AS Level / Year 1

Paper 1 (Edexcel Version)

Set A / Version 1

Question	Scheme		Marks
1	$y = 3x^{-4} + \sqrt{x} - 2x + 1,\ \ x > 0$		
(a)	$\int y\,dx = \dfrac{3x^{-3}}{-3} + \dfrac{x^{\frac{3}{2}}}{3/2} - \dfrac{2x^2}{2} + x \quad \{+c\}$	Attempts to integrate, correct unsimplified integration of three terms (ignore constant here)	M1, A1
	$\int y\,dx = -x^{-3} + \dfrac{2}{3}x^{\frac{3}{2}} - x^2 + x + c$	Fully correct integration with all terms simplified **and** constant of integration present	A1 oe **(3)**
(b)	$\dfrac{dy}{dx} = 3(-4)x^{-5} + \dfrac{1}{2}x^{-\frac{1}{2}} - 2$	Attempts to differentiate, correct unsimplified differentiation of two terms	M1,A1
	$\dfrac{dy}{dx} = -12x^{-5} + \dfrac{1}{2}x^{-\frac{1}{2}} - 2$	Fully correct differentiation with all terms simplified	A1 oe **(3)**
(c)	$\dfrac{d^2y}{dx^2} = -12(-5)x^{-6} + \dfrac{1}{2}\left(-\dfrac{1}{2}\right)x^{-\frac{3}{2}}$	Attempts to differentiate their (b) again	M1
	$\dfrac{d^2y}{dx^2} = 60x^{-6} - \dfrac{1}{4}x^{-\frac{3}{2}}$	Correct simplified second derivative	A1 oe **(2)**
			8

Question 1 Notes

(a) M1 – adds one to the index of any term and divides that term by the new index

2nd A1 – accept equivalent forms, such as $-x^{-3} = -\dfrac{1}{x^3}$ etc.

(b) M1 – multiplies a term by its index and reduces the index by 1

2nd A1 – accept equivalent forms, such as $\dfrac{1}{2}x^{-\frac{1}{2}} = \dfrac{1}{2\sqrt{x}}$

Question	Scheme		Marks
2			
(a)	$\mathrm{f}(x)=4\left(x^2-\frac{5}{4}x-\frac{3}{2}\right)\Rightarrow ...$	Extracts a factor of 4 from all the terms **OR** the first two terms	M1
	$\mathrm{f}(x)=4\left[\left(x-\frac{5}{8}\right)^2-\frac{25}{64}-\frac{3}{2}\right]$	For $(x-a)^2-a^2$ with a = half their 5/4	M1A1
	$\mathrm{f}(x)=4\left(x-\frac{5}{8}\right)^2-\frac{121}{16}$	Correct expression	A1 **(4)**
(b)	$4\left(x-\frac{5}{8}\right)^2=\frac{121}{16}\Rightarrow x-'\frac{5}{8}'=\pm\sqrt{...}$	Attempts to use their (a) to solve the equation **OR** an alternative method	M1
	$x=2$ or $-\frac{3}{4}$	Correct values of x	A1 **(2)**
(c)	$x-1=2,\quad x-1=-\frac{3}{4}\quad\Rightarrow x=...$	Sets x – 1 = their (b)	M1
	$x=3$ or $x=\frac{1}{4}$	Correct values of x ft their (b)	A1ft **(2)**
			8

Question 2 Notes

(a) 1st A1 – correct expression with the 4 extracted oe. If candidates only extract 4 from the first two terms, then the correct expression is $4\left[\left(x-\frac{5}{8}\right)^2-\frac{25}{64}\right]-6$.

(b) **Completing the square:** M1 – uses their (a) to find x by dividing by 4 and square rooting both sides. Must see ±.

Formula: uses a = 4, b = –5 and c = –6 in the formula. If candidates quote the formula before substitution, condone one sign/substitution error. If candidates do not quote the formula before substitution, it must be <u>**completely**</u> correct for the M1.

