

Write your name here	
Surname	Other names
Pearson Edexcel	Centre Number
Level 3 GCE	Candidate Number
Mathematics	
Advanced	
Paper 3: Mechanics	
Friday 15 June 2018 – Afternoon	Paper Reference
Time: 2 hours	9MA0/03
You must have: Mathematical Formulae and Statistical Tables, calculator	Total Marks

Candidates may use any calculator allowed by the regulations of the Joint Council for Qualifications. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided
– *there may be more space than you need.*
- You should show sufficient working to make your methods clear.
Answers without working may not gain full credit.
- Answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 10 questions in this question paper. The total mark for this paper is 100.
- The marks for **each** question are shown in brackets
– *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.

Turn over ►

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SECTION B: MECHANICS

Unless otherwise stated, whenever a numerical value of g is required, take $g = 9.8 \text{ m s}^{-2}$ and give your answer to either 2 significant figures or 3 significant figures.

Answer ALL questions. Write your answers in the spaces provided.

6. At time t seconds, where $t \geq 0$, a particle P moves in the x - y plane in such a way that its velocity v m s^{-1} is given by

$$\mathbf{v} = t^{-\frac{1}{2}}\mathbf{i} - 4t\mathbf{j}$$

When $t = 1$, P is at the point A and when $t = 4$, P is at the point B .

Find the exact distance AB .

(6)

$$\mathbf{x} = 2t^{\frac{1}{2}}\mathbf{i} - 2t^2\mathbf{j} + \mathbf{c}$$

when $t = 1$

$$\mathbf{x}_A = 2\mathbf{i} - 2\mathbf{j} + \mathbf{c}$$

when $t = 4$

$$\mathbf{x}_B = 4\mathbf{i} - 32\mathbf{j} + \mathbf{c}$$

$$AB = \mathbf{x}_B - \mathbf{x}_A$$

$$= 4\mathbf{i} - 32\mathbf{j} - (2\mathbf{i} - 2\mathbf{j})$$

$$= 2\mathbf{i} - 30\mathbf{j}$$

$$\text{distance} = \sqrt{2^2 + 30^2}$$

$$= \underline{\underline{2\sqrt{226}}}$$



7.

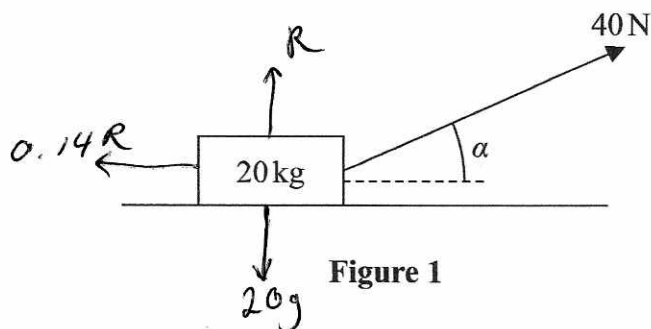


Figure 1

A wooden crate of mass 20 kg is pulled in a straight line along a rough horizontal floor using a handle attached to the crate.

The handle is inclined at an angle α to the floor, as shown in Figure 1, where $\tan \alpha = \frac{3}{4}$

The tension in the handle is 40 N.

The coefficient of friction between the crate and the floor is 0.14

The crate is modelled as a particle and the handle is modelled as a light rod.

Using the model,

- (a) find the acceleration of the crate.

(6)

The crate is now pushed along the same floor using the handle. The handle is again inclined at the same angle α to the floor, and the thrust in the handle is 40 N as shown in Figure 2 below.

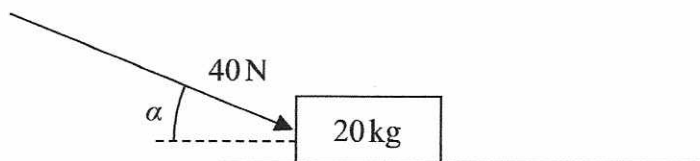
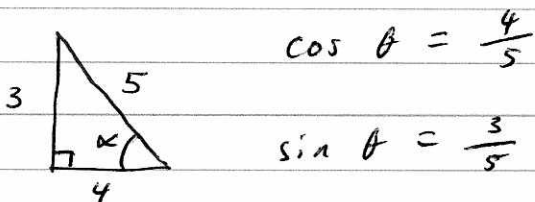


Figure 2

- (b) Explain briefly why the acceleration of the crate would now be less than the acceleration of the crate found in part (a).

