Name:

Level 2 Further Maths

Matrix Transformations

Corbettmaths

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

Revision for this topic

www.corbettmaths.com/more/further-maths/

1. Find where the matrix $\begin{pmatrix} 5 & -2 \\ -1 & 3 \end{pmatrix}$ maps the point $(-3, 1)$

$$\begin{pmatrix} 5 & -2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} -3 \\ 1 \end{pmatrix} = \begin{pmatrix} -17 \\ 6 \end{pmatrix}$$

$(-17, 6)$

(2)

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2. Find where the matrix $\begin{pmatrix} 0 & 4 \\ 2 & -8 \end{pmatrix}$ maps the point $(2, -1)$

$$\begin{pmatrix} 0 & 4 \\ 2 & -8 \end{pmatrix}\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} -4 \\ 12 \end{pmatrix}$$

$(-4, 12)$

(2)

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3. The transformation matrix $\begin{pmatrix} b & -2 \\ -1 & 3 \end{pmatrix}$ maps the point $(5, 1)$ onto the point $(16, c)$

$$\begin{pmatrix} b & -2 \\ -1 & 3 \end{pmatrix}\begin{pmatrix} 5 \\ 1 \end{pmatrix} = \begin{pmatrix} 5b-2 \\ -5+3 \end{pmatrix}$$

$5b - 2 = 16$

$5b = 18$

$b = 3.6$

$b =$ 3.6 $\qquad c =$ -2

(3)

4. The transformation matrix **M** is $\begin{pmatrix} 1 & a \\ -4 & 1 \end{pmatrix}$

The image of the point $(b,\ 3)$ under **M** is $(14,\ -5)$

$$\begin{pmatrix} 1 & a \\ -4 & 1 \end{pmatrix}\begin{pmatrix} b \\ 3 \end{pmatrix} = \begin{pmatrix} b+3a \\ -4b+3 \end{pmatrix}$$

$3a + b = 14$

$3a + 2 = 14$

$3a = 12$

$a = 4$

$-4b + 3 = -5$

$-4b = -8$

$b = 2$

$a =$ ......4...... $b =$ ......2......

(3)

5. The matrix $\begin{pmatrix} 3 & x \\ 6 & -3 \end{pmatrix}$ maps the point $(y,\ 4)$ onto the point $(-14, 24)$

Find the values of x and y

$$\begin{pmatrix} 3 & x \\ 6 & -3 \end{pmatrix}\begin{pmatrix} y \\ 4 \end{pmatrix} = \begin{pmatrix} 3y+4x \\ 6y-12 \end{pmatrix}$$

$6y - 12 = 24$

$6y = 36$

$y = 6$

$18 + 4x = -14$

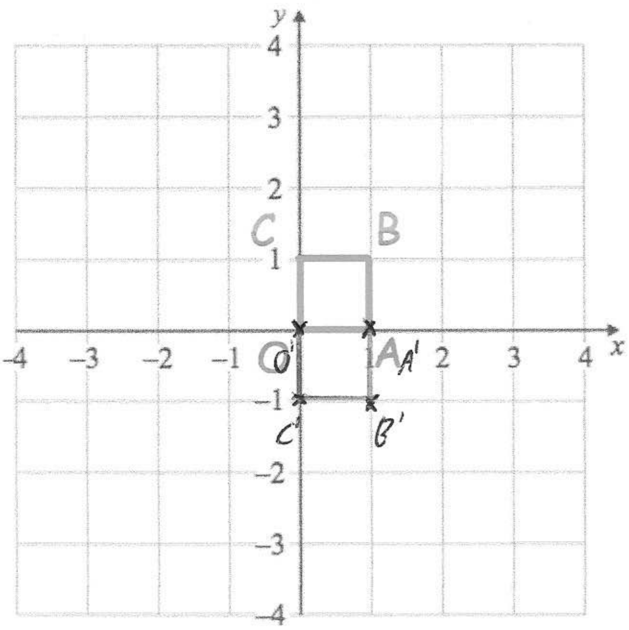
$4x = -32$

$x = -8$

$x =$ ......-8...... $y =$ ......6......

