

A level Mathematics Practice Paper – Binomial expansion – Mark scheme

Question	Scheme	Marks
1(a)	$\left\{ \frac{1}{\sqrt{(9-10x)}} = \right\} (9-10x)^{-\frac{1}{2}}$	B1
	$= (9)^{-\frac{1}{2}} \left(1 - \frac{10x}{9} \right)^{-\frac{1}{2}} = \frac{1}{3} \left(1 - \frac{10x}{9} \right)^{-\frac{1}{2}}$	<u>B1</u>
	$= \left\{ \frac{1}{3} \right\} \left[1 + \left(-\frac{1}{2} \right) (kx) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} (kx)^2 + \dots \right]$	M1
	$= \left\{ \frac{1}{3} \right\} \left[1 + \left(-\frac{1}{2} \right) \left(\frac{-10x}{9} \right) + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2!} \left(\frac{-10x}{9} \right)^2 + \dots \right]$	
	$= \frac{1}{3} \left[1 + \frac{5}{9}x + \frac{25}{54}x^2 + \dots \right]$	
	$= \frac{1}{3} + \frac{5}{27}x; + \frac{25}{162}x^2 + \dots$	A1 A1
		(5)
1(b)	$\frac{3+x}{\sqrt{(9-10x)}} = (3+x)(9-10x)^{-\frac{1}{2}}$	
	$= (3+x) \left(\frac{1}{3} + \frac{5}{27}x + \left\{ \frac{25}{162}x^2 + \dots \right\} \right)$	M1
	$= 1 + \frac{5}{9}x + \frac{25}{54}x^2 + \frac{1}{3}x + \frac{5}{27}x^2 + \dots$	M1
	$= 1 + \frac{8}{9}x + \frac{35}{54}x^2 + \dots$	A1
		(3)
		(8 marks)

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Question	Scheme		Marks
2(a)	$\left\{ (1+kx)^{-4} = 1 + (-4)(kx) + \frac{(-4)(-4-1)}{2!}(kx)^2 + \dots \right\}$		
	Either $(-4)k = -6$ or $(1+kx)^{-4} = 1 + (-4)(kx)$		M1
	leading to $k = \frac{3}{2}$ or 1.5 or $\frac{6}{4}$		A1
			(2)
2(b)	$\frac{(-4)(-5)}{2!}(k)^2$	Either $\frac{(-4)(-5)}{2!}$ or $(k)^2$ or $(kx)^2$	M1
		Either $\frac{(-4)(-5)}{2!}(k)^2$ or $\frac{(-4)(-5)}{2!}(kx)^2$	M1
	$\left\{ A = \frac{(-4)(-5)}{2!} \left(\frac{3}{2} \right)^2 \right\} \Rightarrow A = \frac{45}{2}$	$\frac{45}{2}$ or 22.5	A1
			(3)
			(5 marks)
3	$f(x) = (\dots + \dots)^{-\frac{1}{2}}$		M1
	$= 9^{-\frac{1}{2}} (\dots + \dots)^{\dots}$ $3^{-1}, \frac{1}{3}$ or $\frac{1}{9^{\frac{1}{2}}}$		B1
	$(1+kx^2)^n = 1 + nkx^2 + \dots$	n not a natural number, $k \neq 1$	M1
	$(1+kx^2)^{-\frac{1}{2}} = \dots + \frac{(-\frac{1}{2})(-\frac{3}{2})}{2}(kx^2)^2$	ft their $k \neq 1$	A1 ft
	$\left(1 + \frac{4}{9}x^2 \right)^{-\frac{1}{2}} = 1 - \frac{2}{9}x^2 + \frac{2}{27}x^4$		A1
	$f(x) = \frac{1}{3} - \frac{2}{27}x^2 + \frac{2}{81}x^4$		A1
			(6 marks)

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Question	Scheme	Marks
4(a)	$f(x) = \dots (\dots - \dots x)^{-\frac{1}{2}}$	M1
	$= 6 \times 9^{-\frac{1}{2}} (\dots)$ $\frac{6}{9^{\frac{1}{2}}}, \frac{6}{3}, 2$ or equivalent	B1
	$= \dots \left(1 + \left(-\frac{1}{2}\right)(kx); + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)}{2}(kx)^2 + \frac{\left(-\frac{1}{2}\right)\left(-\frac{3}{2}\right)\left(-\frac{5}{2}\right)}{3!}(kx)^3 + \dots \right)$	M1 A1ft
	$= 2 \left(1 + \frac{2}{9}x + \dots \right)$ or $2 + \frac{4}{9}x$	A1
	$= 2 + \frac{4}{9}x + \frac{4}{27}x^2 + \frac{40}{729}x^3 + \dots$	A1
		(6)
4(b)	$g(x) = 2 - \frac{4}{9}x + \frac{4}{27}x^2 - \frac{40}{729}x^3 + \dots$	B1ft
		(1)
4(c)	$h(x) = 2 + \frac{4}{9}(2x) + \frac{4}{27}(2x)^2 + \frac{40}{729}(2x)^3 + \dots$	M1 A1
	$\left(= 2 + \frac{8}{9}x + \frac{16}{27}x^2 + \frac{320}{729}x^3 + \dots \right)$	
		(2)
		(9 marks)

