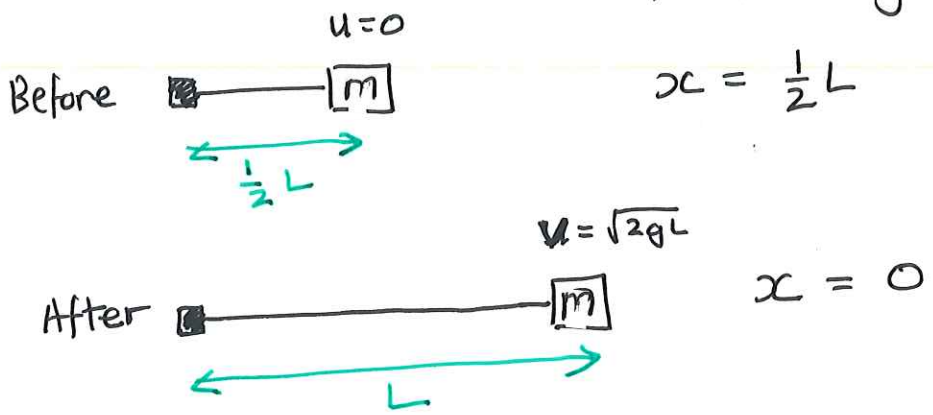
Hooke's Law (Dynamic) + Elastic Energy

Q1)

(a)



Before

Work = 0

GPE = 0

$EPE = \frac{\lambda x^2}{2L}$

KE = 0

After

work = 0

GPE = 0

EPE = 0

$KE = \frac{1}{2}mv^2$

Energy Before = Energy After

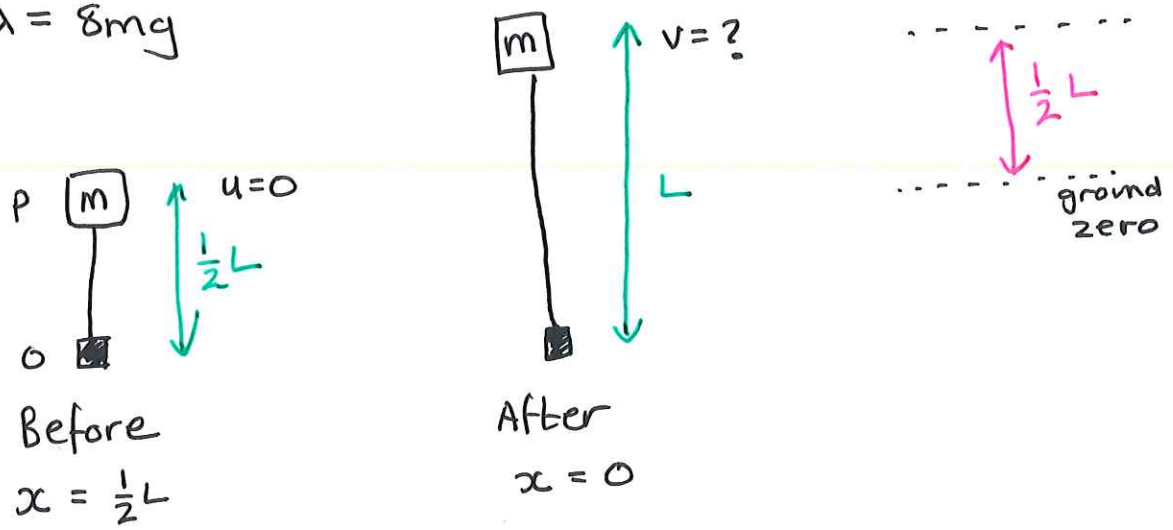
$$\frac{\lambda\left(\frac{1}{2}L\right)^2}{2L} = \frac{1}{2}m\left(\sqrt{2gL}\right)^2$$