(4)

6. The diagram shows the unit square OABC



OABC is transformed by the matrix $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$ to give OA'B'C'

(a) Draw and label OA'B'C'

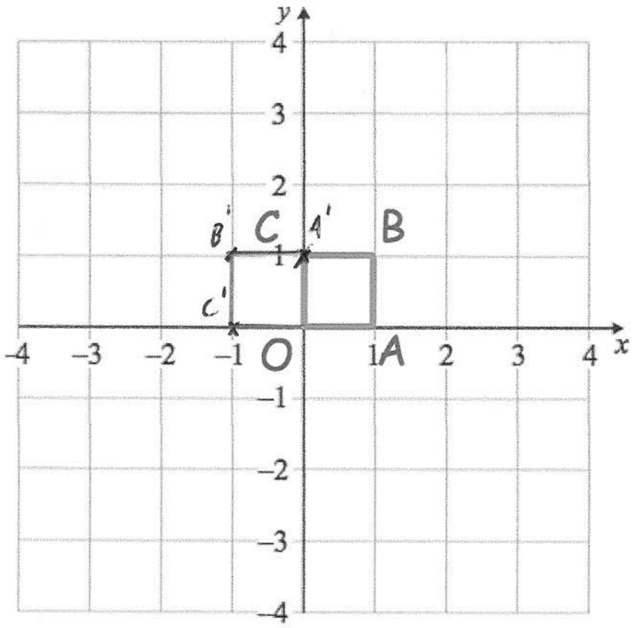
(2)

(b) Describe the transformation fully.

Reflection in the x - axis

(2)

7. The diagram shows the unit square OABC



OABC is transformed by the matrix $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$ to give OA'B'C'

(a) Draw and label OA'B'C'

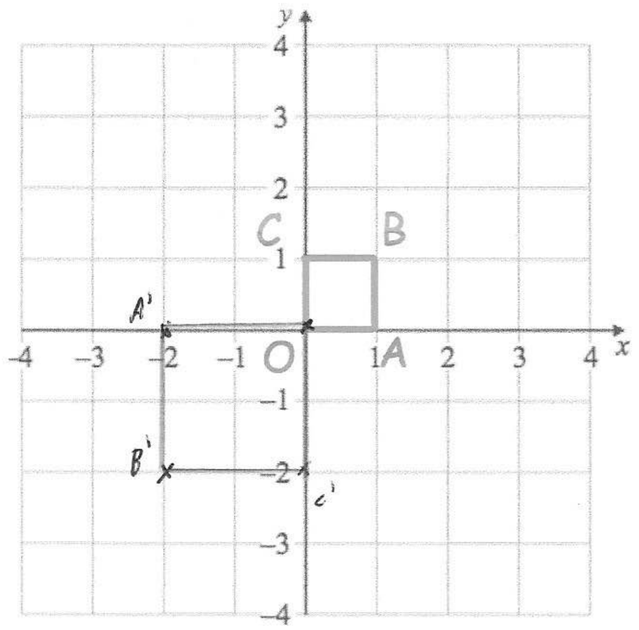
(2)

(b) Describe the transformation fully.

90° rotation anticlockwise about (0,0)

(2)

8. The diagram shows the unit square OABC



OABC is transformed by the matrix $\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$ to give OA'B'C'

(a) Draw and label OA'B'C'

(2)

(b) Describe the transformation fully.

