Year 1 Pure

Chapter 7 ALGEBRA + FUNCTIONS (Answers)

1) $f(x) = 2x^3 - x^2 + px + 6$

a) $(x-1)$ $\therefore$ let $x = 1$

$$2(1)^3 - (1)^2 + 1p + 6 = 0$$

$$2 - 1 + p + 6 = 0$$

$$p = -7$$

b) $(2x+1)$ $\therefore$ let $x = -\frac{1}{2}$

$$2\left(-\tfrac{1}{2}\right)^3 - \left(-\tfrac{1}{2}\right)^2 - 7\left(-\tfrac{1}{2}\right) + 6$$

$$2\left(-\tfrac{1}{8}\right) - \left(\tfrac{1}{4}\right) + \tfrac{7}{2} + 6$$

$$-\tfrac{1}{4} - \tfrac{1}{4} + \tfrac{7}{2} + 6 = \underline{\underline{9}}$$

(2) $f(x) = 2x^3 - 3x^2 - 39x + 20$

a) $(x+4) \therefore$ Let $x = -4$

$$2(-4)^3 - 3(-4)^2 - 39(-4) + 20 = 0$$

$\therefore (x+4)$ is a factor

b)

$$\begin{array}{r|l} & \;\; 2x^2 - 11x + 5 \\ \hline x+4 & 2x^3 - 3x^2 - 39x + 20 \\ - & 2x^3 + 8x^2 \\ \hline & \qquad -11x^2 - 39x \\ - & \qquad -11x^2 - 44x \\ \hline & \qquad\qquad\quad 5x + 20 \\ - & \qquad\qquad\quad 5x - 20 \\ \hline & \qquad\qquad\qquad\quad 0 \end{array}$$

$$(x+4)(2x^2 - 11x + 5)$$

$$= (x+4)(2x-1)(x-5)$$

3

$$f(x) = 3x^3 - 5x^2 - 58x + 40$$

a) $(x-3)$ $\therefore$ let $x = 3$

$$3(3)^3 - 5(3)^2 - 58(3) + 40 = -98$$

b)

$$\begin{array}{r|l} & 3x^2 + 10x - 8 \\ \hline (x-5) & 3x^3 - 5x^2 - 58x + 40 \\ - & 3x^3 - 15x^2 \\ & \quad 10x^2 - 58x \\ - & \quad 10x^2 - 50x \\ & \qquad -8x + 40 \\ - & \qquad -8x + 40 \\ & \qquad\qquad 0 \end{array}$$

$$(x-5)(3x^2 + 10x - 8)$$

$$(x-5)(3x-2)(x+4)$$

$x = 5$ $\quad x = \frac{2}{3}$ $\quad x = -4$

④ $$f(x) = x^4 + 5x^3 + ax + b$$

a) $(x-2)$ $\therefore$ let $x = 2$

$$(2)^4 + 5(2)^3 + 2a + b$$
$$= 16 + 40 + 2a + b$$
$$= 56 + 2a + b$$

$(x+1)$ $\therefore$ let $x = -1$

$$(-1)^4 + 5(-1)^3 - 1a + b$$
$$= 1 - 5 - a + b$$
$$= -4 - a + b$$

hence

$$56 + 2a + b = -4 - a + b$$
$$3a = -60$$
$$a = -20$$

b) $(x+3)$ $\therefore$ let $x = -3$

$$(-3)^4 + 5(-3)^3 - 20(-3) + b = 0$$
$$81 - 135 + 60 + b = 0$$
$$b = -6$$

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$$f(x) = 2x^3 + ax^2 + bx - 6$$

a) $(2x-1)$ $\therefore$ let $x = \frac{1}{2}$

$$2\left(\tfrac{1}{2}\right)^3 + a\left(\tfrac{1}{2}\right)^2 + b\left(\tfrac{1}{2}\right) - 6 = -5$$

$$\frac{1}{4} + \frac{1}{4}a + \frac{1}{2}b - 6 = -5$$

$(\times 4)$

$$1 + a + 2b - 24 = -20$$

$$a + 2b = 3$$

$(x+2)$ $\therefore$ let $x = -2$

$$2(-2)^3 + a(-2)^2 + b(-2) - 6 = 0$$

$$-16 + 4a - 2b - 6 = 0$$

$$4a - 2b = 22$$

$$\begin{array}{rrcl} & 4a - 2b & = & 22 \\ + & a + 2b & = & 3 \\ \hline & 5a & = & 25 \end{array}$$

$$a = 5$$

$$\therefore b = -1$$

5

b)

$$\begin{array}{r|l} & 2x^2 + x - 3 \\ \hline x+2 & 2x^3 + 5x^2 - x - 6 \\ & -\underline{(2x^3 + 4x^2)} \\ & \quad\quad\quad x^2 - x \\ & \quad\quad -\underline{(x^2 + 2x)} \\ & \quad\quad\quad\quad -3x - 6 \\ & \quad\quad\quad -\underline{(-3x - 6)} \\ & \quad\quad\quad\quad\quad\quad 0 \end{array}$$

$$(x+2)(2x^2+x-3)$$

$$(x+2)(2x+3)(x-1)$$

(6) $f(x) = 6x^3 + px^2 + qx + 8$

a) $(2x-1) \therefore$ let $x = \frac{1}{2}$

$$6\left(\tfrac{1}{2}\right)^3 + p\left(\tfrac{1}{2}\right)^2 + q\left(\tfrac{1}{2}\right) + 8 = 0$$

$$\frac{3}{4} + \frac{1}{4}p + \frac{1}{2}q + 8 = 0$$

$(\times 4)$

$$3 + p + 2q + 32 = 0$$

$$p + 2q = -35$$

$(x-1) \therefore$ let $x = 1$

$$6(1)^3 + p(1)^2 + q(1) + 8 = \cancel{0}\ -7$$

$$6 + p + q + 8 = -7$$

$$p + q = -21$$

$$\begin{array}{rl} & p + 2q = -35 \\ - & p + q = -21 \\ \hline & q = -14 \end{array}$$

$\therefore p = -7$

(6)

b)

$$
\begin{array}{r|l}
 & 3x^2 - 2x - 8 \\
\hline
2x-1 & 6x^3 - 7x^2 - 14x + 8 \\
 & -\underline{(6x^3 - 3x^2)} \\
 & 4x^2 - 14x \\
 & -\underline{(4x^2 + 2x)} \\
 & -16x + 8 \\
 & -\underline{(-16x + 8)} \\
 & 0
\end{array}
$$

$$(2x-1)(3x^2-2x-8)$$

$$(2x-1)(3x-4)(x+2)$$

⑦ $(a+b)^2 - (a-b)^2$

$= (a^2 + 2ab + b^2) - (a^2 - 2ab + b^2)$

$= a^2 + 2ab + b^2 - a^2 + 2ab - b^2$

$= 4ab$

$\therefore \quad (a+b)^2 - (a-b)^2 = 4ab$

⑧ $(2m+1)(2n+1)$

$= 4mn + 2m + 2n + 1$

$= 2(2mn + m + n) + 1$

$=$ even $+ 1 =$ odd

⑨

$$y = x^2 + kx + k^2$$

complete the square

$$y = \left(x + \frac{k}{2}\right)^2 - \left(\frac{k}{2}\right)^2 + k^2$$

$$y = \left(x + \frac{k}{2}\right)^2 - \frac{k^2}{4} + k^2$$

$$y = \left(x + \frac{k}{2}\right)^2 + \frac{3}{4}k^2$$

all square numbers are positive

all square numbers are positive

$\therefore$ positive number + positive number > 0

so y must be positive