

Write your name here	
Surname	Other names
Pearson	Centre Number
Edexcel GCE	Candidate Number
A level Mathematics Practice Paper Pure Mathematics - Integration	
You must have: Mathematical Formulae and Statistical Tables (Pink)	Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet ‘Mathematical Formulae and Statistical Tables’ is provided.
- There are 12 questions in this question paper. The total mark for this paper is 119.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.
$$f(x) = \frac{1}{x(3x-1)^2} = \frac{A}{x} + \frac{B}{(3x-1)} + \frac{C}{(3x-1)^2}.$$

(a) Find the values of the constants A , B and C .

(4)

(b) (i) Hence find $\int f(x) \, dx$.

(ii) Find $\int_1^2 f(x) \, dx$, leaving your answer in the form $a + \ln b$, where a and b are constants.

(6)

(Total 10 marks)

2. Use integration to find the exact value of $\int_0^{\frac{\pi}{2}} x \sin 2x \, dx$.

(Total 6 marks)

3. (a) Use integration to find

$$\int \frac{1}{x^3} \ln x \, dx.$$

(5)

(b) Hence calculate

$$\int_1^2 \frac{1}{x^3} \ln x \, dx.$$

(2)

(Total 7 marks)

4. (a) Use integration by parts to find $\int x \sin 3x \, dx$.

(3)

(b) Using your answer to part (a), find $\int x^2 \cos 3x \, dx$.

(3)

(Total 6 marks)

5.

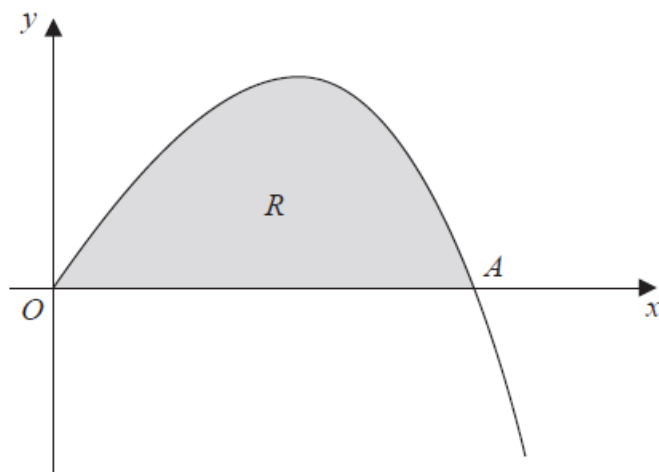


Figure 1

Figure 1 shows a sketch of part of the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$.

The curve meets the x -axis at the origin O and cuts the x -axis at the point A .

(a) Find, in terms of $\ln 2$, the x coordinate of the point A . (2)

(b) Find $\int xe^{\frac{1}{2}x} dx$. (3)

The finite region R , shown shaded in Figure 1, is bounded by the x -axis and the curve with equation $y = 4x - xe^{\frac{1}{2}x}$, $x \geq 0$.

(c) Find, by integration, the exact value for the area of R .
Give your answer in terms of $\ln 2$. (3)

(Total 8 marks)

6.

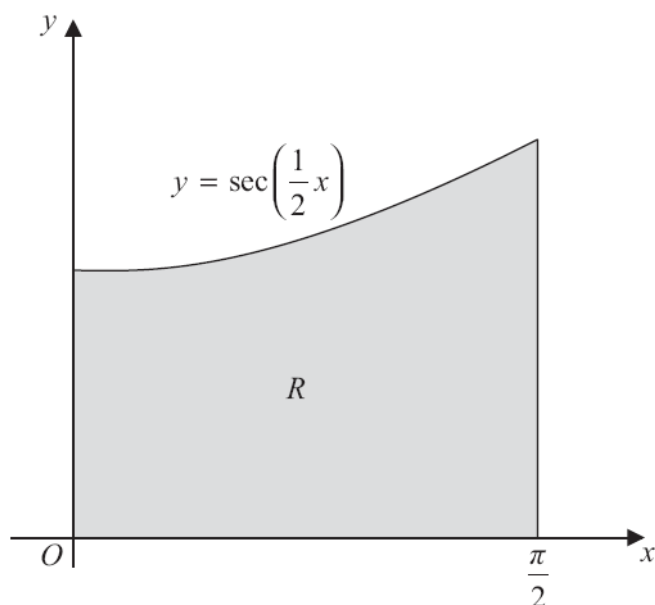


Figure 2

Figure 2 shows the finite region R bounded by the x -axis, the y -axis, the line $x = \frac{\pi}{2}$ and the curve with equation

$$y = \sec\left(\frac{1}{2}x\right), \quad 0 \leq x \leq \frac{\pi}{2}$$

The table shows corresponding values of x and y for $y = \sec\left(\frac{1}{2}x\right)$.

x	0	$\frac{\pi}{6}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
y	1	1.035276		1.414214

(a) Complete the table above giving the missing value of y to 6 decimal places. (1)

(b) Using the trapezium rule, with all of the values of y from the completed table, find an approximation for the area of R , giving your answer to 4 decimal places. (3)

Region R is rotated through 2π radians about the x -axis.

