3)
3(b)	$x \geq \ln\left(\frac{5}{2}\right)$		B1 ft
			(1)
3(c)	$2e^x - 5 = -2 \Rightarrow (x) = \ln\left(\frac{3}{2}\right)$		M1A1
	$(x) = \ln\left(\frac{7}{2}\right)$		B1
			(3)
			(10 marks)

A level Mathematics Practice Paper – Trigonometry (part 3) – Mark scheme

Question	Scheme	Marks	
4(a)	$f(x) > 2$	B1	
		(1)	
4(b)	$fg(x) = e^{\ln x} + 2, = x + 2$	M1 A1	
		(2)	
4(c)	$e^{2x+3} + 2 = 6 \Rightarrow e^{2x+3} = 4$	M1 A1	
	$\Rightarrow 2x + 3 = \ln 4$		
	$\Rightarrow x = \frac{\ln 4 - 3}{2} \quad \text{or} \quad \ln 2 - \frac{3}{2}$	M1 A1	
		(4)	
4(d)	Let $y = e^x + 2 \Rightarrow y - 2 = e^x \Rightarrow \ln(y - 2) = x$	M1	
	$f^{-1}(x) = \ln(x - 2), \quad x > 2.$	A1 B1ft	
		(3)	
4(e)		Shape for $f(x)$	B1
		$(0, 3)$	B1
		Shape for $f^{-1}(x)$	B1
		$(3, 0)$	B1
		(14 marks)	

A level Mathematics Practice Paper – Trigonometry (part 3) – Mark scheme

Question	Scheme	Marks
5(a)	$f(x) > k^2$	B1
		(1)
5(b)	$y = e^{2x} + k^2 \Rightarrow e^{2x} = y - k^2$	M1
	$\Rightarrow x = \frac{1}{2} \ln(y - k^2)$	dM1
	$\Rightarrow f^{-1}(x) = \frac{1}{2} \ln(x - k^2), \quad x > k^2$	A1
		(3)
5(c)	$\ln 2x + \ln 2x^2 + \ln 2x^3 = 6$	M1
	$\Rightarrow \ln 8x^6 = 6$	M1
	$\Rightarrow 8x^6 = e^6 \Rightarrow x = ..$	M1
	$\Rightarrow x = \left(\frac{e}{\sqrt[6]{8}} \right) = \frac{e}{\sqrt{2}} \quad (\text{Ignore any reference to } -\frac{e}{\sqrt{2}})$	A1
		(4)
5(d)	$fg(x) = e^{2 \times \ln(2x)} + k^2$	M1
	$\Rightarrow fg(x) = (2x)^2 + k^2 = 4x^2 + k^2$	A1
		(2)
5(e)	$fg(x) = 2k^2 \Rightarrow 4x^2 + k^2 = 2k^2$	
	$\Rightarrow 4x^2 = k^2 \Rightarrow x = ..$	M1
	$\Rightarrow x = \frac{k}{2} \text{ only}$	A1
		(2)
		(12 marks)

A level Mathematics Practice Paper – Trigonometry (part 3) – Mark scheme

Question	Scheme	Marks
6(a)	20 (mm ²)	B1
		(1)
6(b)	'40' = 20 e ^{1.5t} ⇒ e ^{1.5t} = c	M1
	e ^{1.5t} = $\frac{40}{20} = 2$	A1
	Correct order 1.5t = ln'2' ⇒ t = $\frac{\ln c}{1.5}$	M1
	t = $\frac{\ln 2}{1.5}$ = (awrt 0.46)	A1
	12.28 or 28 (minutes}	A1
		(5)
		(6 marks)
7(a)	(θ =) 20	B1
		(1)
7(b)	Sub t = 40, θ = 70 ⇒ 70 = 120 – 100 e ^{-40λ}	
	⇒ e ^{-40λ} = 0.5	M1A1
	⇒ λ = $\frac{\ln 2}{40}$	M1A1
		(4)
7(c)	θ = 100 ⇒ T = $\frac{\ln 0.2}{- \text{their '}\lambda\text{'}}$	M1
	T = awrt 93	A1
		(2)
		(7 marks)