$$\frac{\lambda\frac{1}{4}L^2}{2L} = mgL$$

$$\lambda\frac{1}{8}L = mgL$$

$$\lambda = 8mg$$

(b) $\lambda = 8mg$



Before

Work = 0

GPE = 0

$EPE = \frac{\lambda x^2}{2L}$

KE = 0

After

Work = 0

GPE = mgh

EPE = 0

$KE = \frac{1}{2}mv^2$

Energy before = Energy after

$$\frac{(8mg)(\frac{1}{2}L)^2}{2L} = \frac{1}{2}mv^2 + mg(\tfrac{1}{2}L)$$

$$\frac{(8mg)\frac{1}{4}L^2}{2L} = \frac{1}{2}mv^2 + \frac{1}{2}mgL$$

$$mgL = \frac{1}{2}mv^2 + \frac{1}{2}mgL$$

$\div m \times 2$

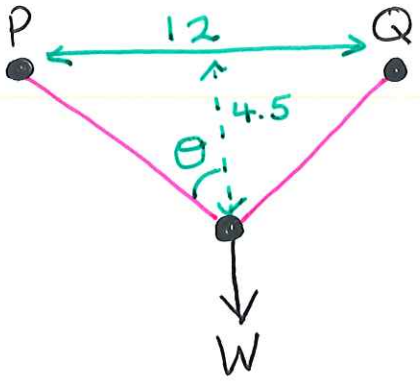
$$2gL = v^2 + gL$$

$$v^2 = gL$$

$$v = \sqrt{gL}$$

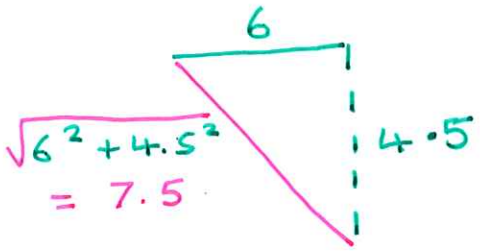
Q2)

(a)



Natural length 8

$\lambda = 80$



$x = 7.5 - 4 = 3.5$

$$T = \frac{\lambda x}{L}$$

$$T = \frac{80(3.5)}{4}$$

$$T = \underline{\underline{84\ N}}$$

(b) $$EPE = \frac{\lambda x^2}{2L}$$

$$= \frac{80(3.5)^2}{2(4)}$$

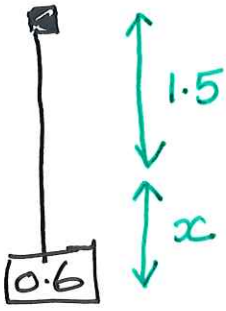
$$= \underline{\underline{245}}$$

Q3) Natural length $= 1.5$ $\lambda = 14.7$

(a)

Before

After



Before

Work $= 0$

KE $= 0$

EPE $= 0$

GPE $= mgh$

After

Work $= 0$

KE $= 0$

EPE $= \frac{\lambda x^2}{2L}$

GPE $= 0$

Energy before = Energy after

$$0.6g(x+1.5) = \frac{14.7x^2}{2(1.5)}$$

$$1.8g(x+1.5) = 14.7x^2$$

$$17.64x + 26.46 = 14.7x^2$$

$$14.7x^2 - 17.64x - 26.46 = 0$$

$$x = 2.07 \quad \text{or} \quad x = -0.86969$$

$$OA = 1.5 + 2.07 = \underline{3.57}$$

(Q3)

(b) $F = ma$

$$0.6g - T = 0.6a$$

$$0.6g - \frac{\lambda x}{L} = 0.6a$$

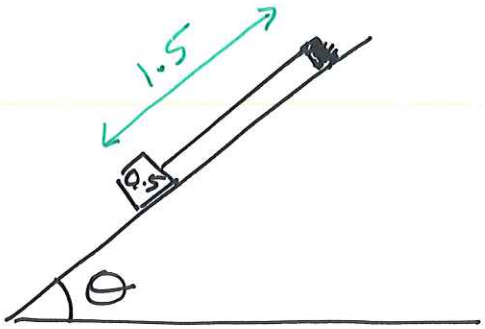
$$0.6g - \frac{14.7(2.07)}{1.5} = 0.6a$$

$$a = 24.0$$

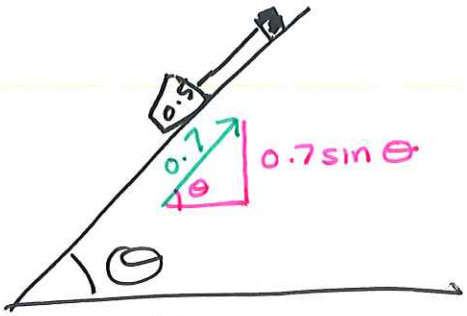
(Q4) $L = 0.9$ $\mu = 0.15$ $\lambda = ?$

$\sin\theta = \frac{3}{5}$

$\cos\theta = \frac{4}{5}$



Before



After

Before	After
Work = 0	Work = Friction Force (μR) × Distance (0.7)
GPE = 0	GPE = mgh
KE = 0	KE = 0
EPE = $\frac{\lambda x^2}{2L}$	EPE = $\frac{\lambda x^2}{2L}$

