Year 2 Pure Chapter 9: Differentiation – Exam Questions (Total Marks 67)

Q1.

Differentiate with respect to x

(a) $\ln (x^2 + 3x + 5)$

(2)

(b) $\dfrac{\cos x}{x^2}$

(3)

(Total 5 marks)

Q2.

At time t seconds the radius of a sphere is r cm, its volume is V cm^3 and its surface area is S cm^2.

[*You are given that* $V = \frac{4}{3}\pi r^3$ *and that* $S = 4\pi r^2$]

The volume of the sphere is increasing uniformly at a constant rate of 3 cm^3 s^{-1}.

(a) Find $\frac{dr}{dt}$ when the radius of the sphere is 4 cm, giving your answer to 3 significant figures.

(4)

(b) Find the rate at which the surface area of the sphere is increasing when the radius is 4 cm.

(2)

(Total 6 marks)

Q3.

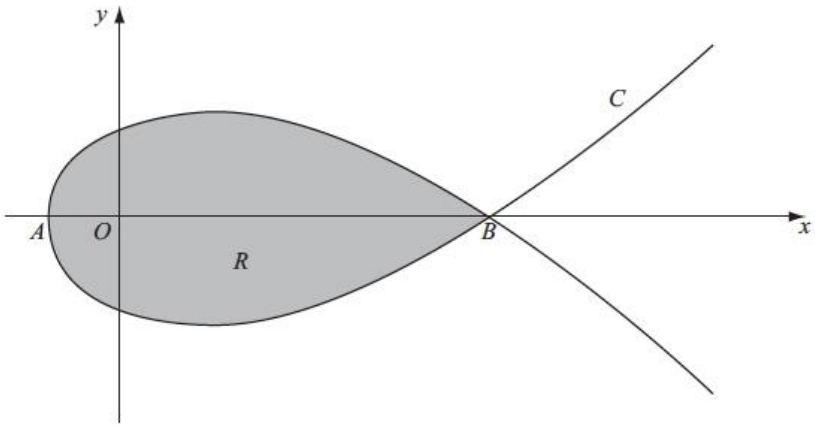

Figure 2

Figure 2 shows a sketch of the curve C with parametric equations

$$x = 5t^2 - 4, \qquad y = t(9 - t^2)$$

The curve C cuts the x-axis at the points A and B.

(a) Find the x-coordinate at the point A and the x-coordinate at the point B.

(3)

The region R, as shown shaded in Figure 2, is enclosed by the loop of the curve.

(b) Use integration to find the area of R.

(6)

(Total 9 marks)

Q4.

A curve is described by the equation

$$x^3 - 4y^2 = 12xy.$$

(a) Find the coordinates of the two points on the curve where $x = -8$.

(3)

(b) Find the gradient of the curve at each of these points.

(6)

(Total 9 marks)

Q5.

The curve C has parametric equations

$x = 3t - 4, y = 5 - \frac{6}{t}, \quad t > 0$

(a) Find $\frac{dy}{dx}$ in terms of t

(2)

The point P lies on C where $t = \frac{1}{2}$

(b) Find the equation of the tangent to C at the point P. Give your answer in the form $y = px + q$, where p and q are integers to be determined.

(3)

(c) Show that the cartesian equation for C can be written in the form

$y = \frac{ax + b}{x + 4}, \quad x > -4$

where a and b are integers to be determined.

(3)

(Total for question = 8 marks)

Q6.

A curve *C* has equation

$$y = x^2 e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of *C*.

(3)

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Determine the nature of each turning point of the curve *C*.

(2)

(Total for question = 10 marks)

Q7.

A curve *C* has parametric equations

$$x = \sin^2 t, \quad y = 2\tan t, \quad 0 \leqslant t < \frac{\pi}{2}$$

(a) Find $\frac{dy}{dx}$ in terms of *t*.

(4)

The tangent to *C* at the point where $t = \frac{\pi}{3}$ cuts the *x*-axis at the point *P*.

(b) Find the *x*-coordinate of *P*.

(6)
(Total 10 marks)

Q8.

$$x^2 + y^2 + 10x + 2y - 4xy = 10$$

(a) Find $\frac{dy}{dx}$ in terms of x and y, fully simplifying your answer.

(5)

(b) Find the values of y for which $\frac{dy}{dx} = 0$

(5)

(Total 10 marks)