**GCE A Level Mathematics (9MA0) – Shadow Paper (Set 1)**

**9MA0-32 Mechanics**

**June 2022 Shadow Paper mark scheme**

**Please note that this mark scheme is not the one used by examiners for making scripts. It is intended more as a guide, indicating where marks are given for correct answers. As such, it may not show follow-through marks (marks that are awarded despite errors being made) or special cases.**

**It should also be noted that for many questions, there may be alternative methods of finding correct solutions that are not shown here – they will be covered in the formal mark scheme from the original paper.**

**This document is intended for guidance only and may differ significantly from the examiners' final mark scheme for the original paper, which was published in August 2022.**

| **Guidance on the use of codes within this document** |
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| M1 – method mark. This mark is generally given for an appropriate method in the context of the question. This mark is given for showing your working and may be awarded even if working is incorrect.<br><br>A1 – accuracy mark. This mark is generally given for a correct answer following correct working.<br><br>B1 – working mark. This mark is usually given when working and the answer cannot easily be separated.<br><br>Some questions require all working to be shown; in such questions, no marks will be given for an answer with no working (even if it is a correct answer). |

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| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **1(a)** | Put $t = 2$ in **v** and use Pythagoras: $\sqrt{24^2 + \left(9\sqrt{2}\right)^2}$ | M1 | 3.1a |
| | $\sqrt{738}$ or $3\sqrt{82}$ or 27.2 or better $(\text{m s}^{-1})$ | A1 | 1.1b |
| | | **(2)** | |
| **1(b)** | Differentiate **v** wrt $t$ to obtain **a**<br>Both powers decreasing by 1. Allow a column vector | M1 | 3.4 |
| | $12t\mathbf{i} - \frac{9}{2}t^{-\frac{1}{2}}\mathbf{j}$ oe $(\text{m s}^{-2})$ isw<br>cao. Do not accept a column vector here. | A1 | 1.1b |
| | | **(2)** | |
| **1(c)** | Integrate **v** wrt $t$ to obtain **r**<br>Both powers increasing by 1 | M1 | 3.4 |
| | $\mathbf{r} = 2t^3\mathbf{i} - 6t^{\frac{3}{2}}\mathbf{j}(+\mathbf{C})$ | A1 | 1.1b |
| | $2\mathbf{i} - 4\mathbf{j} = 2\times1^3\mathbf{i} - 6\times1^{\frac{3}{2}}\mathbf{j} + \mathbf{C} \Rightarrow \mathbf{C} = 2\mathbf{j}$<br>**C** does not need to be found and this is a method mark, so allow slips. | M1 | 3.1a |
| | $2\times4^3\mathbf{i} - \left(6\times4^{\frac{3}{2}} + 2\right)\mathbf{j} + \mathbf{C} = 128\mathbf{i} - 46\mathbf{j}$<br>isw, e.g. if they go on to find the distance. | A1 | 1.1b |
| | | **(4)** | |
| | | | **(8 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **2(a)(i)** | Resolve vertically | M1 | 3.1b |
| | $F$ acting UP the plane: $F\sin\alpha + 100\cos\alpha = 10g$ **OR** $F$ acting DOWN the plane: $-F\sin\alpha + 100\cos\alpha = 10g$<br><br>Other possible equations from which $X$ would need to be eliminated to give an equation in $F$ **only** to earn the M mark are shown below. The equation in $F$ **only** must then be correct to earn the A mark.<br><br>Possible equations:<br>Resolve perpendicular to the plane<br>$100 = 10g\cos\alpha + X\sin\alpha$<br>$100 = 10g\cos\alpha + X\sin\alpha$(leads to $X$ = ?? with $g = 9.8$)<br>$F$ acting UP the plane: $F + X\cos\alpha = 10g\sin\alpha$ **OR** $F$ acting DOWN the plane: $-F + X\cos\alpha = 10g\sin\alpha$ | A1 | 1.1b |
| | $F = 47.5$ (N) (note; this is from $g = 9.8$ so do **not** accept $F = \frac{95}{2}$)<br>**N.B.** If sin and cos are interchanged in all equations, this leads to an answer of 9.8 in the wrong direction and can only score<br>(a) (i)M1A0A0 (ii) A0 | A1 | 1.1b |
