AS Mathematics SAMs (2017) Paper 1: Pure Mathematics Mark Scheme

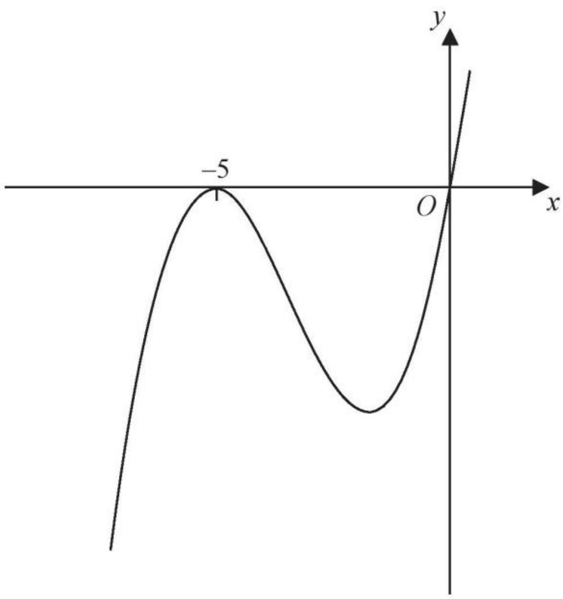
For mark scheme with alternative versions and examiners' notes, please see the full pdf document of AS Mathematics SAMs on the Edexcel website.

Qn	Scheme	Marks
1.	Use $y = mx + c$ with both (3, 1) and (4, -2) and attempt to find m or c	M1
	$m = -3$	A1
	$c = 10$ so $y = -3x + 10$	A1 (3)
		(3 marks)
2.	Attempt to differentiate	M1
	$\frac{dy}{dx} = 4x - 12$	A1
	Substitutes $x = 5 \Rightarrow \frac{dy}{dx} = ...$	M1
	$\Rightarrow \frac{dy}{dx} = 8$	A1 ft (4)
		(4 marks)
3. (a)	Attempts $\overrightarrow{AB} = \overrightarrow{OB} - \overrightarrow{OA}$ or similar	M1
	$\overrightarrow{AB} = 5\mathbf{i} + 10\mathbf{j}$	A1 (2)
(b)	Finds length using Pythagoras: $\lvert AB \rvert = \sqrt{(5)^2 + (10)^2}$	M1
	$\lvert AB \rvert = 5\sqrt{5}$	A1ft (2)
		(4 marks)

Qn	Scheme	Marks
4. (a)	States or uses $f(+3) = 0$	M1
	$4(3)^3 - 12(3)^2 + 2(3) - 6 = 108 - 108 + 6 - 6 = 0$ and so $(x - 3)$ is a factor	A1 (2)
(b)	Begins division or factorisation so $4x^3 - 12x^2 + 2x - 6 = (x-3)(4x^2 + ...)$	M1
	$4x^3 - 12x^2 + 2x - 6 = (x-3)(4x^2 + 2)$	A1
	Considers the roots of their quadratic function using completion of square or discriminant	M1
	$4x^2 + 2 = 0$ has no real roots with a reason (e.g. negative number does not have a real square root, or $4x^2 + 2 > 0$ for all x)	A1 (4)
	So $x = 3$ is the only real root of $f(x) = 0$.*	
		(6 marks)
5.	$f(x) = 2x + 3 + 12x^{-2}$	B1
	Attempts to integrate	M1
	$\int \left(2x + 3 + \frac{12}{x^2}\right)dx = x^2 + 3x - \frac{12}{x}$	A1
	$= ((2\sqrt{2})^2 + 3(2\sqrt{2}) - \frac{12\sqrt{2}}{2 \times 2}) - (-8)$	M1
	$= 16 + 3\sqrt{2}$ (*)	A1 (5)
		(5 marks)
6.	Considers $\frac{3(x+h)^2 - 3x^2}{h}$	B1
	Expands $3(x+h)^2 = 3x^2 + 6xh + 3h^2$	M1
	so gradient $= \frac{6xh + 3h^2}{h} = 6x + 3h$ or $\frac{6x\delta x + 3(\delta x)^2}{\delta x} = 6x + 3\delta x$	A1
	States as $h \to 0$, gradient $\to 6x$ so in the limit derivative $= 6x$ (*)	A1 (4)