A level Mathematics Practice Paper – Trigonometry (part 3) – Mark scheme

Question	Scheme	Marks
8(a)	$S_{\infty} = \frac{20}{1 - \frac{7}{8}} ; = 160$	M1A1
		(2)
8(b)	$S_{12} = \frac{20(1 - (\frac{7}{8})^{12})}{1 - \frac{7}{8}} ; = 127.77324...$	M1A1
		(2)
8(c)	$160 - \frac{20(1 - (\frac{7}{8})^N)}{1 - \frac{7}{8}} < 0.5$	M1
	$160\left(\frac{7}{8}\right)^N < (0.5) \text{ or } \left(\frac{7}{8}\right)^N < \left(\frac{0.5}{160}\right)$	dM1
	$N \log\left(\frac{7}{8}\right) < \log\left(\frac{0.5}{160}\right)$	M1
	$N > \frac{\log\left(\frac{0.5}{160}\right)}{\log\left(\frac{7}{8}\right)} = 43.19823... \Rightarrow N = 44$	A1 cso
		(4)
		(8 marks)
9(a)	$a + ar = 34 \text{ or } \frac{a(1-r^2)}{(1-r)} = 34 \text{ or } \frac{a(r^2-1)}{(r-1)} = 34 ; \quad \frac{a}{1-r} = 162$	B1 B1
	Eliminate a to give $(1+r)(1-r) = \frac{17}{81} \text{ or } 1-r^2 = \frac{34}{162} .. \quad (\text{not a cubic})$	aM1
	(and so $r^2 = \frac{64}{81}$ and) $r = \frac{8}{9}$ only	aA1
		(4)
9(b)	Substitute their $r = \frac{8}{9} \quad (0 < r < 1)$ to give $a =$	bM1
	$a = 18$	bA1
		(2)
9(c)	$\frac{42(1 - \frac{6}{7}^n)}{1 - \frac{6}{7}} > 290 \quad (\text{For trial and improvement approach see notes below})$	M1
	to obtain So $(\frac{6}{7})^n < (\frac{4}{294})$ or equivalent e.g. $(\frac{7}{6})^n > (\frac{294}{4})$ or $(\frac{6}{7})^n < (\frac{2}{147})$	A1
	So $n > \frac{\log\left(\frac{4}{294}\right)}{\log\left(\frac{6}{7}\right)} \text{ or } \log_{\frac{6}{7}}\left(\frac{4}{294}\right)$ or equivalent but must be log of positive quantity	M1
	(i.e. $n > 27.9$) so $n = 28$	A1
		(4)
		(10 marks)

A level Mathematics Practice Paper – Trigonometry (part 3) – Mark scheme

Question	Scheme	Marks
10(a)	(£) 19500	B1
		(1)
10(b)	$9500 = 17000e^{-0.25t} + 2000e^{-0.5t} + 500$	
	$17e^{-0.25t} + 2e^{-0.5t} = 9$	
	$(\times e^{0.5t}) \Rightarrow 17e^{0.25t} + 2 = 9e^{0.5t}$	
	$0 = 9e^{0.5t} - 17e^{0.25t} - 2$	M1
	$0 = (9e^{0.25t} + 1)(e^{0.25t} - 2)$	M1
	$e^{0.25t} = 2$	A1
	$t = 4 \ln(2) \text{ oe}$	A1
		(4)
10(c)	$\left(\frac{dV}{dt}\right) = -4250e^{-0.25t} - 1000e^{-0.5t}$	M1A1
	When $t = 8$ Decrease = 593 (£/year)	M1A1
		(4)
		(9 marks)
11(a)	Subs $D = 15$ and $t = 4$ $x = 15e^{-0.2 \times 4} = 6.740 \text{ (mg)}$	M1 A1
		(2)
11(b)	$15e^{-0.2 \times 7} + 15e^{-0.2 \times 2} = 13.754 \text{ (mg)}$	M1 A1*
		(2)
11(c)	$15e^{-0.2 \times T} + 15e^{-0.2 \times (T+5)} = 7.5$	M1
	$15e^{-0.2 \times T} + 15e^{-0.2 \times T} e^{-1} = 7.5$	
	$15e^{-0.2 \times T} (1 + e^{-1}) = 7.5 \Rightarrow e^{-0.2 \times T} = \frac{7.5}{15(1 + e^{-1})}$	dM1
	$T = -5 \ln \left(\frac{7.5}{15(1 + e^{-1})} \right) = 5 \ln \left(2 + \frac{2}{e} \right)$	A1 A1
		(4)
		(8 marks)

A level Mathematics Practice Paper – Trigonometry (part 3) – Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	C3 June 2014	2	6.3	Solving exponential and ln equations	1.1b
2	C3 2013	6	6.3, 6.4	Exponentials and logarithms	1.1b, 3.1a
3	C3 2015	2	6.1, 2.9, 6.5	The exponential graph and modulus leading to solving equations	1.1b, 2.1, 2.2a
4	C3 2012	6	2.8, 6.1, 6.3	Composite and inverse functions, Exponentials and logarithms	1.1b, 2.5
5	C3 June 2014R	6	2.8, 6.1, 6.3, 6.4	Functions, Logarithms	1.1b, 3.1a
6	C3 Jan 2012	3	6.3, 6.7	Exponentials and logarithms	1.1b, 3.2a, 3.4
7	C3 2015	4	6.1, 6.3	Exponential functions and related equations	1.1b, 3.4
8	C2 2014	6	4.5, 6.3 and 6.5	Geometric series, Exponentials and logarithms	1.1b, 2.2a and 2.4
9	C2 2015	5	4.5, 6.5, 2.4	Geometric series, Exponentials and logarithms	1.1b
10	C3 Jan 2013	8	6.2, 6.3, 6.7	Exponentials and logarithms	1.1b, 3.1a, 3.2a, 3.4
11	C3 2016	9	6.1, 6.3, 6.5, 6.7	Exponential problem	1.1b, 2.1, 3.1b, 3.4