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Question	Scheme	Marks
5(a)	$\frac{1}{(2-5x)^2} = (2-5x)^{-2} = \underline{(2)^{-2}} \left(1 - \frac{5x}{2}\right)^{-2} = \underline{\frac{1}{4}} \left(1 - \frac{5x}{2}\right)^{-2}$	
	$\underline{(2)^{-2}} \text{ or } \underline{\frac{1}{4}}$	B1
	$= \left\{ \frac{1}{4} \right\} \left[1 + (-2)(**x) + \frac{(-2)(-3)}{2!} (**x)^2 + \dots \right]$	M1 A1ft
	$= \left\{ \frac{1}{4} \right\} \left[1 + (-2) \left(-\frac{5x}{2} \right) + \frac{(-2)(-3)}{2!} \left(-\frac{5x}{2} \right)^2 + \dots \right]$	
	$= \frac{1}{4} \left[1 + 5x; + \frac{75}{4} x^2 + \dots \right]$	
	$= \frac{1}{4} + \frac{5}{4} x; + \frac{75}{16} x^2 + \dots$	A1 A1
		(5)
5(b)	$\left\{ \frac{2+kx}{(2-5x)^2} \right\} = (2+kx) \left(\frac{1}{4} + \frac{5}{4} x + \left\{ \frac{75}{16} x^2 + \dots \right\} \right)$	M1
	<i>Can be implied by later work even in part (c).</i>	
	x terms: $\frac{2(5x)}{4} + \frac{kx}{4} = \frac{7x}{4}$	
	giving, $10 + k = 7 \Rightarrow \underline{k = -3}$	A1
		(2)
5(c)	$x^2 \text{ terms: } \frac{150x^2}{16} + \frac{5kx^2}{4}$	M1
	So, $A = \frac{75}{8} + \frac{5(-3)}{4} = \frac{75}{8} - \frac{15}{4} = \underline{\frac{45}{8}}$	
	$\underline{\frac{45}{8}}$ or $5\frac{5}{8}$ or <u>5.625</u>	A1
		(2)
		(9 marks)

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Question	Scheme	Marks
6(a)	$(2-3x)^{-2} = 2^{-2} \left(1 - \frac{3}{2}x\right)^{-2}$	B1
	$\left(1 - \frac{3}{2}x\right)^{-2} = 1 + (-2)\left(-\frac{3}{2}x\right) + \frac{-2 \cdot -3}{1 \cdot 2} \left(-\frac{3}{2}x\right)^2 + \frac{-2 \cdot -3 \cdot -4}{1 \cdot 2 \cdot 3} \left(-\frac{3}{2}x\right)^3 + \dots$	M1 A1
	$= 1 + 3x + \frac{27}{4}x^2 + \frac{27}{2}x^3 + \dots$	
	$(2-3x)^{-2} = \frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2 + \frac{27}{8}x^3 + \dots$	M1 A1
		(5)
6(b)	$f(x) = (a+bx) \left(\frac{1}{4} + \frac{3}{4}x + \frac{27}{16}x^2 + \frac{27}{8}x^3 + \dots \right)$	
	Coefficient of x ; $\frac{3a}{4} + \frac{b}{4} = 0 \quad (3a+b=0)$	M1
	Coefficient of x^2 ; $\frac{27a}{16} + \frac{3b}{4} = \frac{9}{16} \quad (9a+4b=3)$ A1 either correct	M1 A1
	Leading to $a = -1, b = 3$	M1 A1
		(5)
6(c)	Coefficient of x^3 is $\frac{27a}{8} + \frac{27b}{16} = \frac{27}{8} \times (-1) + \frac{27}{16} \times 3$	M1 A1ft
	$= \frac{27}{16}$ cao	A1
		(3)
		(13 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C4 June 2014R	1	4.1	Binomial expansion	1.1b, 2.1
2	C4 June 2014	2	4.1	Binomial expansion	1.1b
3	C4 2011	2	4.1	Binomial expansion	1.1b
4	C4 2012	3	4.1	Binomial expansion	1.1b, 2.2a
5	C4 Jan 2012	3	4.1	Binomial expansion	1.1b, 2.1
6	C4 Jan 2011	5	4.1	Binomial expansion	1.1b, 2.2a