FM1 – Mechanics Chapter 3
Exam Questions:
Hooke's Law (Dynamics) and Elastic Energy

Q1.

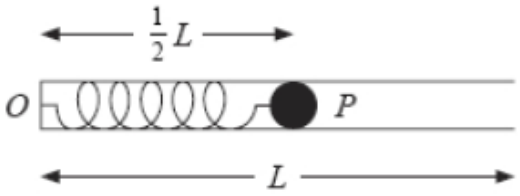

Figure 1

A light elastic spring, of natural length L and modulus of elasticity λ, has a particle P of mass m attached to one end. The other end of the spring is fixed to a point O on the closed end of a fixed smooth hollow tube of length L.

The tube is placed horizontally and P is held inside the tube with $OP = \frac{1}{2}L$, as shown in Figure 1. The particle P is released and passes through the open end of the tube with speed $\sqrt{(2gL)}$.

(a) Show that $\lambda = 8mg$.

(4)

The tube is now fixed vertically and P is held inside the tube with $OP = \frac{1}{2}L$ and P above O. The particle P is released and passes through the open top of the tube with speed u.

(b) Find u.

(5)

(Total 9 marks)

Q2.

A light elastic string has natural length 8 m and modulus of elasticity 80 N.

The ends of the string are attached to fixed points *P* and *Q* which are on the same horizontal level and 12 m apart. A particle is attached to the mid-point of the string and hangs in equilibrium at a point 4.5 m below *PQ*.

(a) Calculate the weight of the particle.

(6)

(b) Calculate the elastic energy in the string when the particle is in this position.

(3)

(Total 9 marks)

Q3.

One end of a light elastic string, of natural length 1.5m and modulus of elasticity 14.7N, is attached to a fixed point O on a ceiling. A particle P of mass 0.6kg is attached to the free end of the string. The particle is held at O and released from rest. The particle comes to instantaneous rest for the first time at the point A.

Find

(a) the distance OA,

(6)

(b) the magnitude of the instantaneous acceleration of P at A.

(3)

(Total for question = 9 marks)

Q4.

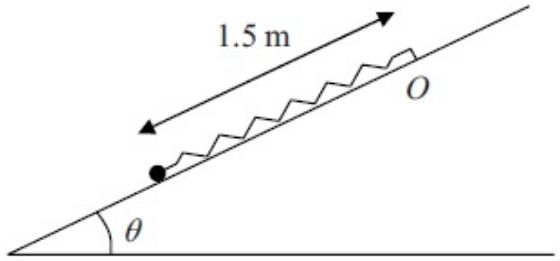

Figure 2

A particle of mass 0.5 kg is attached to one end of a light elastic spring of natural length 0.9 m and modulus of elasticity
λ newtons. The other end of the spring is attached to a fixed point O on a rough plane which is inclined at an angle θ to the horizontal, where $\sin\theta = \frac{3}{5}$. The coefficient of friction between the particle and the plane is 0.15. The particle is held on the plane at a point which is 1.5 m down the line of greatest slope from O, as shown in Figure 2. The particle is released from rest and first comes to rest again after moving 0.7 m up the plane.

Find the value of λ.

(9)

(Total 9 marks)

Q5.

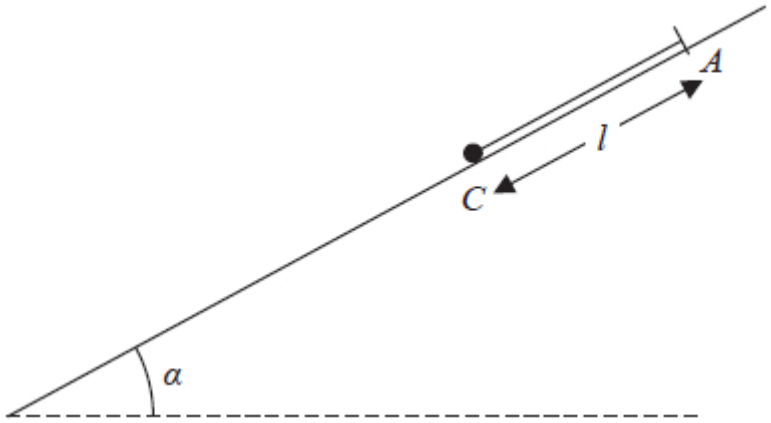

Figure 3

One end of a light elastic string, of natural length l and modulus of elasticity $3mg$, is fixed to a point A on a fixed plane inclined at an angle α to the horizontal, where $\sin \alpha = \frac{3}{5}$

A small ball of mass $2m$ is attached to the free end of the string. The ball is held at a point C on the plane, where C is below A and $AC = l$ as shown in Figure 3. The string is parallel to a line of greatest slope of the plane. The ball is released from rest. In an initial model the plane is assumed to be smooth.

