

1)

$$\int \frac{2x}{x^2+4} dx$$

$$u = x^2 + 4$$

$$\frac{du}{dx} = 2x$$

$$\int \frac{2x}{x^2+4} \frac{dx}{\frac{du}{dx}} du$$

$$\frac{dx}{du} = \frac{1}{2x}$$

$$\int \frac{2x}{u} \cdot \frac{1}{2x} du$$

$$\int \frac{1}{u} du$$

$$\ln u + C$$

$$\underline{\underline{\ln(x^2 + 4) + C}}$$

2)

$$\int \sin^3 x \cos x dx$$

$$u = \sin x$$

$$\frac{du}{dx} = \cos x$$

$$\int u^3 \cos x \frac{dx}{\frac{du}{dx}} du$$

$$\frac{dx}{du} = \frac{1}{\cos x}$$

$$\int u^3 \cos x \frac{1}{\cos x} du$$

$$\int u^3 du$$

$$\frac{1}{4} u^4 + C$$

$$\underline{\underline{\frac{1}{4} \sin^4 x + C}}$$

4/

$$\int \frac{e^{3x}}{1+e^x} dx$$

$$u = 1 + e^x$$

$$\frac{du}{dx} = e^x$$

$$\int \frac{(e^x)^3}{u} \frac{dx}{du} du$$

$$\frac{dx}{du} = \frac{1}{e^x}$$

$$\int \frac{(e^x)^3}{u} \cdot \frac{1}{e^x} du$$

$$\int \frac{(e^x)^2}{u} du$$

$$e^x = u - 1$$

$$\int \frac{(u-1)^2}{u} du$$

$$\int \frac{u^2 - 2u + 1}{u} du$$

$$\int u - 2 + \frac{1}{u} du$$

$$\frac{1}{2} u^2 - 2u + \ln u + C$$

$$\frac{1}{2} (1 + e^x)^2 - 2(1 + e^x) + \ln(1 + e^x) + C$$

$$\frac{1}{2} + \frac{1}{2} e^{2x} + 2 - 2e^x + \ln(1 + e^x) + C$$

$$-\frac{3}{2} + \frac{1}{2} e^{2x} + \ln(1 + e^x) + C$$

6)

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-x^2}} dx$$

$$x = \sin u$$

$$\frac{dx}{du} = \cos u$$

$$\int_0^{\frac{1}{2}} \frac{1}{\sqrt{1-\sin^2 u}} dx$$

$$x = \sin\left(\frac{1}{2}\right)$$

$$\frac{1}{2} = \sin u$$

$$\int_0^{\frac{1}{6}\pi} \frac{1}{\sqrt{1-\sin^2 u}} \frac{dx}{du} du$$

$$u = \frac{1}{6}\pi$$

$$0 = \sin u$$

$$\int_0^{\frac{1}{6}\pi} \frac{1}{\sqrt{\cos^2 u}} \cdot \cos u du$$

$$u = 0$$

$$\int_0^{\frac{1}{6}\pi} \frac{\cos u}{\cos u} du$$

$$\int_0^{\frac{1}{6}\pi} 1 du$$

$$\int_0^{\frac{1}{6}\pi} [u]_0^{\frac{1}{6}\pi}$$

$$\frac{1}{6}\pi - 0$$

$$\underline{\underline{\frac{1}{6}\pi}}$$