① a) The students did not find all the values of $\cos 2x$

b) $2x = 150$ and $2x = 360 - 150 = 210^\circ$

$x = 75^\circ$ and $x = 105^\circ$

② $|a| = \sqrt{4^2 + (-7)^2} = \sqrt{65}$

$\frac{1}{\sqrt{65}}(4i - 7j)$

③ $\frac{(6\sqrt{3} - 4)(8 + \sqrt{3})}{(8 - \sqrt{3})(8 + \sqrt{3})} = \frac{48\sqrt{3} + 18 - 32 - 4\sqrt{3}}{64 + 8\sqrt{3} - 8\sqrt{3} - 3}$

$= \frac{44\sqrt{3} - 14}{61}$

$= \frac{44}{61}\sqrt{3} - \frac{14}{61}$

$p = \frac{44}{61}$

$q = \frac{14}{61}$

④ $1 + 3x^2 + x^3 < (1+x)^3$

a)

$$1 + 3x^2 + x^3 < 1 + 3x + 3x^2 + x^3$$

$$0 < 3x$$

$$\therefore \quad x > 0$$

b) Let $x = 1$

$$1 + 3(-1)^2 + (-1)^3 = 1 + 3 - 1 = 3$$

$$1 + 3(-1) + 3(-1)^2 + (-1)^3 = 1 - 3 + 3 - 1 = 0$$

$$3 \cancel{<} 0$$

5

$$h'(x) = 15x^1 x^{1/2} - 40x^{-1/2}$$

$$h'(x) = 15x^{3/2} - 40x^{-1/2}$$

+1 to the power
÷ by new power

$$h(x) = \frac{15x^{5/2}}{5/2} - \frac{40x^{1/2}}{1/2} + c$$

$$h(x) = 6x^{5/2} - 80x^{1/2} + c$$

$$(4, 19)$$

$$19 = 6(4)^{5/2} - 80(4)^{1/2} + c$$

$$c = -13$$

$$h(x) = 6x^{5/2} - 80x^{1/2} - 13$$

(6)