Qn	Scheme	Marks
		(4 marks)
7. (a)	$\left(2-\frac{x}{2}\right)^7 = 2^7 + \binom{7}{1}2^6.\left(-\frac{x}{2}\right) + \binom{7}{2}2^5.\left(-\frac{x}{2}\right)^2 + \ldots$	M1
	$\left(2-\frac{x}{2}\right)^7 = 128 + \ldots$	B1
	$\left(2-\frac{x}{2}\right)^7 = \ldots -224x + \ldots$	A1
	$\left(2-\frac{x}{2}\right)^7 = \ldots + \ldots + 168x^2 \ (+ \ldots)$	A1 (4)
(b)	Solve $\left(2-\frac{x}{2}\right) = 1.995$ so $x = 0.01$ and state that 0.01 would be substituted for x into the expansion	B1 (1) **(5 marks)**
8. (a)	Finds third angle of triangle and uses or states $\frac{x}{\sin 60°} = \frac{30}{\sin"50°"}$	M1
	So $x = \frac{30\sin 60°}{\sin 50°} \quad (= 33.9)$	A1
	Area = $\frac{1}{2}\times 30\times x\times\sin 70°$ or $\frac{1}{2}\times 30\times y\times\sin 60$	M1
	= 478 m^2	A1 ft (4)
(b)	Plausible reason, e.g. because the angles and the side length are not given to four significant figures or the lawn may not be flat	B1 (1) **(5 marks)**

Qn	Scheme	Marks
9.	Uses $\sin^2 x = 1 - \cos^2 x \Rightarrow\ 12(1-\cos^2 x) + 7\cos x - 13 = 0$	M1
	$\Rightarrow 12\cos^2 x - 7\cos x + 1 = 0$	A1
	Uses solution of quadratic to give $\cos x =$	M1
	Uses inverse cosine on their values, giving two correct follow-through values	M1
	$\Rightarrow x = \ 430.5°,\ 435.5°$	A1 (5)
		(5 marks)
10.	Realises that $k = 0$ will give no real roots as equation becomes $3 = 0$ (proof by contradiction)	B1
	(For $k \neq 0$) quadratic has no real roots provided $b^2 < 4ac$ so $16k^2 < 12k$	M1
	$4k(4k-3) < 0$ with attempt at solution	M1
	So $0 < k < \frac{3}{4}$, which together with $k = 0$ gives $0 \leq k < \frac{3}{4}$ (*)	A1 (4)
		(4 marks)
11. (a)	Since x and y are positive, their square roots are real and so $(\sqrt{x} - \sqrt{y})^2 \geq 0$ giving $x - 2\sqrt{x}\sqrt{y} + y \geq 0$	M1
	$\therefore\ 2\sqrt{xy} \leq x + y$ provided x and y are positive and so $\sqrt{xy} \leq \dfrac{x+y}{2}$ *	A1 (2)
(b)	Let $x = -3$ and $y = -5$; then LHS = $\sqrt{15}$ and RHS= -4 so as $\sqrt{15} > -4$, result does not apply	B1 (1)
		(3 marks)

Qn	Scheme	Marks
12. (a)	$2^{2x}+2^4$ is wrong in line 2 - it should be $2^{2x}\times 2^4$	B1
	In line 4, 2^4 has been replaced by 8 instead of by 16	B1 (2)
(b)	$2^{2x+4}-9(2^x)=0$ $2^{2x}\times 2^4-9(2^x)=0$ Let $2^x=y$ $16y^2-9y=0$	M1
	$y=\frac{9}{16}$ or $y=0$ So $x=\log_2\left(\frac{9}{16}\right)$ or $\dfrac{\log\left(\frac{9}{16}\right)}{\log 2}$ with no second answer.	A1 (2)
		(4 marks)
13. (a)	$x^3+10x^2+25x=x(x^2+10x+25)$	M1
	$=x(x+5)^2$	A1 (2)
(b)	 A cubic with correct orientation	M1
	Curve passes through the origin (0, 0) and touches at (−5, 0) (see note below for ft)	A1 ft (2)
(c)	Curve has been translated a to the left	M1
	$a=-2$	A1 ft
	$a=3$	A1 ft (3)
		(7 marks)