Factorising: attempts to write the expression as a product of two factors with $(4x + a)(x + b)$ present.

(c) A1 - Correct x ft their (b).

Question	Scheme		Marks
3			
(a)	$2^{\frac{2x+1}{2}} \times 2^{3(y-3)} = 2^{2\left(\frac{y+x}{3}\right)} \times 2^{x-1}$	**<u>Attempts</u>** to write **all** of the terms in the form 2^a	M1
	$\Rightarrow \dfrac{2x+1}{2} + 3y - 9 = \dfrac{2y+2x}{3} + x - 1$	Equates powers	dM1
	$\Rightarrow 6x + 3 + 18y - 54 = 4y + 4x + 6x - 6$		
	$\Rightarrow y = \dfrac{4x+45}{14}$	Correct expression for y in terms of x	A1 oe **(3)**
(b)	$7x^2 + 42\left(\dfrac{4x+45}{14}\right) = 130$	Substitutes to form an equation in x (or y) only	M1
	$7x^2 + 12x + 5 = 0$	Correct 3TQ ft their (a)	A1ft oe
	$(7x+5)(x+1) = 0 \Rightarrow x = -\dfrac{5}{7}$ or $x = -1$	Attempts to solve their 3TQ for x (or y)	dM1A1
	$\Rightarrow y = \dfrac{4(-1)+45}{14}, y = \dfrac{4(-5/7)+45}{14}$	Attempts to use their x (or y) to find y (or x)	dM1
	$\Rightarrow y = \dfrac{41}{14}$ when $x = -1$ or $y = \dfrac{295}{98}$ when $x = -\dfrac{5}{7}$	Correct values of y	A1 **(6)**
			9
	Question 3 Notes		

(b) 1st M1 - Substitutes their y from (a) into the quadratic in (b) to eliminate x **OR** attempts to eliminate x.

1st A1 – correct 3TQ ft their (a). A correct 3TQ for y is $1372y^2 - 8148y + 12095 = 0$ but this can vary if their (a) is wrong.

Question	Scheme		Marks
4			
Way 1	{Let f(x) = a, $a \in \mathbb{R}$ }		
	$\mathrm{f}'(x) = \lim\limits_{h\to 0}\left(\dfrac{\mathrm{f}(x+h)-\mathrm{f}(x)}{h}\right) = \lim\limits_{h\to 0}\left(\dfrac{a-a}{h}\right) = 0$	Uses limit definition of the derivative	M1A1
	$\therefore \mathrm{f}'(x) = 0$ OR $\therefore$ the derivative of a constant function is 0	<u>**See notes**</u>	A1A1 **(4)**
Way 2	{Let f(x) = a, $a \in \mathbb{R}$ }		
	$\mathrm{f}'(x) = \lim\limits_{x\to c}\left(\dfrac{\mathrm{f}(x)-\mathrm{f}(c)}{x-c}\right) = \lim\limits_{x\to c}\left(\dfrac{a-a}{x-c}\right) = 0$	Uses limit definition of the derivative	M1A1
	$\therefore \mathrm{f}'(x) = 0$ OR $\therefore$ the derivative of a constant function is 0	<u>**See notes**</u>	A1A1 **(4)**
			4

Question 4 Notes

<u>Way 1:</u>

1st M1 – <u>uses</u> the limit definition of the derivative (**must be correct**) in the context of a constant function.

1st A1 – obtains limit = 0

2nd A1 – a suitable conclusion

3rd A1 – this is a clarity mark for a clear, well-structured proof.