$$8 - 7\cos x = 6\sin^2 x$$

$$8 - 7\cos x = 6(1 - \cos^2 x)$$

$$8 - 7\cos x = 6 - 6\cos^2 x$$

$$6\cos^2 x - 7\cos x + 2 = 0$$

$$x = \frac{2}{3} \text{ or } \frac{1}{2}$$

$\cos x = \frac{2}{3}$ $\qquad$ $\cos x = \frac{1}{2}$

$x = 48.2^\circ$ $\qquad$ $x = 60^\circ$

$x = 311.8^\circ$ $\qquad$ $x = 300^\circ$

7

a) $(1+3x)^8$

$a = 1$

$b = 3x$

$n = 8$

$$a^8 \qquad (1)^8$$

$$+\ 8a^7b \qquad +\ 8(1)^7(3x)$$

$$+\ 28a^6b^2 \qquad +\ 28(1)^6(3x)^2$$

$$+\ 56a^5b^3 \qquad +\ 56(1)^5(3x)^3$$

$$= 1 + 24x + 252x^2 + 1512x^3$$

b) $1.03^8 = (1+3x)^8$

$1.03 = 1 + 3x$

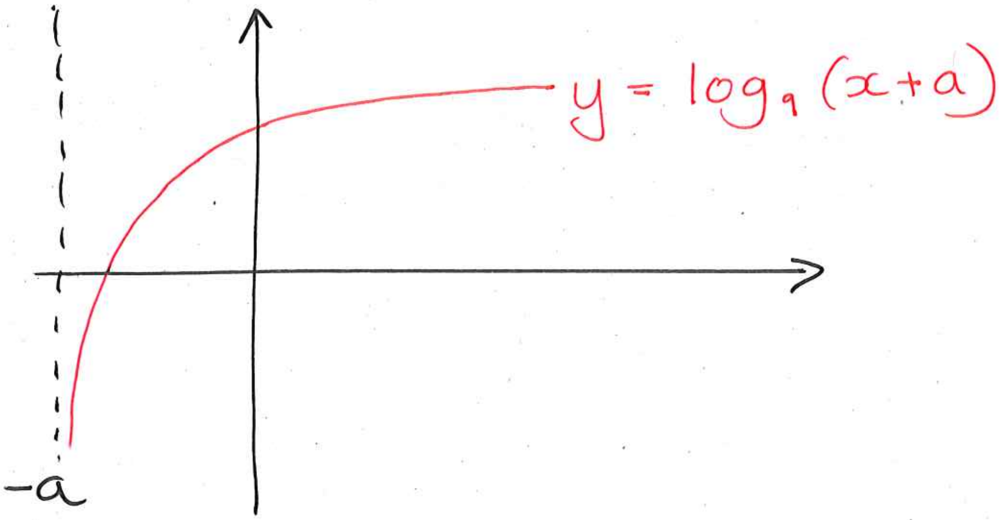
$x = 0.01$

$$1 + 24(0.01) + 252(0.01)^2 + 1512(0.01)^3$$

$$= 1 + 0.24 + 0.0252 + 0.001512$$

$$= 1.2667 \text{ (5sf)}$$

⑧



$y = 0$

$0 = \log_9(x+a)$

$9^0 = x + a$

$1 = x + a$

$x = 1 - a$

$(1-a, 0)$

$x = 0$

$y = \log_9(a)$

$(0, \log_9 a)$

b) $y = \log_9(x+a)^2$

$y = 2\log_9(x+a)$

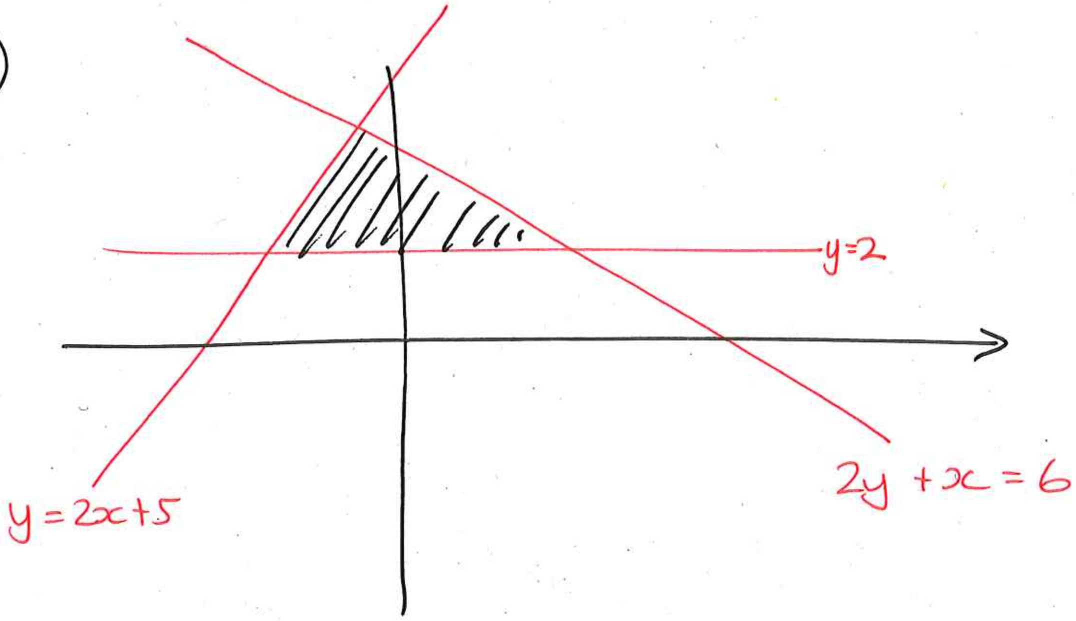
if $f(x) = \log_9(x+a)$

then $2f(x) = 2\log(x+a)$

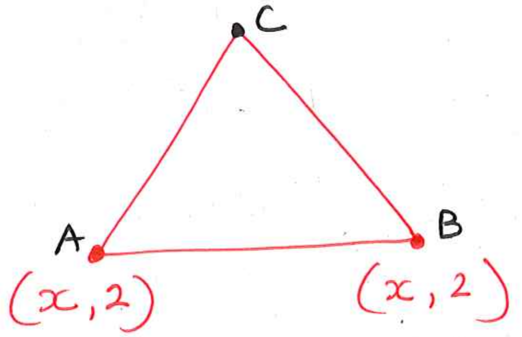
$\therefore$ $(x, 2y)$ stretch parallel to y-axis