(c) Use calculus to find the exact volume of the solid formed. (4)

(Total 8 marks)

7. (a) Express $\frac{5}{(x-1)(3x+2)}$ in partial fractions. (3)

(b) Hence find $\int \frac{5}{(x-1)(3x+2)} dx$, where $x > 1$. (3)

- (c) Find the particular solution of the differential equation

$$(x-1)(3x+2) \frac{dy}{dx} = 5y, \quad x > 1,$$

for which $y = 8$ at $x = 2$. Give your answer in the form $y = f(x)$.

(6)

(Total 12 marks)

8. (i) Given that $y > 0$, find

$$\int \frac{3y-4}{y(3y+2)} dy.$$

(6)

- (ii) (a) Use the substitution $x = 4\sin^2\theta$ to show that

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx = \lambda \int_0^{\frac{\pi}{3}} \sin^2\theta d\theta,$$

where λ is a constant to be determined.

(5)

- (b) Hence use integration to find

$$\int_0^3 \sqrt{\left(\frac{x}{4-x}\right)} dx,$$

giving your answer in the form $a\pi + b$, where a and b are exact constants.

(4)

(Total 15 marks)

9.

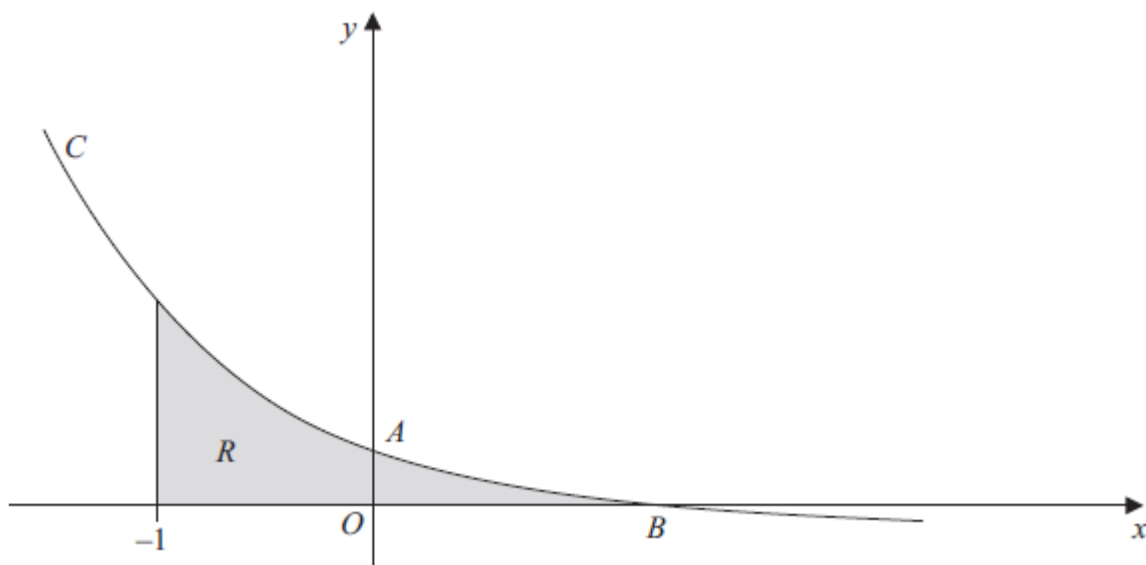


Figure 3

Figure 3 shows a sketch of part of the curve C with parametric equations

$$x = 1 - \frac{1}{2}t, \quad y = 2^t - 1.$$

The curve crosses the y -axis at the point A and crosses the x -axis at the point B .

- (a) Show that A has coordinates $(0, 3)$. (2)
- (b) Find the x -coordinate of the point B . (2)
- (c) Find an equation of the normal to C at the point A . (5)

The region R , as shown shaded in Figure 3, is bounded by the curve C , the line $x = -1$ and the x -axis.

- (d) Use integration to find the exact area of R . (6)

(Total 15 marks)

10.