<u>Way 2:</u>

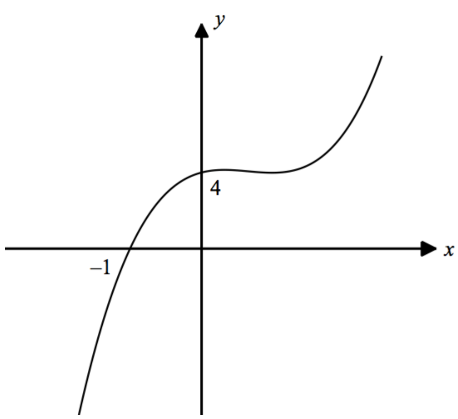
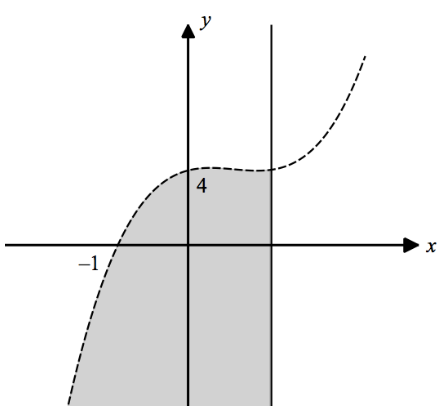
1st M1 – <u>uses</u> the limit definition of the derivative (**must be correct**) in the context of a constant function.

1st A1 – obtains limit = 0

2nd A1 – a suitable conclusion

3rd A1 – this is a clarity mark for a clear, well-structured proof.

Question	Scheme		Marks
5			
(a/i)	$$\begin{array}{r} x^2+x+4 \\ x-3\overline{)1x^3-2x^2+1x+4} \\ \underline{1x^3-3x^2} \\ 1x^2+1x \\ \underline{1x^2-3x} \\ +4x+4 \\ \underline{+4x-12} \\ 16 \end{array}$$ so the remainder is 16	Uses algebraic division to find the remainder	M1A1 **(2)**
(a/ii)	$$\begin{array}{r} x^2-3x+4 \\ x+1\overline{)1x^3-2x^2+1x+4} \\ \underline{1x^3+1x^2} \\ -3x^2+1x \\ -\underline{3x^2-3x} \\ +4x+4 \\ \underline{+4x+4} \\ 0 \end{array}$$ so the remainder is 0	Uses algebraic division to find the remainder	M1A1 **(2)**
(b)	$\mathrm{f}(x)=(x+1)(x^2-3x+4)$		
	So one solution is – 1	Identifies the real solution	B1
	For $x^2-3x+4=0$, $b^2-4ac=9-4(4)=-7<0$ So it has no real solutions. $\therefore$ f(x) = 0 has only one real solution	Shows that the other quadratic factor has no real roots (see notes for alternatives)	B1 **(2)**

(c)	$f'(x) = 3x^2 - 4x + 1$	Attempts to differentiate f(x)	M1
	$f'(x) = 0 \Rightarrow (3x-1)(x-1) = 0$	Sets their derivative = 0 and attempts to solve the 3TQ	dM1
	$\Rightarrow x = \frac{1}{3}, x = 1$	Correct x coordinates of the turning points	A1
	When $x = 1$, $f(1) = 1^3 - 2(1)^2 + 1 + 4 = 4$ When $x = \frac{1}{3}$, $f\left(\frac{1}{3}\right) = \left(\frac{1}{3}\right)^3 - 2\left(\frac{1}{3}\right)^2 + \frac{1}{3} + 4 = \frac{112}{27}$	Attempts to find the y coordinates of the turning points	dM1
	So coordinates are $(1,4)$, $\left(\frac{1}{3}, \frac{112}{27}\right)$	Correct coordinates	A1 **(5)**
(d)		Correct shape	B1
		Correct intersections	B1 **(2)**
(e)		f(x) as a dashed line	B1
		$x \leq 1$ as a solid line	B1
		Correct region shaded *NB: no need to show coordinates of intersection with axes	B1 **(3)**
			16

	Question 5 Notes

(a/i) and (a/ii): M1 – for an attempt to use long division. First term in quotient must be correct in each case.