9

a)



b)



$y = 2$

$2 = 2x + 5$

$x = -\frac{3}{2}$

$(-\frac{3}{2}, 2)$

$y = 2$

$2(2) + x = 6$

$x = 2$

$(2, 2)$

$-3/2$ $\longleftrightarrow$ 2

3.5

point C

$$2(2x+5) + x = 6$$

$$4x + 10 + x = 6$$

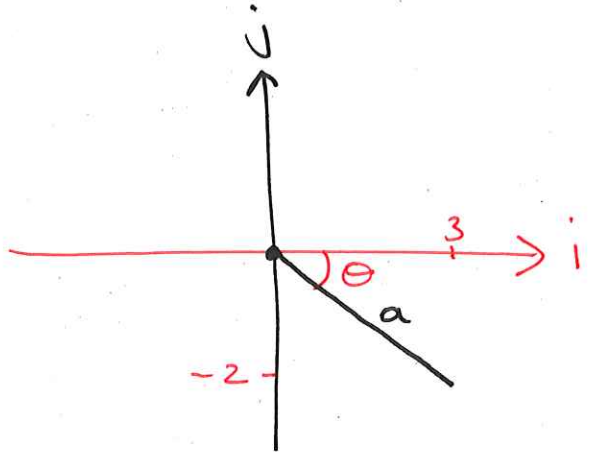
$$5x = -4$$

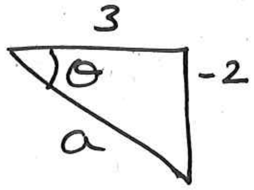
$$x = -\frac{4}{5} \quad \therefore y = 3.4$$

$$\text{Area} = \frac{1}{2} \times 3.5 \times (3.4 - 2) = 2.45$$

(10)

a)





$\tan\theta = \frac{-2}{3}$

$\theta = -33.7°$

b) $F = ma$

$(8+p)i + (-10+q)j = 6(3i - 2j)$

$(8+p)i + (-10+q)j = 18i - 12j$

$8 + p = 18 \quad \therefore p = 10$

$-10 + q = -12 \quad q = -2$

c) $R = (8+10)i + (-10+-2)j$

$R = 18i + -12j$

$|R| = \sqrt{18^2 + (-12)^2} = 6\sqrt{13}$

11) $f(x) = x^3 - 7x^2 - 24x + 18$

$f'(x) = 3x^2 - 14x - 24$

$y = 3x^2 - 14x - 24$

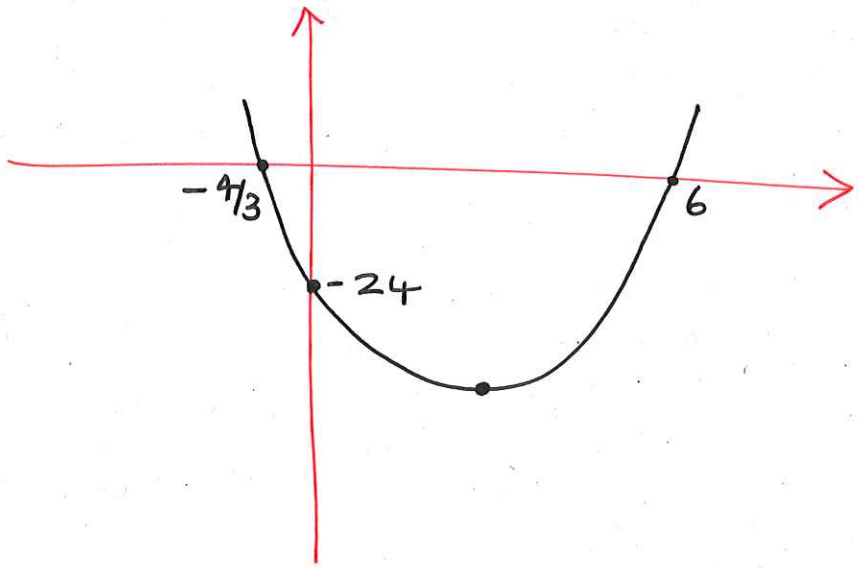
$y = (3x+4)(x-6)$

when $y = 0$

$x = -\frac{4}{3}$ and 6

when $x = 0$

$y = -24$



min point

$y = 3x^2 - 14x - 24$

$\frac{dy}{dx} = 6x - 14$

$0 = 6x - 14$

$x = \frac{14}{6}$

$\therefore y = \frac{-121}{3}$

(12)