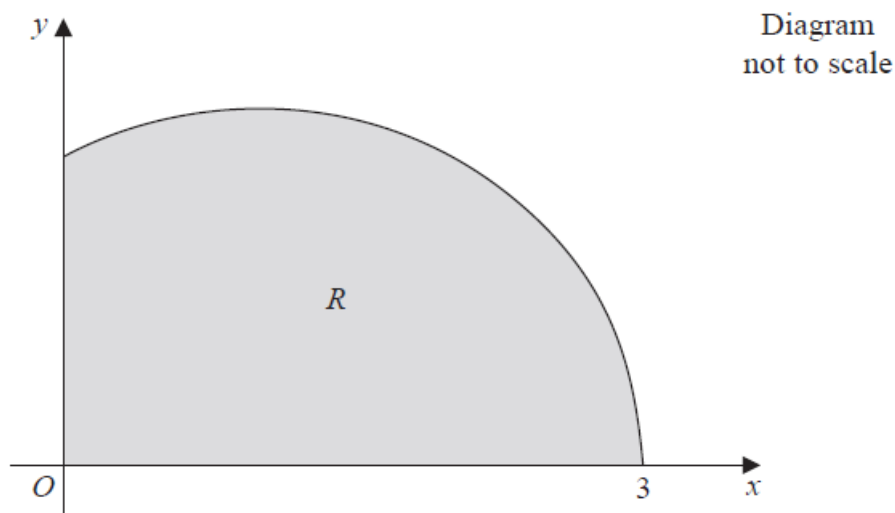


Figure 4

Figure 4 shows a sketch of the curve with equation $y = \sqrt{(3-x)(x+1)}$, $0 \leq x \leq 3$.

The finite region R , shown shaded in Figure 4, is bounded by the curve, the x -axis, and the y -axis.

(a) Use the substitution $x = 1 + 2 \sin \theta$ to show that

$$\int_0^3 \sqrt{(3-x)(x+1)} \, dx = k \int_{-\frac{\pi}{6}}^{\frac{\pi}{2}} \cos^2 \theta \, d\theta,$$

where k is a constant to be determined.

(5)

(b) Hence find, by integration, the exact area of R .

(3)

(Total 8 marks)

11.

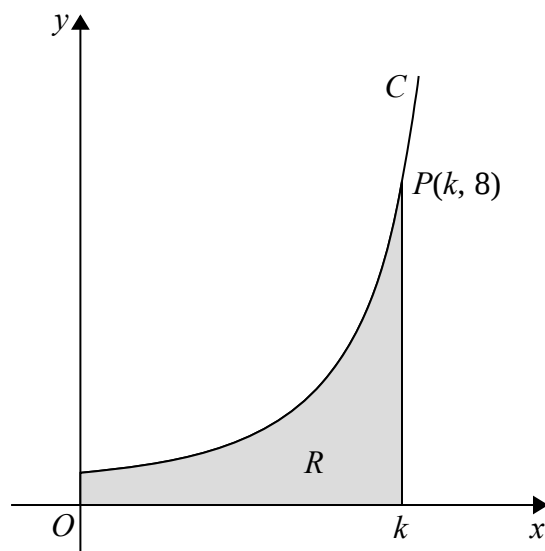


Diagram not
drawn to scale

Figure 5

Figure 5 shows a sketch of part of the curve C with parametric equations

$$x = 3\theta \sin \theta, \quad y = \sec^3 \theta, \quad 0 \leq \theta < \frac{\pi}{2}$$

The point $P(k, 8)$ lies on C , where k is a constant.

(a) Find the exact value of k .

(2)

The finite region R , shown shaded in Figure 5, is bounded by the curve C , the y -axis, the x -axis and the line with equation $x = k$.

(b) Show that the area of R can be expressed in the form

$$\lambda \int_a^b (q \sec^2 q + \tan q \sec^2 q) dq$$

where λ , a and b are constants to be determined.

(4)

(c) Hence use integration to find the exact value of the area of R .

(6)

(Total 12 marks)

12. a) Express $\frac{1}{P(5-P)}$ in partial fractions.

(3)

A team of conservationists is studying the population of meerkats on a nature reserve. The population is modelled by the differential equation

$$\frac{dP}{dt} = \frac{1}{15}P(5-P), \quad t \geq 0,$$

where P , in thousands, is the population of meerkats and t is the time measured in years since the study began.

Given that when $t = 0$, $P = 1$,

- (b) solve the differential equation, giving your answer in the form,

$$P = \frac{a}{b + ce^{\frac{1}{3}t}}$$

where a , b and c are integers.

(8)

- (c) Hence show that the population cannot exceed 5000.

(1)

(Total 12 marks)

TOTAL FOR PAPER: 119 MARKS