

Year 2 Pure Chapter 12: Vectors – Exam Questions (Answers)

1 Coordinates of M are $(3, 5, 4)$

Distance from M to C

$$\begin{aligned} &= \sqrt{(5-3)^2 + (8-5)^2 + (7-4)^2} \\ &= \sqrt{4+9+9} = \sqrt{22} \end{aligned}$$

2 Distance from P to Q

$$\begin{aligned} &= \sqrt{((a-2)-2)^2 + (6-3)^2 + (7-a)^2} \\ &= \sqrt{a^2 - 8a + 16 + 9 + 49 - 14a + a^2} \\ &= \sqrt{2a^2 - 22a + 74} = \sqrt{14} \end{aligned}$$

$$2a^2 - 22a + 74 = 14$$

$$a^2 - 11a + 30 = 0$$

$$(a-5)(a-6) = 0$$

$$a = 5 \text{ or } a = 6$$

$$3 \quad |\overrightarrow{AB}| = \sqrt{3^2 + t^2 + 5^2} = \sqrt{t^2 + 34}$$

$$\sqrt{t^2 + 34} = 5\sqrt{2}$$

$$t^2 + 34 = 50$$

$$t^2 = 16$$

$$t = 4 \quad (\text{since } t > 0)$$

$$\text{So } \overrightarrow{AB} = -3\mathbf{i} + 4\mathbf{j} + 5\mathbf{k}$$

$$6\mathbf{i} - 8\mathbf{j} - \frac{5}{2}t\mathbf{k} = 6\mathbf{i} - 8\mathbf{j} - 10\mathbf{k}$$

$$= -2\overrightarrow{AB}$$

$$\text{So } \overrightarrow{AB} \text{ is parallel to } 6\mathbf{i} - 8\mathbf{j} - \frac{5}{2}t\mathbf{k}$$

4 a Let O be the fixed origin.

$$\overrightarrow{PQ} = \overrightarrow{OQ} - \overrightarrow{OP} = -3\mathbf{i} - 8\mathbf{j} + 3\mathbf{k}$$

$$\overrightarrow{PR} = \overrightarrow{OR} - \overrightarrow{OP} = -3\mathbf{i} - 9\mathbf{j} + 8\mathbf{k}$$

$$\overrightarrow{QR} = \overrightarrow{OR} - \overrightarrow{OQ} = -\mathbf{j} + 5\mathbf{k}$$

b $|\overrightarrow{PQ}| = \sqrt{9 + 64 + 9} = \sqrt{82}$

$$|\overrightarrow{PR}| = \sqrt{9 + 81 + 64} = \sqrt{154}$$

$$|\overrightarrow{QR}| = \sqrt{1 + 25} = \sqrt{26}$$

$$\cos \angle QPR = \frac{82 + 154 - 26}{2 \times \sqrt{82} \times \sqrt{154}} = 0.9343\dots$$

$$\angle QPR = 20.87\dots^\circ$$

Area of triangle

$$= \frac{1}{2} \times \sqrt{82} \times \sqrt{154} \sin 20.87\dots^\circ$$

$$= 20.0 \text{ (1 d.p.)}$$

5 a $\overrightarrow{DE} = \overrightarrow{OE} - \overrightarrow{OD} = 4\mathbf{i} + 3\mathbf{j} + 4\mathbf{k}$

$$\overrightarrow{EF} = \overrightarrow{OF} - \overrightarrow{OE} = -3\mathbf{i} - 4\mathbf{j} + 4\mathbf{k}$$

$$\overrightarrow{FD} = \overrightarrow{OD} - \overrightarrow{OF} = -\mathbf{i} + \mathbf{j} - 8\mathbf{k}$$

b $|\overrightarrow{DE}| = \sqrt{16 + 9 + 16} = \sqrt{41}$

$$|\overrightarrow{EF}| = \sqrt{9 + 16 + 16} = \sqrt{41}$$

$$|\overrightarrow{FD}| = \sqrt{1 + 1 + 64} = \sqrt{66}$$

c Two sides are equal in length so the triangle is isosceles.

Total force on particle

$$= \mathbf{F} = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3$$

$$= ((b+1)\mathbf{i} + (4-b)\mathbf{j} + (7-b)\mathbf{k})\text{N}$$

$$\begin{aligned} |\mathbf{F}| &= \sqrt{(b+1)^2 + (4-b)^2 + (7-b)^2} \\ &= \sqrt{b^2 + 2b + 1 + 16 - 8b + b^2 + 49 - 14b + b^2} \\ &= \sqrt{3b^2 - 20b + 66} \end{aligned}$$

$$|\mathbf{F}| = m|\mathbf{a}|$$

$$\Rightarrow \sqrt{3b^2 - 20b + 66} = 2 \times 3.5 = 7$$

$$3b^2 - 20b + 66 = 49$$

$$3b^2 - 20b + 17 = 0$$

$$(b-1)(3b-17) = 0$$

$$b = 1 \text{ or } b = \frac{17}{3}$$