a) $x + 6 = x^2 - 8x + 20$

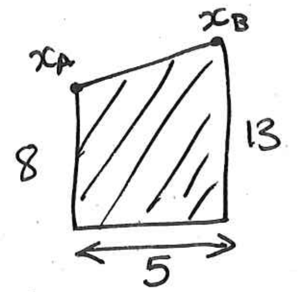
$$0 = x^2 - 9x + 14$$

$$x = 2 \text{ and } 7$$

$$y = 8 \text{ and } 13$$

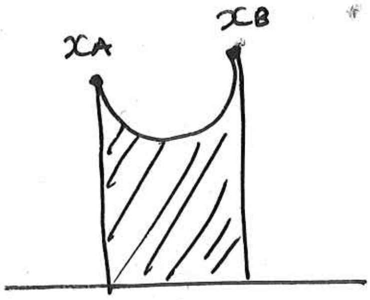
$$A(2, 8) \quad B(7, 13)$$

b)



$$\frac{1}{2}(8+13)5 = 52.2$$

c)



$$\int_2^7 x^2 - 8x + 20$$

$$\left[\frac{x^3}{3} - \frac{8x^2}{2} + 20x\right]_2^7 = \frac{95}{3}$$

d) $52.2 - \frac{95}{3} = 20.8$

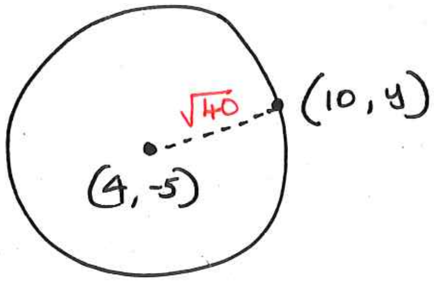
(13) a) $(x - -4)^2 - 16 + (y+5)^2 - 25 + 1 = 0$

$(x+4)^2 + (y+5)^2 = 40$

centre $(4, -5)$

radius $\sqrt{40}$

b)



$\sqrt{40}^2 = (10-4)^2 + (y - -5)^2$

$40 = 36 + (y+5)^2$

$4 = (y+5)^2$

$2 = y+5$ or $-2 = y+5$

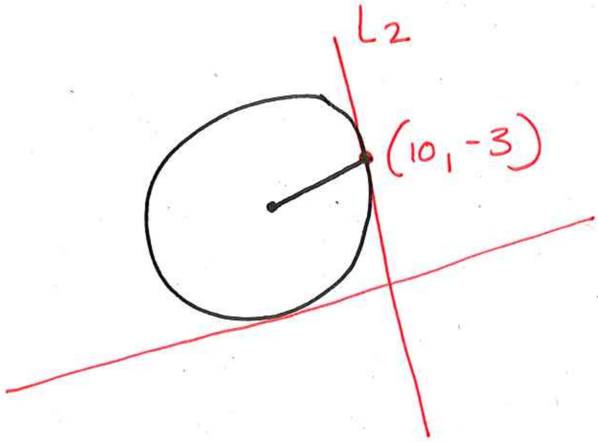
$y = -3$ $\quad y = -7$

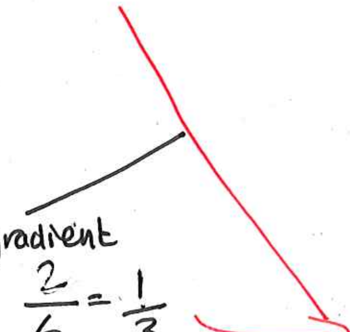
gradient +

so $y = -3$

13

c)