(2)



$$\cos \theta = \frac{4}{5}$$

$$\sin \theta = \frac{3}{5}$$

Forces Up = Forces Down

$$R + 40 \sin \theta = 20g$$

$$R + 40 \left(\frac{3}{5} \right) = 20g$$

$$R = 20g - 24$$



Question 7 continued

$$F = ma$$

$$40 \cos \alpha - 0.14R = 20a$$

$$a = \frac{40\left(\frac{4}{5}\right) - 0.14(20g - 24)}{20}$$

$$= 0.396 \quad (3\text{sf}) \quad \text{ms}^{-2}$$

$$= 0.40 \quad (2\text{sf}) \quad \text{ms}^{-2}$$

b/ The acceleration would be less than in (a).
 R would increase $\rightarrow$ Max Friction would increase.



8. [In this question $\mathbf{i}$ and $\mathbf{j}$ are horizontal unit vectors due east and due north respectively and position vectors are given relative to the fixed point O .]

A particle P moves with constant acceleration.

At time $t = 0$, the particle is at O and is moving with velocity $(2\mathbf{i} - 3\mathbf{j}) \text{ ms}^{-1}$

At time $t = 2$ seconds, P is at the point A with position vector $(7\mathbf{i} - 10\mathbf{j}) \text{ m}$.

- (a) Show that the magnitude of the acceleration of P is 2.5 ms^{-2}

(4)

At the instant when P leaves the point A , the acceleration of P changes so that P now moves with constant acceleration $(4\mathbf{i} + 8.8\mathbf{j}) \text{ ms}^{-2}$

At the instant when P reaches the point B , the direction of motion of P is north east.

- (b) Find the time it takes for P to travel from A to B .

(4)

$$\begin{aligned} a/ \quad s &= 7\mathbf{i} - 10\mathbf{j} \\ u &= 2\mathbf{i} - 3\mathbf{j} \\ v & \\ a &= ? \\ t &= 2 \end{aligned}$$

$$s = ut + \frac{1}{2}at^2$$

$$7\mathbf{i} - 10\mathbf{j} = 2(2\mathbf{i} - 3\mathbf{j}) + \frac{1}{2}a(2)^2$$

$$7\mathbf{i} - 10\mathbf{j} = 4\mathbf{i} - 6\mathbf{j} + 2a$$

$$3\mathbf{i} - 4\mathbf{j} = 2a$$

$$a = \frac{3}{2}\mathbf{i} - 2\mathbf{j} \text{ ms}^{-2}$$

$$= \sqrt{\left(\frac{3}{2}\right)^2 + 2^2}$$

$$= \underline{\underline{2.5 \text{ ms}^{-2}}}$$

$$\begin{aligned} b/ \quad A \rightarrow B \quad v &= u + at \\ &= 2\mathbf{i} - 3\mathbf{j} + 2\left(\frac{3}{2}\mathbf{i} - 2\mathbf{j}\right) \\ &= 2\mathbf{i} - 3\mathbf{j} + 3\mathbf{i} - 4\mathbf{j} \\ &= 5\mathbf{i} - 7\mathbf{j} \end{aligned}$$



Question 8 continued

3

$$u = 5i - 7j$$

$$v = k(i + j)$$

$$a = 4i + 8.8j$$

$$t = ?$$

$$v = u + at$$

$$k(i + j) = 5i - 7j + t(4i + 8.8j)$$

$$i// \quad k = 5 + 4t$$

$$j// \quad k = -7 + 8.8t$$

$$5 + 4t = -7 + 8.8t$$

$$12 = 4.8t$$

$$t = \frac{12}{4.8}$$

$$t = \frac{12}{4.8}$$

$$= 2.5 \text{ seconds}$$



0 5 8 2 5 0 1 0 2 5 2 6

9.