(b) 1st B1 – identifies the correct real root = – 1

2nd B1 – shows clearly that the other quadratic factor has no real roots **AND** gives the appropriate conclusions:

- discriminant is less than 0 so no real roots/completing the square demands sqrt of negative number/oe
- so the quadratic factor has no real roots
- so f(x) only has one real root (*)

The scheme shows the use of the discriminant, but completing the square can be used, by showing that it demands the sqrt of a negative number to have solutions/the minimum point lies above the x axis. Other methods should be accepted provided they are complete

NB: both B marks can be obtained together. In particular, some candidates may phrase (*) as: "so only –1 solves f(x) = 0" owtte, which is B1 B1.

(c) 1st M1 – attempts to differentiate the expression

2nd M1 – sets their derivative (provided it is a 3TQ) = 0 AND attempts to solve the 3TQ

1st A1 – correct x coordinates of the turning points

3rd M1 – substitutes their x coordinates into f to find the corresponding y coordinates. This is dependent on previous M marks.

2nd A1 – correct **coordinates** of the turning points. Must be in coordinate form.

(d) 1st B1 – correct shape of the cubic: one real root **and** both turning points above the x axes **and** in the region of positive x.

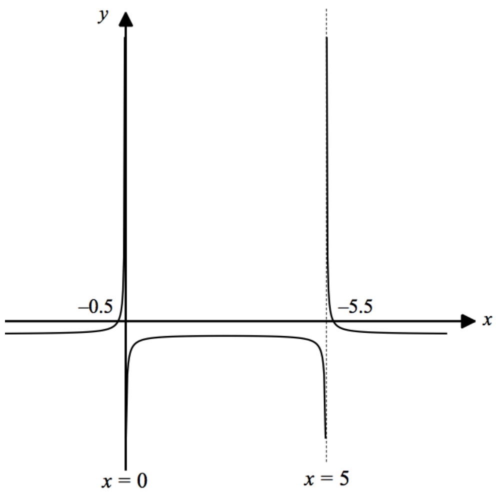
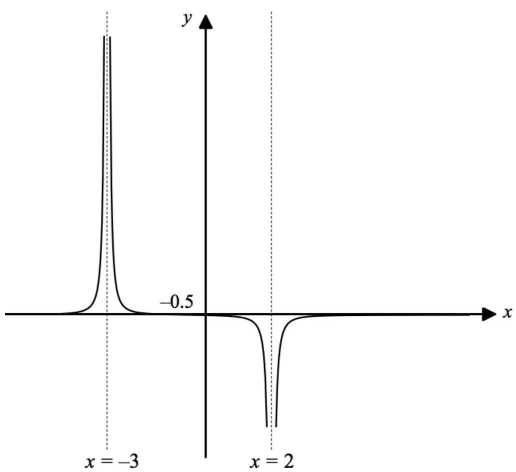
2nd B1 – correct intersections with the coordinate axes

(e) 1st B1 – f(x) drawn a dashed line

2nd B1 – $x \leq 1$ as a solid line

3rd B1 – correct region shaded

NB: no need to show coordinates of intersection, as this is not asked by the q.

Question	Scheme		Marks
6			
(a)		Horizontal translation of ± 3	B1
		Completely correct graph, with correct coordinates of intersection and vertical asymptotes	B1
			(2)
(b)		Two 'sections' of the graph with the correct shape	B1
		Completely correct graph, with correct x coordinates of intersection and vertical asymptotes	B1
			(2)
			4

Question 6 Notes

(a) 1st B1 – horizontal translation of $y = \mathrm{f}(x)$ by ± 3 units. There must be some indication that the translation is by ± 3 units for this mark, i.e. a correct x coordinate of intersection/vertical asymptote or a direct statement.

2nd B1 – fully correct graph (translation by + 3 units) with correct coordinates of intersection with the x axis and the correct equations of any vertical asymptotes.

(b) 1st B1 – two 'sections' of the graph with the correct shape (the asymptotes divide the graph into three sections).

2nd B1 – completely correct graph, with correct coordinates of intersection with the x axis and the correct equations of any vertical asymptotes.