| | | **(3)** | |
| **2(a)(ii)** | Up the plane (Allow up or upwards or an arrow ↗, but must appear as the answer to (a) (ii) not just on the diagram.) | A1 | 2.2a |
| | | **(1)** | |
| **2(b)** | **N.B.** If they use $R = 100$ in this part, the maximum they can score is M1A1M0A0M0A0; if they use $F = 9.8$ in this part, the maximum they can score is M1A1M0A0M0A0 | | |
| | Equation of motion down the plane | M1 | 2.1 |
| | $10g\sin\alpha - F = 10a =$ Allow $(-a)$ instead of $a$ | A1 | 1.1b |
| | Resolve perpendicular to the plane | M1 | 3.1b |
| | $R = 10g\cos\alpha$ | A1 | 1.1b |
| | $F = 0.25R$ seen | M1 | 3.4 |
| | $10g\sin\alpha - 0.25 \times 10g\cos\alpha = 10a$<br>$8g - 1.5g = 10a$<br>$a = 6.37\ (\text{m s}^{-2})$ or $0.65g$ | A1 | 1.1b |
| | | **(6)** | |
| | | | **(10 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **3(a)** | $(2\mathbf{i} - 3\mathbf{j}) + (\alpha\mathbf{i} + \beta\mathbf{j}) = (2 + \alpha)\mathbf{i} + (-3 + \beta)\mathbf{j})$ | M1 | 3.4 |
| | Use **ratios** to obtain an equation in $\alpha$ and $\beta$ **only** | M1 | 2.1 |
| | $\frac{2+\alpha}{-3+\beta} = \frac{6}{4}$ | A1 | 1.1b |
| | Reaches $4\alpha - 6\beta + 26 = 0$ *<br>Allow $4\alpha - 6\beta + 26 = 0$ but nothing else. | A1* | 1.1b |
| | | **(4)** | |
| **(b)** | $\alpha = 4 \Rightarrow \beta = 7$; Resultant force $6\mathbf{i} + 4\mathbf{j}$ (N) | M1 | 3.1a |
| | $6\mathbf{i} + 4\mathbf{j} = 5\mathbf{a}$ **OR** $\lvert 6\mathbf{i} + 4\mathbf{j}\rvert = 5a$ | M1 | 1.1b |
| | Use of $\mathbf{r} = \mathbf{u}t + \frac{1}{2}\mathbf{a}t^2$ with $\mathbf{u} = \mathbf{0}$, their $\mathbf{a}$ and $t = 10$:<br>**Or** they may integrate their $\mathbf{a}$ twice with $\mathbf{u} = \mathbf{0}$ and put $t = 10$ :<br>$\mathbf{r} = \frac{1}{2} \times \frac{1}{5}(6\mathbf{i} + 4\mathbf{j}) \times 10^2 = 60\mathbf{i} + 40\mathbf{j}$ | DM1 | 2.1 |
| | $\sqrt{60^2 + 40^2}$ | M1 | 1.1b |
| | **ALTERNATIVE 1** for last two M marks:<br>Use of $s = ut + \frac{1}{2}at^2$, with $u = 0$, their $a$ and $t = 10$ : DM1<br>$s = \frac{1}{2}\sqrt{1.2^2 + 0.8^2} \times 10^2$<br>Use of Pythagoras to find mag of $\mathbf{a}$ : $a = \sqrt{1.2^2 + 0.8^2}$ M1 | | |
| | **ALTERNATIVE 2** for last two M marks:<br>Use of $s = ut + \frac{1}{2}at^2$, with $u = 0$, their $a$ and $t = 10$ : DM1<br>$s = \frac{1}{2} \times \frac{\sqrt{6^2 + 4^2}}{5} \times 10^2$<br>Use of Pythagoras to find $\lvert 6\mathbf{i} + 4\mathbf{j}\rvert$ : $= \sqrt{6^2 + 4^2}$ M1 | | |
| | $AB = 20\sqrt{13}\,(\text{m})$ or 72.1 (m) or better | A1 | 1.1b |
| | | **(5)** | |
| | | | **(9 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(a)** | The horizontal component of $T$ acts to the left and since the **only** other horizontal force is friction, it must act to the right oe | B1 | 2.4 |
| | | **(1)** | |
| **4(b)** | Take moments about $A$ or any other complete method to obtain **an equation in $T$, $M$ and $\theta$ only.** (see possible equations below that they may use) | M1 | 3.1b |
| | $2xT = 2Mgx\cos\theta + 4Mg \times 1.8x\cos\theta$<br>(A0 if $x$'s missing) | A1 | 1.1b |
| | Other possible equations but $F$ and $R$ would need to be eliminated.<br>$(\nwarrow), R\cos\theta + T = F\sin\theta + Mg\cos\theta + 2Mg\cos\theta$<br>$(\nearrow), R\sin\theta + F\cos\theta = Mg\sin\theta + 2Mg\sin\theta$<br>$(\rightarrow),\ F = T\sin\theta$<br>$\text{M}(B), R.2a\cos\theta = Mga\cos\theta + 2Mg \times 0.5a\cos\theta + F.2a\sin\theta$<br>$\text{M}(G), Fa\sin\theta + Ta = Ra\cos\theta + 2Mg \times 0.5a\cos\theta$<br>$\text{M}(C), R \times 1.5a\cos\theta = T \times 0.5a + Mg \times 0.5a\cos\theta + F \times 1.5a\sin\theta$ | | |
| | $T = \frac{23}{5}Mg\cos\theta$ *<br>Given answer correctly obtained with no wrong working seen | A1* | 1.1b |
| | | **(3)** | |
| **4(c)** | e.g. Resolve vertically | M1 | 3.4 |