Energy Before = Energy After

$$\frac{\lambda(0.6)^2}{2(0.9)} = 0.15(0.5g\cos\theta)0.7 + 0.5g\,0.7\sin\theta + \frac{\lambda(0.1)^2}{2(0.9)}$$

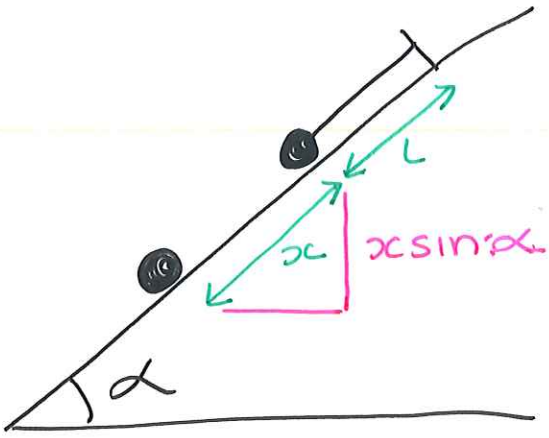
$$\frac{1}{5}\lambda = 0.4116 + 2.058 + \frac{1}{180}\lambda$$

$$\frac{7}{36}\lambda = 2.4696$$

$$\lambda = 12.7$$

Q5) Natural Length = L $\quad \lambda = 3mg$ $\quad \sin\alpha = \frac{3}{5}$ $\quad \cos\alpha = \frac{4}{5}$

(a)



Before	After
Work = 0	Work = 0
GPE = mgh	GPE = 0
KE = 0	EPE = $\frac{\lambda x^2}{2L}$
EPE = 0	KE = 0

Energy before = Energy after

$$2mgx\sin\alpha = \frac{3mgx^2}{2L}$$

$$\frac{6}{5}mgx = \frac{3mgx^2}{2L}$$

$$12mgxL = 15mgx^2$$

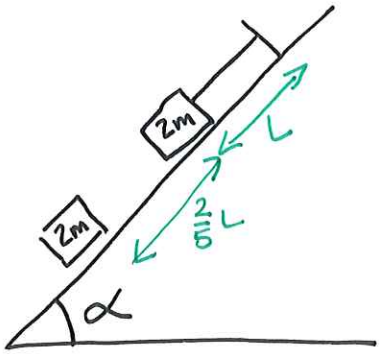
$\div 3mgx$

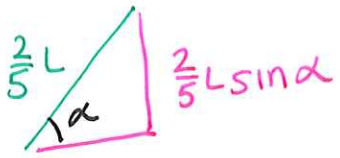
$$4L = 5x$$

$$x = \frac{4L}{5}$$

Q5.)

(b) Natural length = L $\quad \lambda = 3mg$ $\quad \sin\alpha = \frac{3}{5}$ $\quad \cos\alpha = \frac{4}{5}$





Before

Work = 0

GPE = mgh

KE = 0

EPE = 0

After

Work = Fiction Force x Distance

$\mu R \times (\frac{2}{5}L)$

GPE = 0

KE = 0

EPE = $\frac{\lambda x^2}{2L}$

Energy before = Energy After

$$2mg\left(\tfrac{2}{5}L\sin\alpha\right) = \mu\underbrace{(2mg\cos\alpha)}_{\left(\frac{2}{5}L\right)} + \frac{3mg\left(\frac{2}{5}L\right)^2}{2L}$$

$$\frac{12}{25}mgL = \frac{16mgL\mu}{25} + \frac{6}{25}mgL$$

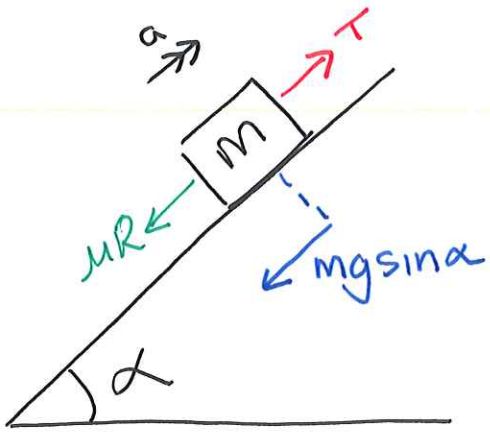
$\div mgL$
$\times 25$

$$12 = 16\mu + 6$$

$$\mu = \frac{6}{16} = \frac{3}{8}$$

Q6)

(a)