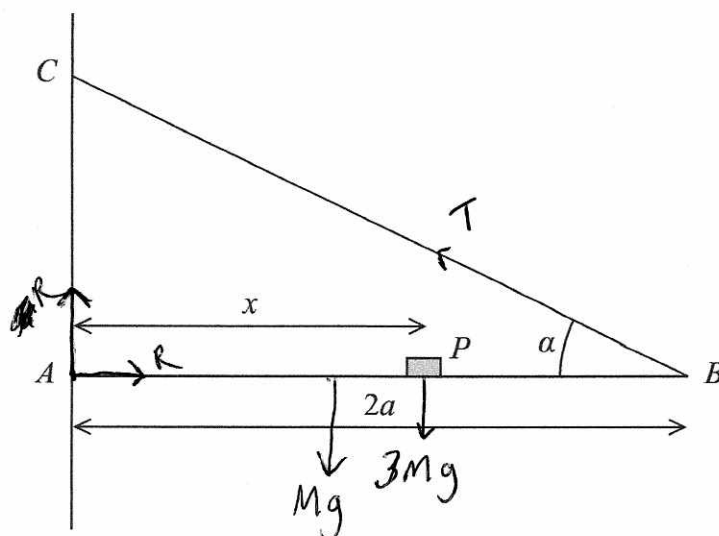


Figure 3

A plank, AB , of mass M and length $2a$, rests with its end A against a rough vertical wall. The plank is held in a horizontal position by a rope. One end of the rope is attached to the plank at B and the other end is attached to the wall at the point C , which is vertically above A .

A small block of mass $3M$ is placed on the plank at the point P , where $AP = x$. The plank is in equilibrium in a vertical plane which is perpendicular to the wall.

The angle between the rope and the plank is α , where $\tan \alpha = \frac{3}{4}$, as shown in Figure 3.

The plank is modelled as a uniform rod, the block is modelled as a particle and the rope is modelled as a light inextensible string.

- (a) Using the model, show that the tension in the rope is $\frac{5Mg(3x + a)}{6a}$ (3)

The magnitude of the horizontal component of the force exerted on the plank at A by the wall is $2Mg$.

- (b) Find x in terms of a . (2)

The force exerted on the plank at A by the wall acts in a direction which makes an angle β with the horizontal.

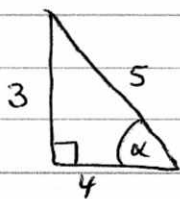
- (c) Find the value of $\tan \beta$ (5)

The rope will break if the tension in it exceeds $5Mg$.

- (d) Explain how this will restrict the possible positions of P . You must justify your answer carefully. (3)



Question 9 continued



$$\sin \alpha = \frac{3}{5}$$

$$\cos \alpha = \frac{4}{5}$$

a/ Taking moments about A

$$a(mg) + x(3mg) = 2a(T \sin \alpha)$$

$$aMg + 3xMg = 2a \frac{3}{5} T$$

$$Mg(a + 3x) = \frac{6a}{5} T$$

$$5Mg(3x + a) = 6aT$$

$$\frac{5Mg(3x + a)}{6a} = T$$

b/ Resolving horizontally:

$$2Mg = T \cos \alpha$$

$$2Mg = \frac{4}{5} T$$

$$\frac{5}{2} Mg = T$$

$$\frac{5Mg(3x + a)}{6a} = \frac{5}{2} Mg$$

$$3x + a = 3a$$

$$3x = 2a$$

$$x = \frac{2}{3} a$$



Question 9 continued

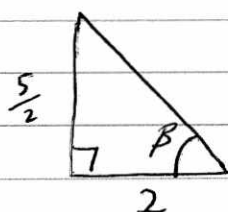
c/ Taking moments about B:

$$2a \cdot R_v = \frac{4}{3}a(3Mg) + a(Mg)$$

$$2a R_v = 4a Mg + a Mg$$

$$2a R_v = 5a Mg$$

$$R_v = \frac{5}{2} Mg$$



$$\begin{aligned} \tan \beta &= \frac{2.5}{2} \\ &= \frac{5}{4} \end{aligned}$$

d/

$$\text{Tension} = \frac{5Mg(3x+a)}{6a}$$

$$x = \frac{2}{3}a$$

$$= 5Mg$$

$$\text{Break if } \frac{5Mg(3x+a)}{6a} > 5Mg$$

$$\frac{(3x+a)}{6a} > 1$$

$$3x + a > 6a$$

$$3x > 5a$$

$$x > \frac{5}{3}a$$

P cannot be placed more than $\frac{5}{3}a$ along the plank from A otherwise the rope will break.



10.