Question	Scheme		Marks
7			
(a)	$(a-b)^2 \geq 0$	Seen or implied	M1
	$a^2 - 2ab + b^2 \geq 0$ $\Rightarrow a^2 + b^2 \geq 2ab$	Convincing proof with all steps shown	A1 **(2)**
(b)	Let $a = 3^x$ and $b = 3^{-x}$	Seen or implied	M1
	$\Rightarrow 3^{2x} + 3^{-2x} \geq 2\left(3^x\right)\left(3^{-x}\right)$	Substitutes a and b in	dM1
	$\Rightarrow 9^x + 9^{-x} \geq 2$	Convincing proof with all steps shown	A1 **(3)**
(c)	$9^{2x} + 1 = 2\left(9^x\right) \Rightarrow 9^{2x} - 2\left(9^x\right) + 1 = 0 \Rightarrow 9^x = ...$	Forms the correct 3TQ **AND** attempts to solve it	M1
	$\left(9^x - 1\right)^2 = 0$ $\Rightarrow 9^x = 1$ $\Rightarrow x = 0$	Correct value of x	A1 **(2)**
			7

Question 7 Notes

(b) 1st M1 – chooses correct values of a and b (seen or implied)

2nd M1 – correct substitution of values

A1 – complete and convincing proof with all steps seen without errors

(c) M1 - Forms the correct 3TQ **AND** attempts to solve it explicitly or through the use of a substitution

A1 – correct value of x

Correct answer only = **2/2**

Question	Scheme		Marks
8			
(a)	$m_{AB}=\frac{-5-2}{5--2}=-1$	Attempts to find the gradient of AB	M1
	gradient of bisector $=1$	Correct gradient of bisector	A1
	Midpoint of AB $=\left(\frac{3}{2},-\frac{3}{2}\right)$	Correct midpoint of AB	B1
	$y+\frac{3}{2}=x-\frac{3}{2}\quad\{\Rightarrow y=x-3\}$	Correct equation of perpendicular bisector	A1 oe **(4)**
(b/i)	Gradient of $l=\frac{3}{4}$	Attempts to find gradient of l	M1
	Normal to C at B is $y+5=-\frac{4}{3}(x-5)$	Correct equation of normal to C at B	A1
	$x-3+5=-\frac{4}{3}(x-5)\Rightarrow x=...$	Attempts to solve their normal and (a) simultaneously	dM1
	$\Rightarrow x=2$	Correct x (or y) coordinate of the centre of C	A1
	$\Rightarrow y='2'-3$	Attempts to find the y (or x) coordinate of the centre of C	dM1
	$(2,-1)$	Correct centre	A1 **(6)**
(b/ii)	$r^2=(2--2)^2+(-1-2)^2$	Attempts to use Pythagoras to find the radius	M1A1
	$r=5$	Correct radius of the circle	A1 **(3)**
(c)	$(x-2)^2+(y+1)^2=25$	Correct LHS, Correct RHS	B1ft B1ft **(2)**
			7

	Question 8 Notes

(a) 1st M1 – attempts to find the gradient of AB. Condone one sign error, but order of subtraction of coordinates must be consistent.

1st A1 – correct gradient of the **perpendicular bisector**

B1 – correct coordinates of midpoint. Seen or implied at any stage

2nd A1 – correct equation of perpendicular bisector in any form

(b/i) 1st M1 – attempts to find the gradient of l, i.e. by re-arranging to the form $y = mx + c$

1st A1 – correct equation of the normal to C at B

2nd M1 – attempts to solve the equation of their perpendicular bisector with their normal simultaneously for x (or y)

2nd A1 – one correct coordinate of the centre of C

3rd M1 – attempts to find the other coordinate of the centre of C

3rd A1 – correct centre

(b/ii) 1st M1 – attempts to find radius of the circle using Pythagoras. The expression may be different if they use the coordinates of B

1st A1 – correct expression for r or r^2 (need not be simplified)

2nd A1 – correct radius of the circle

(c) 1st B1 – correct LHS of the equation of the circle ft their centre

2nd B1 – correct RHS of the equation ft their radius

Isw if candidates expand the brackets. If candidates give the equation in the form $ax^2 + bx + ay^2 + cy + d = 0$ (oe), this is OK but check clearly before awarding ft.