Qn	Scheme	Marks
14. (a)	$\log_{10} P = mt + c$	M1
	$\log_{10} P = \frac{1}{200}t + 5$	A1 (2)
(b)	As $P = ab^t$ then $\log_{10} P = t\log_{10} b + \log_{10} a$	M1
	$\log_{10} b = \frac{1}{200}$ or $\log_{10} a = 5$	M1
	so a = 100 000 **or** b = 1.0116	A1
	both a = 100 000 **and** b = 1.0116 (awrt 1.01)	A1 (4)
(c)	(i) The initial population	B1
	(ii) The proportional increase of population each year	B1 (2)
(d)	(i) 300 000, to nearest hundred thousand	B1
	(ii) Uses $200000 = ab^t$ with their values of a and b or $\log_{10} 200000 = \frac{1}{200}t + 5$ and rearranges to give t =	M1
	60.2 years to 3 sf	A1 ft (3)
(e)	Any two valid reasons, e.g. • 100 years is a long time and population may be affected by wars and disease • Inaccuracies in measuring gradient may result in widely different estimates • Population growth may not be proportional to population size • The model predicts unlimited growth	B2 (2)
		(13 marks)

Qn	Scheme	Marks
15.	Finds $\frac{\mathrm{d}y}{\mathrm{d}x} = 8x - 6$	M1
	Gradient of curve at P is –2	M1
	Normal gradient is $\frac{-1}{m} = \frac{1}{2}$	M1
	Thus equation of normal is $(y - 2) = \frac{1}{2}\left(x - \frac{1}{2}\right)$ or $4y = 2x + 7$	A1
	Eliminate y between $= \frac{1}{2}x + \ln(2x)$ and normal equation to give an equation in x	M1
	Solve $\ln 2x = \frac{7}{4}$ so $x = \frac{1}{2}\mathrm{e}^{\frac{7}{4}}$	M1
	Subsitute to find a value for y	M1
	Point Q is $\left(\frac{1}{2}\mathrm{e}^{\frac{7}{4}}, \frac{1}{4}\mathrm{e}^{\frac{7}{4}}, \frac{7}{4}\right)$	A1 (8)
		(8 marks)

Qn	Scheme	Marks
16. (a)	Sets $2xy+\frac{\pi x^2}{2}=250$	B1
	Obtains $y=\frac{250-\frac{\pi x^2}{2}}{2x}$ and substitute into P	M1
	Uses $P=2x+2y+\pi x$ with their y substituted	M1
	$P=2x+\frac{250}{x}-\frac{\pi x^2}{2x}+\pi x=2x+\frac{250}{x}+\frac{\pi x}{2}$ *	A1 (4)
(b)	$x>0$ and $y>0$ (distance) $\Rightarrow \frac{250-\frac{\pi x^2}{2}}{2x}>0$ or $250-\frac{\pi x^2}{2}>0$	M1
	As x and y are distances they are positive, so $0<x<\sqrt{\frac{500}{\pi}}$ *	A1 (2)
	Differentiates P with negative index correct in $\frac{\mathrm{d}P}{\mathrm{d}x}$; $x^{-1}\rightarrow x^{-2}$	M1
	$\frac{\mathrm{d}P}{\mathrm{d}x}=2-\frac{250}{x^2}+\frac{\pi}{2}$	A1
	Sets $\frac{\mathrm{d}P}{\mathrm{d}x}=0$ and proceeds to $x=$	M1
	Substitutes their x into $P=2x+\frac{250}{x}+\frac{\pi x}{2}$ to give perimeter = 59.8m.	A1 (4)
		(10 marks)

Qn	Scheme	Marks
17. (a)	Finds circle equation $(x \pm 2)^2 + (y\ \mu\ 6)^2 = (10 \pm (-2))^2 + (11\ \mu\ 6)^2$	M1
	Checks whether (10, 1) satisfies their circle equation	M1
	Obtains $(x + 2)^2 + (y - 6)^2 = 13^2$ and checks that $(10 + 2)^2 + (1 - 6)^2 = 13^2$ and so states that (10, 1) lies on C (*)	A1 (3)
(b)	Finds radius gradient $\dfrac{11-6}{10-(-2)}$ or $\dfrac{1-6}{10-(-2)}$ (m)	M1
	Finds gradient perpendicular to their radius using $-\dfrac{1}{m}$	M1
	Finds (equation and) y-intercept of tangent (see note below)	M1
	Obtains a correct value for y-intercept of their tangent, i.e. 35 or –23	A1
	Deduces gradient of second tangent	M1
	Finds (equation and) y-intercept of second tangent	M1
	So obtains distance $PQ = 35 + 23 = 58$ (*)	A1 (7)
		(10 marks)