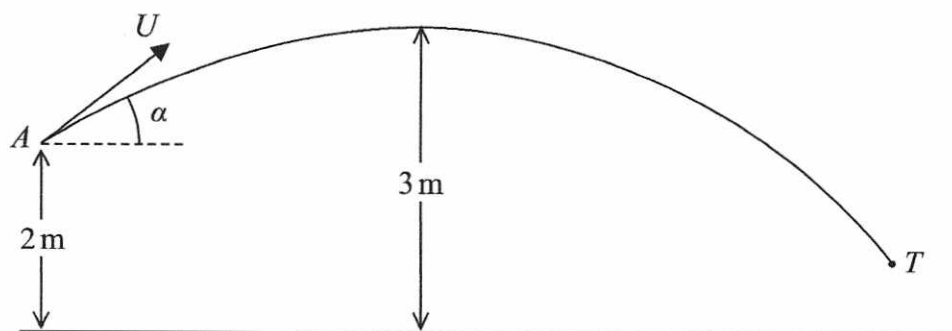


Figure 4

A boy throws a ball at a target. At the instant when the ball leaves the boy's hand at the point A , the ball is 2 m above horizontal ground and is moving with speed U at an angle α above the horizontal.

In the subsequent motion, the highest point reached by the ball is 3 m above the ground. The target is modelled as being the point T , as shown in Figure 4. The ball is modelled as a particle moving freely under gravity.

Using the model,

(a) show that $U^2 = \frac{2g}{\sin^2 \alpha}$. (2)

The point T is at a horizontal distance of 20 m from A and is at a height of 0.75 m above the ground. The ball reaches T without hitting the ground.

(b) Find the size of the angle α (9)

(c) State one limitation of the model that could affect your answer to part (b). (1)

(d) Find the time taken for the ball to travel from A to T . (3)

vertical motion: $s = 1$

$$u = U \sin \alpha$$

$$v = 0$$

$$a = -9.8 \quad -g$$

$$t$$

$$v^2 = u^2 + 2as$$

$$0 = (U \sin \alpha)^2 - 2g$$

$$2g = U^2 \sin^2 \alpha$$

$$U^2 = \frac{2g}{\sin^2 \alpha}$$

$$U^2 = \frac{2g}{\sin^2 \alpha}$$



Question 10 continued

b/ Horizontal motion: $s = 20$

$$u = U \cos \alpha$$

$$v = U \cos \alpha$$

$$a = 0$$

$$t = t$$

$$s = ut + \frac{1}{2}at^2$$

$$20 = (U \cos \alpha) t$$

$$t = \frac{20}{U \cos \alpha}$$

Vertical motion: $s = -1.25$

$$u = U \sin \alpha$$

$$v =$$

$$a = -g$$

$$t = t$$

$$-1.25 = (U \sin \alpha) t - \frac{1}{2} g t^2$$

$$-1.25 = U \sin \alpha \left(\frac{20}{U \cos \alpha} \right) - \frac{1}{2} g \left(\frac{20}{U \cos \alpha} \right)^2$$

$$-1.25 = 20 \tan \alpha - \frac{200g}{(U^2) \cos^2 \alpha}$$

$$-1.25 = 20 \tan \alpha - \frac{200g}{\sin^2 \alpha \cdot \cos^2 \alpha}$$

$$-1.25 = 20 \tan \alpha - \frac{100}{\cot^2 \alpha}$$

$$-1.25 = 20 \tan \alpha - 100 \tan^2 \alpha$$

$$100 \tan^2 \alpha - 20 \tan \alpha - 1.25 = 0$$

$$400 \tan^2 \alpha - 80 \tan \alpha - 5 = 0$$

$$(100 \tan \alpha + 5)(4 \tan \alpha - 1) = 0$$

$$\tan \alpha = -\frac{1}{20}$$

$$\alpha = -2.9$$

$$\tan \alpha = \frac{1}{4}$$

$$\alpha = 14$$

α must be between 0 and 90 $\therefore \alpha = 14^\circ$



Question 10 continued

c/ There is no air resistance in the model.

$$d/ \quad u^2 = \frac{2g}{\sin^2 \alpha}$$

$$= \frac{2g}{\sin^2(14^\circ)}$$

$$u^2 = 333.2$$

$$u = 18.25$$

$$t = \frac{20}{18 \cos 14^\circ}$$

$$= 1.13 \text{ (3sf) s}$$

$$= 1.1 \text{ (2sf) s}$$