Question	Scheme		Marks
9			
(a)	$\mathrm{p}(\theta)=\dfrac{9+16\left(1-\sin^2\theta\right)}{5+4\sin\theta}$	Uses $\cos^2\theta=1-\sin^2\theta$	M1
	$=\dfrac{25-16\sin^2\theta}{5+4\sin\theta}$		
	$=\dfrac{(5-4\sin\theta)(5+4\sin\theta)}{5+4\sin\theta}$	Uses the difference of two squares	dM1
	$=5-4\sin\theta \quad \{a=5,\ b=-4\}$	Correct proof with no errors seen	A1 **(3)**
(b) **Way 1**	$10\left(5-4\sin(\theta-30°)\right)=54 \Rightarrow \sin(\theta-30°)=...$	Attempts to find an expression for $\sin(\theta-30°)$ using their (a)	M1
	$\sin(\theta-30°)=-0.1$ $\Rightarrow \theta-30°=\sin^{-1}(-0.1)=-5.739...°$	Attempts to find principal value of $\sin(\theta-30°)$ using their '0.1'	dM1A1
	$\theta-30°=-5.739...,185.73...° \Rightarrow \theta=...$	Attempts to use their principal value to find the values of θ in range	dM1
	$\theta=24.26°,215.74°$	Correct solutions to 2 dp	A1 **(5)**
(b) **Way 2**	$\{\text{Let } X=\theta-30°,\}$ $9+16\cos^2 X=5.4(5+4\sin X)$ $\Rightarrow 16\cos^2 X-21.6\sin X-18=0$ $\Rightarrow 16\left(1-\sin^2 X\right)-21.6\sin X-18=0$	Attempts to form a quadratic in **ONE** trigonometric variable	M1
	$16\sin^2 X+21.6\sin X+2=0$		
	$\Rightarrow(10\sin X+1)(4\sin X+5)=0$ $\Rightarrow \sin X=-0.1$ $\Rightarrow X=\sin^{-1}(-0.1)=-5.739...°$	Attempts to find principal value of $\sin X$ using their '0.1'	dM1A1
	$\theta-30°=-5.739...,185.73...° \Rightarrow \theta=...$	Attempts to use their principal value to find the values of θ in range	dM1
	$\theta=24.26°,215.74°$	Correct solutions to 2 dp	A1 **(5)**

(c)	Max. value = 9	B1
	Occurs when $\sin\theta = -1 \Rightarrow \theta = 270°$	M1A1 **(3)**
		11
	Question 9 Notes	

(a) 1st M1 – uses the identity $\cos^2\theta = 1 - \sin^2\theta$

2nd M1 – uses the difference of two squares on their expression

A1 – complete and convincing proof with no errors seen.

(b) Way 1:

1st M1 – attempts to find an expression for $\sin(\theta - 30°)$ using their (a) and the information given

2nd M1 – attempts to find the principal value of $\sin(\theta - 30°)$ using their '–0.1'. NB: their '–0.1' must be in [-1,1].

1st A1 – correct principal value of –5.739…

3rd M1 – attempts to find other value in range **AND** then adds 30 to these values. Ignore any additional values that appear at this stage (e.g. 354.26…)

2nd A1 – correct solutions to two decimal places with no extra solutions seen inside or outside the range.

Way 2:

1st M1 – attempts to form a 3TQ in one trigonometric variable

The rest is as Way 1.

(c) B1 – correct max. value. No ft here.