| | $(\uparrow)\quad R = 6Mg - T\cos\theta = 6Mg - \left(\frac{23}{5}Mg\cos\theta\right)\cos\theta = 6Mg - \frac{368}{125}Mg$ | A1 | 1.1b |
| | $= \frac{382}{125}Mg$ *<br>Given answer correctly obtained with no wrong working seen | A1* | 1.1b |
| | | **(3)** | |
| | Other possible equations but $F$ would need to be eliminated. | | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(d)** | Find an equation containing $F$ e.g. Resolve horizontally | M1 | 3.4 |
| | $(\rightarrow),\ F = T\sin\theta\ = \frac{276}{125}Mg$ | A1 | 1.1b |
| | Other possible equations<br>$\mu R = T\sin\theta$ | | |
| | $F = \mu R$ used i.e. both $F$ and $R$ are substituted.<br>Must be used i.e M0 if merely quoting it. | M1 | 3.1b |
| | $\mu = \frac{138}{191}$ *<br>Given answer correctly obtained with no wrong working seen | A1* | 2.2a |
| | | **(4)** | |
| | | | **(11 marks)** |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5(a)** | Using horizontal motion | M1 | 3.3 |
| | **Whole Motion** **Half way** | | |
| | $U\cos\theta \times t = 150$ **OR** $U\cos\theta \times t = 75$ | A1 | 1.1b |
| | Using vertical motion | M1 | 3.4 |
| | $U\sin\theta \times t - \frac{1}{2}gt^2 = 0$ **OR** $0 = U\sin\theta - gt$ | A1 | 1.1b |
| | Attempt to solve problem by eliminating $t$ | DM1 | 3.1b |
| | $U^2\sin\theta\cos\theta = 735$ *<br>Given answer correctly obtained, with no wrong working seen. | A1* | 2.2a |
| | | **(6)** | |
| | **N.B.** No credit given if they use the given answer from (b). | | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5(b)** | Using vertical motion **OR** conservation of energy | M1 | 3.4 |
| | $0 = U^2\sin^2\theta - 2g\times 20$ $\quad\quad$ $\frac{1}{2}mU^2 - \frac{1}{2}m\left(U\cos\theta\right)^2 = mg\times 20$ | A1 | 1.1b |
| | **ALTERNATIVE 1:**<br>If $t$ is time to top: use of $20 = \frac{1}{2}gt^2$ oe $\left(t = \frac{10\sqrt{2}}{7}\right)$ to obtain an equation in $U$ and $\theta$ only M1<br>$U\sin\theta = 14\sqrt{2}$ or $U\cos\theta = \frac{105}{2\sqrt{2}}$ A1<br><br>**ALTERNATIVE 2:**<br>If $t$ is time to top:<br>use of : $20 = U\sin\theta\times\ t - \frac{1}{2}gt^2$ with $t = \frac{75}{U\cos\theta}$ substituted to obtain an equation in $U$ and $\alpha$ only : M1<br>$20 = U\sin\theta\times\frac{75}{U\cos\theta} - \frac{1}{2}g\left(\frac{75}{U\cos\theta}\right)^2$ A1 | | |
| | Attempt to solve problem by eliminating $\theta$:<br>e.g. $U\sin\theta = 14\sqrt{2} \Rightarrow U\cos\theta = \frac{105}{2\sqrt{2}}$ , from part (a) or from using $t = \frac{10\sqrt{2}}{7}$, then square and add to give result<br><br>**OR:** $U^2\sin^2\theta = 40g = 392$ and $U^2\sin\theta\cos\theta = 735$, divide to give $\tan\theta = \frac{8}{15}$ then $\sin^2\theta = \frac{64}{289}$, hence result<br><br>**OR in ALTERNATIVE 2**: sub for $U^2$ using part (a), to give $\tan\theta = \frac{8}{15}$ then $\sin^2\theta = \frac{64}{289}$ hence result<br>**N.B**. Just stating that $\sin^2\theta = \frac{64}{289}$ with no working is DM0A0. | DM1 | 3.1b |
| | $U^2 = \frac{14161}{8}$ *<br>Given answer correctly obtained, with no wrong working seen. | A1* | 2.2a |
| | **N.B.** Verification (i.e. starting with $U^2 = \frac{14161}{8}$ and trying to work backwards) is not an acceptable method for this question. | | |
| | | **(4)** | |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **5(c)** | $V$, since air resistance has to be overcome, or just 'because of <u>air resistance</u>' isw | B1 | 3.5a |
| | | **(1)** | |
| **5(d)** | e.g. wind effects, more accurate value of $g$, spin of ball, size of ball, shape of ball, dimensions of ball, not a particle, variable acceleration, surface area of ball, humidity. Allow wind resistance.<br>(Ignore any mention of air resistance or drag) | B1 | 3.5c |
| | | **(1)** | |
| | | | **(12 marks)** |