Stationary Point – Section Test (Answers)

1.

$y = x^2 - 3x + 1$

$\frac{dy}{dx} = 2x - 3$

For a decreasing function,

$\frac{dy}{dx} < 0$

$2x - 3 < 0$

$2x < 3$

$x < 1.5$

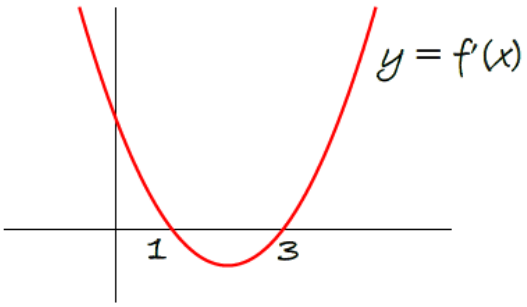
2.

$y = x^3 - 6x^2 + 9x + 4$

$\frac{dy}{dx} = 3x^2 - 12x + 9$

$= 3(x^2 - 4x + 3)$

$= 3(x - 1)(x - 3)$



The function is increasing when $\frac{dy}{dx} > 0$.

This is the case for $x < 1$ and $x > 3$.

3.

$y = x^3 \quad \Rightarrow \frac{dy}{dx} = 3x^2$

$\frac{dy}{dx} = 0$ when $x = 0$, so $\frac{dy}{dx}$ is not positive for all values of x. So function (i) is not an increasing function for all values of x.

$y = x^3 + x \quad \Rightarrow \frac{dy}{dx} = 3x^2 + 1$

Since x^2 is positive for all values of x, $\frac{dy}{dx}$ is positive for all values of x.

So function (ii) is an increasing function for all values of x.

4.

$y = 3x - x^3 + 1$

$\frac{dy}{dx} = 3 - 3x^2$

At stationary points, $\frac{dy}{dx} = 0 \Rightarrow 3 - 3x^2 = 0$

$\Rightarrow x^2 = 1 \Rightarrow x = \pm 1$

x	$x < -1$	$x = -1$	$-1 < x < 1$	$x = 1$	$x > 1$
$\frac{dy}{dx}$	-ve ↘	0 —	+ve ↗	0 —	-ve ↘

There is a minimum point at $x = -1$

There is a maximum point at $x = 1$.

5.

From the working above, there is a minimum point at $x = -1$ and a maximum point at $x = 1$, so the graph must be either Q or S.

When $x = 0$, $y = 1$ so the graph passes through $(0, 1)$, so the correct graph is Q.

6.

$y = (x + 2)(x + 3)$

$= x^2 + 5x + 6$

$\frac{dy}{dx} = 2x + 5$

$\frac{d^2y}{dx^2} = 2$

7.

$y = 3x^4 - 4x^3 + 1 \Rightarrow \frac{dy}{dx} = 12x^3 - 12x^2$

At stationary points, $12x^3 - 12x^2 = 0$

$x^3 - x^2 = 0$

$x^2(x - 1) = 0$

$x = 0$ or 1

When $x = 0$, $y = 1$

When $x = 1$, $y = 0$

x	$x < 0$	$x = 0$	$0 < x < 1$	$x = 1$	$x > 1$
$\frac{dy}{dx}$	-ve ↘	0 —	-ve ↘	0 —	+ve ↗

So, there is a point of inflection at $x = 0$, and a minimum point at $x = 1$.

There is no maximum point.

8.

From the working above, there is a minimum point at $x = 1$.

9.

$$y = 3x^4 - 4x^3 + 1$$

$$\frac{dy}{dx} = 12x^3 - 12x^2$$

$$\frac{d^2y}{dx^2} = 36x^2 - 24x$$

10.

When $x = 0$, $y = 1$

When $x = 1$, $y = 0$

From the working above, (0, 1) is a point of inflection, and (1, 0) is a minimum point.

This is graph R.