M1 – states/implies max. occurs when $\sin\theta = -1$

A1 – correct value of θ

Question	Scheme		Marks
10			
(a)	$E = 2^0 \mathrm{e}^{\left(\frac{0+4}{10}\right)} = 1.491...$	Substitutes 0 into expression for E	M1
	Josh was earning £1.49 **per hour** {when the video was released}	Correct answer in context	A1 **(2)**
(b)	$\ln 1024 = \ln\left(2^t \mathrm{e}^{\left(\frac{t+4}{10}\right)}\right)$	Logs both sides	M1
	$\ln 1024 = \ln\left(2^t\right) + \ln\left(\mathrm{e}^{\left(\frac{t+4}{10}\right)}\right)$	Uses addition-multiplication property of logs	dM1
	$10\ln 2 = t\ln 2 + \left(\frac{t+4}{10}\right)$	Uses power property of logs on RHS **(ignore LHS) and** that ln(e) = 1	dM1
	$100\ln 2 - 4 = t(10\ln 2 + 1) \Rightarrow t = ...$	Attempts to make t the subject	dM1
	$t = \dfrac{100\ln 2 - 4}{10\ln 2 + 1}$ $\{a = 100, b = -4, c = 10, d = 1\}$	Correct expression	A1 **(5)**
(c)	the model / E will {still} be increasing	Correct idea	B1 **(1)**
			8

Question 10 Notes

(a) M1 – substitutes 0 into the expression A1 – correct earnings in context. Must include correct time period.

(b) 1st M1 – takes natural logs to both sides

2nd M1 – uses the addition-multiplication property of logs to separate the terms

3rd M1 – uses the power rule for logs on the RHS **AND** that ln(e) = 1. Ignore the LHS for this mark

4th M1 – attempts to make t the subject. Again, ignore if candidates still have ln(1024) here

A1 – correct expression

(c) B1 – idea that the model predicts that Josh's earnings will still be increasing

Question	Scheme		Marks
11			
	$\mathrm{f}(x)=2x-\dfrac{2\sqrt{x}}{1/2}+c=2x-4\sqrt{x}+c$	Attempts to integrate to find f(x)	M1
	$\mathrm{f}(1)=-2\Rightarrow 2-4+c=-2\Rightarrow c=...$	Attempts to find constant of integration	dM1
	$\mathrm{f}(x)=2x-4\sqrt{x}$	Correct f(x)	A1
	$2x=4\sqrt{x}\Rightarrow 4x^2-16x=0$ $\Rightarrow\{x=0,\}x=4$	Attempts to find out where their f crosses the x axis (i.e. the (upper) bound for integration)	dM1A1
	Signed area = $\displaystyle\int_0^4\left(2x-4\sqrt{x}\right)dx=\left[x^2-\frac{8\sqrt{x^3}}{3}\right]_0^4$	Correct expression for the area with their 4 ; correct integration	dM1A1
	$=\left(4^2-\dfrac{8\sqrt{4^3}}{3}\right)\{-0\}=16-\dfrac{64}{3}=-\dfrac{16}{3}$	Substitutes limits the correct way around and computes their definite integral	dM1 A1ft
	Area = $\dfrac{16}{3}$ {units^2}	Correct area	A1
			10

Question 11 Notes

1st M1 – **attempts** integrates $\mathrm{f}'(x)$ wrt x to find f(x). Must see a constant of integration.

2nd M1 – substitutes the point (–1, 2) into their f to find the constant of integration. Must sub coordinates in the right way around. Dependent on 1st M1

2nd A1 – correct expression for f

3rd M1 – attempts to find the points where f crosses the x axis; in particular, we are looking for an attempt to find the upper limit of integration (the lower limit is obvious). Dependent on previous M marks

3rd A1 – identifies upper limit as 4

4th M1 – correct expression for the area with 0 as the lower limit and **their** 4 as the upper limit. Dependent on previous M marks

4th A1 – correct integration. No ft.

5th M1 – attempts to find the integral and substitutes the limits in the right way around

5th A1 – correct definite integration ft their integral and limits

6th A1 – correct area

ALTERNATIVE:

The integral $\int_4^0 \mathrm{f}(x)\,dx = \frac{16}{3}$. Candidates may use this approach, but for the final A1, they should make a clear statement that this gives the AREA as reversing the limits changes the sign of the integral. If this statement is not made, withhold the final A1.