Natural length = 3L

$\mu = \frac{1}{3}$ $\lambda = 3mg$

$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

$x = 3L - L = 2L$

$T = \frac{\lambda x}{L}$

$T = \frac{3mg(2L)}{3L}$

$T = 2mg$

Friction force

$= \mu R$

$= \frac{1}{3} mg\cos\alpha$

$= \frac{4}{15} mg$

$F = ma$

$2mg - \frac{4}{15}mg - mg\sin\alpha = ma$

$\div m$
$\times 15$

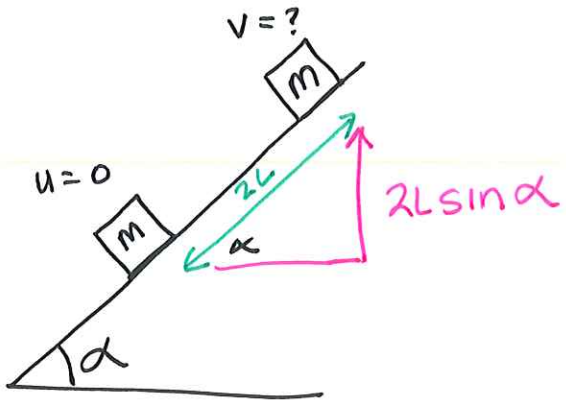
$30g - 4g - 9g = 15a$

$17g = 15a$

$\frac{17g}{15} = a$

Q6)

(b)



Natural length = 3L

$\mu = \frac{1}{3}$ $\lambda = 3mg$

$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

Before

Work = 0

GPE = 0

$EPE = \frac{\lambda x^2}{2L}$

KE = 0

After

Work = $\mu R \times$ Distance

GPE = mgh

EPE = 0

$KE = \frac{1}{2}mv^2$

Energy before = Energy after

$$\frac{3mg(2L)^2}{2(3L)} = \frac{1}{3}(mg\cos\alpha)2L + mg(2L\sin\alpha) + \frac{1}{2}mv^2$$

$$2mgL = \frac{8}{15}mgL + \frac{6}{5}mgL + \frac{1}{2}mv^2$$

$\div m$ $\times 15$ $\quad 30gL = 8gL + 18gL + 7.5v^2$

$$4gL = 7.5v^2$$

$$8gL = 15v^2$$

$$v^2 = \frac{8}{15}gL$$

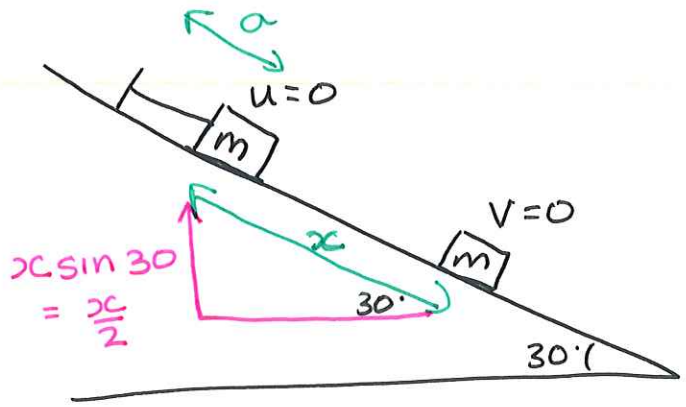
$$v = \sqrt{\frac{8}{15}gL}$$

Q7)

(a)

$\lambda = 6mg$

$L = a$



Before	After
Work = 0	Work = 0
GPE = mgh	GPE = 0
EPE = 0	EPE = $\frac{\lambda x^2}{2L}$
KE = 0	KE = 0

Energy before = Energy after

$$mg\frac{x}{2} = \frac{6mg\,x^2}{2a}$$

$$mg \times a = 6mg \times x^2$$

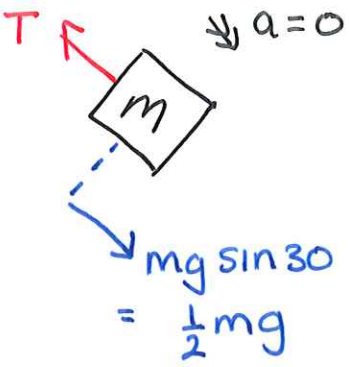
$$xa = 6x^2$$

$$x = \frac{a}{6}$$

$$AC = a + \frac{a}{6} = \frac{7a}{6}$$

Q7)