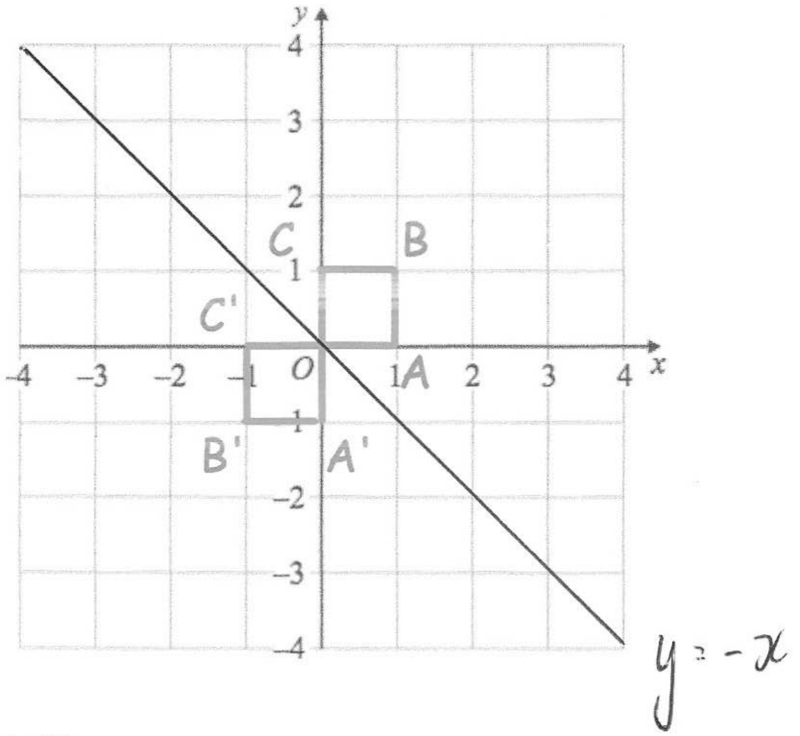
Enlargement with scale factor −2, centre (0,0)

(2)

9. The unit square OABC has vertices

O(0, 0) A(1, 0) B(1, 1) C(0, 1)

OABC is mapped to OA'B'C' under transformation matrix **M**



Work out matrix **M**

$$\begin{pmatrix} a & b \\ c & d \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$

$a = 0$

$c = -1$

$$\begin{pmatrix} 0 & b \\ -1 & d \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$$

$b = -1$

$d = 0$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$$

(3)

10. The point $(4, a)$ is invariant when transformed by the matrix $\begin{pmatrix} -5 & -2 \\ 3 & 2 \end{pmatrix}$

Find the value of a

$$\begin{pmatrix} -5 & -2 \\ 3 & 2 \end{pmatrix}\begin{pmatrix} 4 \\ a \end{pmatrix} = \begin{pmatrix} -20 - 2a \\ 12 + 2a \end{pmatrix}$$

$-20 - 2a = 4$

$-24 = 2a$

$a = -12$

$12 + 2a$

$12 + (-24) = -12$

$a =$ -12

(4)

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11. The transformation matrix $\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}$ maps the point $(2, -4)$ onto the point $(10, 40)$

Find the values of a and b

$$\begin{pmatrix} a & b \\ -a & 2b \end{pmatrix}\begin{pmatrix} 2 \\ -4 \end{pmatrix} = \begin{pmatrix} 2a - 4b \\ -2a - 8b \end{pmatrix}$$

$2a - 4b = 10$

add $-2a - 8b = 40$

$-12b = 50$

$b = -4\frac{1}{6}$

$2a - (-16\frac{2}{3}) = 10$

$2a = -6\frac{2}{3}$

$a = -3\frac{1}{3}$

$a =$ $-3\frac{1}{3}$ $b =$ $-4\frac{1}{6}$

(5)

12. The transformation matrix $\begin{pmatrix} 3a & 4b \\ b & a \end{pmatrix}$ maps the point $(1, -2)$ onto the point $(-2, -1)$

Find the values of a and b

$$\begin{pmatrix} 3a & 4b \\ b & a \end{pmatrix}\begin{pmatrix} 1 \\ -2 \end{pmatrix} = \begin{pmatrix} 3a - 8b \\ b - 2a \end{pmatrix}$$