(a) Find the distance that the ball moves before first coming to instantaneous rest.

(5)

In a refined model the plane is assumed to be rough. The coefficient of friction between the ball and the plane is μ. The ball first comes to instantaneous rest after moving a distance $\frac{2}{5}l$.

(b) Find the value of μ.

(6)

(Total 11 marks)

Q6.

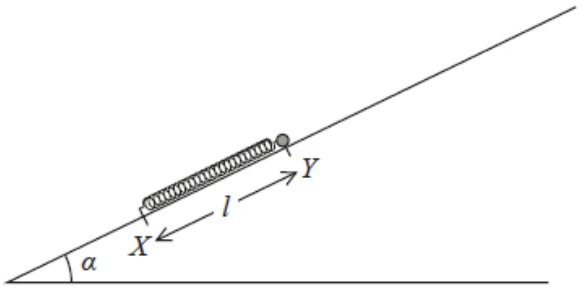

Figure 2

A light elastic spring has natural length 3*l* and modulus of elasticity 3*mg*.

One end of the spring is attached to a fixed point *X* on a rough inclined plane.

The other end of the spring is attached to a package *P* of mass *m*.

The plane is inclined to the horizontal at an angle α where $\tan\alpha = \frac{3}{4}$

The package is initially held at the point *Y* on the plane, where *XY* = *l*. The point *Y* is higher than *X* and *XY* is a line of greatest slope of the plane, as shown in Figure 2.

The package is released from rest at *Y* and moves up the plane.

The coefficient of friction between *P* and the plane is $\frac{1}{3}$

By modelling *P* as a particle,

(a) show that the acceleration of *P* at the instant when *P* is released from rest is $\frac{17}{15}g$

(5)

(b) find, in terms of *g* and *l*, the speed of *P* at the instant when the spring first reaches its natural length of 3*l*.

(6)

(Total for question = 11 marks)

Q7.

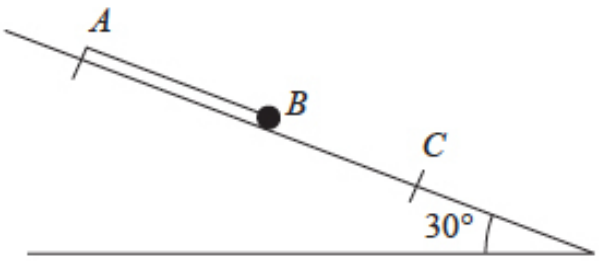

Figure 2

One end *A* of a light elastic string, of natural length a and modulus of elasticity 6*mg*, is fixed at a point on a smooth plane inclined at 30° to the horizontal. A small ball *B* of mass *m* is attached to the other end of the string. Initially *B* is held at rest with the string lying along a line of greatest slope of the plane, with *B* below *A* and *AB* = *a*. The ball is released and comes to instantaneous rest at a point *C* on the plane, as shown in Figure 2. Find

(a) the length *AC*,

(5)

(b) the greatest speed attained by *B* as it moves from its initial position to *C*.

(7)

(Total 12 marks)

Q8.

A spring of natural length a has one end attached to a fixed point A. The other end of the spring is attached to a package P of mass m.
The package P is held at rest at the point B, which is vertically below A such that $AB = 3a$.
After being released from rest at B, the package P first comes to instantaneous rest at A.
Air resistance is modelled as being negligible.

By modelling the spring as being light and modelling P as a particle,

(a) show that the modulus of elasticity of the spring is $2mg$

(5)

(b) (i) Show that P attains its maximum speed when the extension of the spring is $\frac{1}{2}a$

(ii) Use the principle of conservation of mechanical energy to find the maximum speed, giving your answer in terms of a and g.

(6)

In reality, the spring is not light.

(c) State one way in which this would affect your energy equation in part (b).

(1)

(Total for question = 12 marks)