(b) $F = ma$



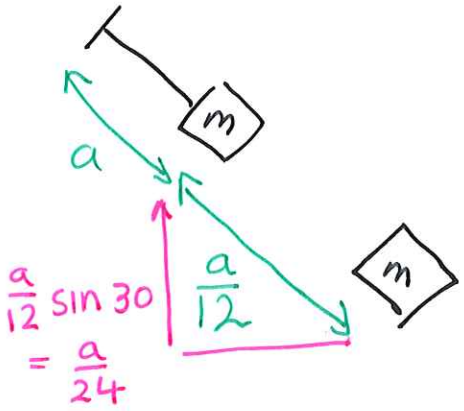
$T = \frac{1}{2}mg$

$T = \frac{\lambda x}{L}$

$\frac{1}{2}mg = \frac{6mg(x)}{a}$

$mga = 12mgx$

$\frac{a}{12} = x$



Before

Work = 0

GPE = mgh

EPE = 0

KE = 0

After

work = 0

GPE = 0

$EPE = \frac{\lambda x^2}{2L}$

$KE = \frac{1}{2}mv^2$

Energy before = Energy after

$mg\left(\frac{a}{24}\right) = \frac{6mg\left(\frac{a}{12}\right)^2}{2a} + \frac{1}{2}mv^2$

$mga = \frac{1}{2}mga + 12mv^2$

$ga = \frac{1}{2}ga + 12v^2$

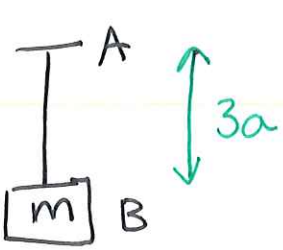
$\frac{1}{2}ga = 12v^2$

$\sqrt{\frac{ga}{24}} = v$

$v = \frac{\sqrt{6ga}}{12}$

Q8)

(a) Natural length = a



Before

Work = 0

GPE = 0

$EPE = \frac{\lambda x^2}{2L}$

KE = 0

After

Work = 0

GPE = mgh

$EPE = \frac{\lambda x^2}{2L}$

KE = 0

Energy before = Energy after

$$\frac{\lambda(2a)^2}{2(a)} = mg(3a) + \frac{\lambda(a)^2}{2a}$$

$$\lambda 2a = 3mga + \frac{\lambda a}{2}$$

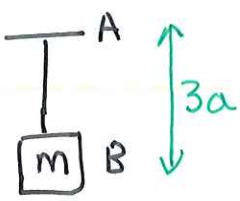
$$\frac{3\lambda a}{2} = 3mga$$

$$\lambda = 2mg$$

Q8) $L = a \quad \lambda = 2mg$

(b)

(i)



max speed

$a = 0$

$\therefore T = mg$

$$T = \frac{\lambda x}{L}$$

$$T = \frac{2mgx}{a}$$

$$mg = \frac{2mgx}{a}$$

$$\frac{a}{2} = x$$

(ii)

Before

work = 0

GPE = 0

$EPE = \frac{\lambda x^2}{2L}$

KE = 0

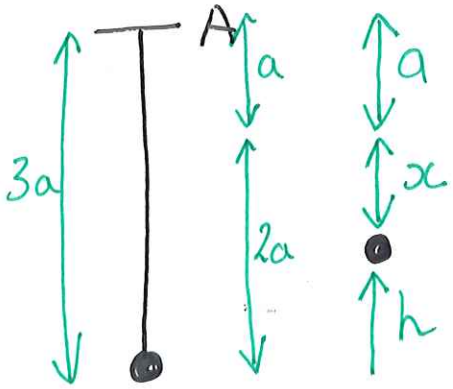
After

work = 0

GPE = mgh

$EPE = \frac{\lambda x^2}{2L}$

$KE = \frac{1}{2}mv^2$



$h = 2a - x$

Q8)

(b)

ii) Energy before = Energy after

$$\frac{2mg(2a)^2}{2a} = \frac{1}{2}mv^2 + mg(2a-x) + \frac{2mg(x)^2}{2a}$$

$$4mga = \frac{1}{2}mv^2 + 2amg - xmg + \frac{mgx^2}{a}$$

$x = \frac{a}{2}$

$$4mga = \frac{1}{2}mv^2 + 2amg - \frac{a}{2}mg + \frac{mg(\frac{a}{2})^2}{a}$$

$\div m$

$$4ga = \frac{1}{2}v^2 + 2ag - \frac{ag}{2} + \frac{ga}{4}$$

$\times 4$

$$16ga = 2v^2 + 8ag - 2ag + ga$$

$$9ga = 2v^2$$

$$v^2 = \sqrt{\frac{9ga}{2}}$$

$$v^2 = 3\sqrt{\frac{ga}{2}}$$

(c) If the spring has mass then GPE of the spring would need to be 'included'.