$3a - 8b = -2$

$\times 8$ $b - 2a = -1$

add $-16a + 8b = -8$

$-13a = -10$

$a = \frac{10}{13}$

$b = \frac{7}{13}$

$a = $ $\frac{10}{13}$ $b = $ $\frac{7}{13}$

(5)

13. The transformation matrix $\begin{pmatrix} -a & 2b \\ -8b & 3a \end{pmatrix}$ maps the point $(-1, -2)$ onto the point $(-12, 32)$

Find the values of a and b

$$\begin{pmatrix} -a & 2b \\ -8b & 3a \end{pmatrix}\begin{pmatrix} -1 \\ -2 \end{pmatrix} = \begin{pmatrix} a - 4b \\ 8b - 6a \end{pmatrix}$$

$a - 4b = -12$

$8b - 6a = 32$

$6a - 24b = -72$

add $8b - 6a = 32$

$-16b = -40$

$b = 2.5$

$20 - 6a = 32$

$-6a = 12$

$a = -2$

$a = $ -2 $b = $ 2.5

(5)

14. Work out the matrix that transforms the unit square by a 90° anticlockwise rotation about O

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(2)

15. Work out the matrix that transforms the unit square by a 180° rotation about O.

$$\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

(2)

16. Work out the matrix that transforms the unit square by a reflection in the y-axis.

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

....

(2)

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17. Work out the matrix that transforms the unit square by a reflection in line $y = x$

$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

....

(2)

18. Describe fully the **single** transformation represented by $\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$

A rotation of 90° clockwise about the origin.

(3)

19. Describe fully the **single** transformation represented by $\begin{pmatrix} 5 & 0 \\ 0 & 5 \end{pmatrix}$

Enlargement of scale factor 5 , centre (0,0)

(3)

20. The transformation matrix $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ maps point P to point Q.

The transformation matrix $\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$ maps point Q to point R.

Point P is $(-3, 2)$.

Work out the coordinates of point R.

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} -3 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -2 \\ -3 \end{pmatrix}$$

$(-2, -3)$

(3)

21. The transformation matrix $\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ maps point P to point Q.

The transformation matrix $\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}$ maps point Q to point R.

Point R is $(-6, -15)$.

Work out the coordinates of point P. (x, y)

$$\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} y \\ x \end{pmatrix}$$

$$\begin{pmatrix} -3 & 0 \\ 0 & -3 \end{pmatrix}\begin{pmatrix} y \\ x \end{pmatrix} = \begin{pmatrix} -3y \\ -3x \end{pmatrix}$$

$-3y = -6$ $\quad$ $-3x = -15$

$y = 2$ $\quad$ $x = 5$

$(5, 2)$

(5)

$$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

22. The unit square OABC is transformed by a reflection in the y-axis followed by enlargement scale factor 3, centre the origin. $\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}$

What is the matrix of the combined transformation?

$$\begin{pmatrix} 3 & 0 \\ 0 & 3 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$$

$\begin{pmatrix} -3 & 0 \\ 0 & 3 \end{pmatrix}$

(4)

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23. Here are two transformation:

A: A rotation of 180° $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$

B: A reflection in the x-axis $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

Work out the single matrix which represents the combined transformation A followed by B.

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

$\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$

(4)

24. The transformation matrix **M** represents a 270° clockwise rotation about the origin.

(a) Write down matrix **M**

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

(2)

(b) Describe fully the **single** transformation represented by **M**^{3}

90° clockwise rotation about the origin

(2)

(c) Write down the matrix for the **single** transformation represented by **M**^{3}

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

(2)

25. The unit square is transformed by matrix **Q** followed by matrix **R** followed by the matrix **S**

This is equivalent to transforming the unit square by the identity matrix.

Matrix **R** represents a rotation.

Matrices **Q** and **S** represent reflections.

Write down three possible matrices for **Q**, **R** and **S**

Possible solution:

Q - reflection in the y-axis

R - rotation 180° about O.

S - reflection in the x-axis

Q = $\begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$ **R** = $\begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$ **S** = $\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

(6)