gradient

$\frac{2}{6} = \frac{1}{3}$

gradient $= -3$

$y = -3x + c$

$(10, -3)$

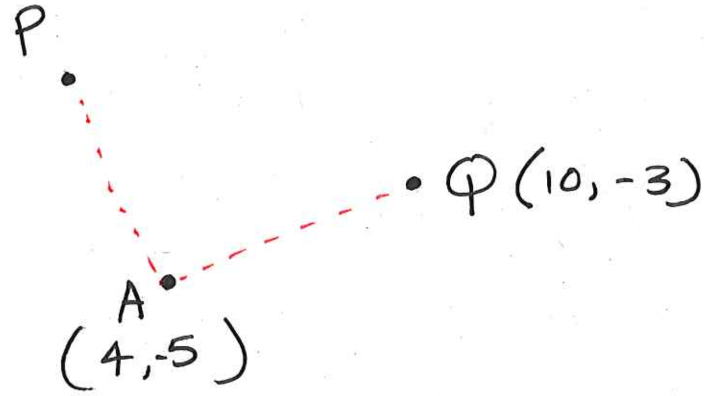
$-3 = -3(10) + c$

$c = 27$

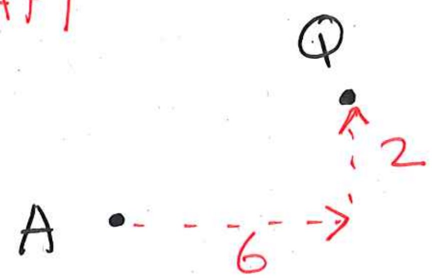
$y = -3x + 27$

13

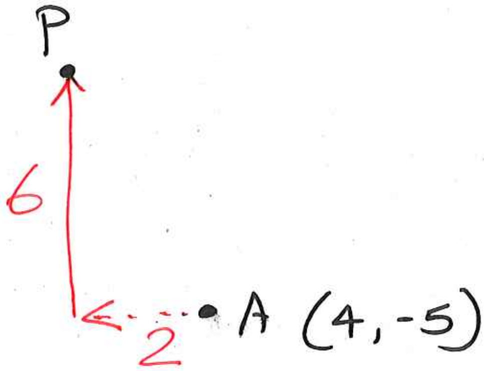
d)



$|AQ| = |AP|$



hence



$\therefore$ P $(2, 1)$

L_1 gradient $= \frac{1}{3}$

$$y = \frac{1}{3}x + c$$

$(2, 1)$

$$1 = \frac{1}{3}(2) + c$$

$$c = \frac{1}{3}$$

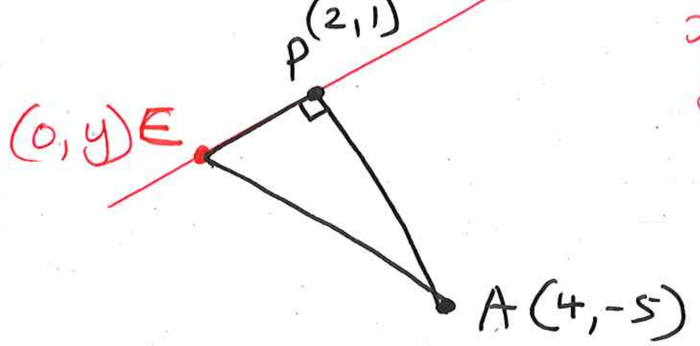
$$y = \frac{1}{3}x + \frac{1}{3}$$

(13)

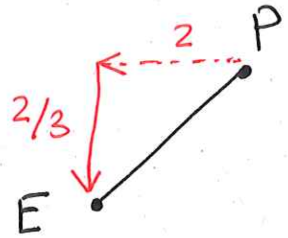
e)

$L_1 \; y = \frac{1}{3}x + \frac{1}{3}$

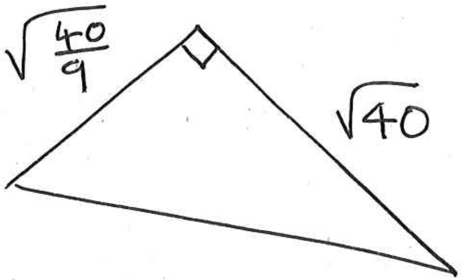
$x = 0$
$y = 1/3$



$PA = \text{radius} = \sqrt{40}$



$\sqrt{2^2 + (2/3)^2} = \sqrt{\frac{40}{9}}$



$\text{Area} = \frac{1}{2} \times \sqrt{\frac{40}{9}} \times \sqrt{40}$

$= \frac{20}{3}$