AS and A level Further Mathematics Practice Paper – Matrix algebra (part 2) – Mark scheme

Question	Scheme	Marks
1(a)	Rotation of 45 degrees anticlockwise, about the origin	B1B1 **(2)**
1(b)	$\begin{pmatrix} -2 & 0 \\ 0 & -2 \end{pmatrix}$	B1 **(1)**
1(c)	$\frac{224}{16}(=14)$ $\det \mathbf{M} = 3\times3 - k\times -2 = 14$ $k = 2.5$	B1 M1 A1 **(3)**
		(6 marks)
2(a)	$\mathbf{A} = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, $\mathbf{B} = \begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}$ $\mathbf{P} = \mathbf{AB} \left\{= \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} 2 & 3 \\ 1 & 4 \end{pmatrix}\right\}$ $\mathbf{P} = \begin{pmatrix} 1 & 4 \\ -2 & -3 \end{pmatrix}$	 M1 A1 **(2)**
2(b)	$\det \mathbf{P} = 1(-3) - (4)(-2)\ \{= -3 + 8 = 5\}$ $\text{Area}(T) = \underline{\frac{24}{5}}\ (\text{units})^2$	M1 dM1 A1ft **(3)**
2(c)	$\mathbf{QP} = \mathbf{I} \Rightarrow \mathbf{QPP^{-1}} = \mathbf{IP^{-1}} \Rightarrow \mathbf{Q} = \mathbf{P^{-1}}$ $\mathbf{Q} = \mathbf{P^{-1}} = \frac{1}{5}\begin{pmatrix} -3 & -4 \\ 2 & 1 \end{pmatrix}$	 M1 A1ft **(2)**
		(7 marks)

AS and A level Further Mathematics Practice Paper – Matrix algebra (part 2) – Mark scheme

Question	Scheme	Marks
3(a)	Determinant: $2 - 3a = 0$ and solve for $a =$ So $a = \frac{2}{3}$ or equivalent	M1 A1 **(2)**
3(b)	Determinant: $(1\times 2)-(3\times -1)=5 \quad (\Delta)$ $\mathrm{Y}^{-1} = \frac{1}{5}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix} \qquad \left[= \begin{pmatrix} 0.4 & 0.2 \\ -0.6 & 0.2 \end{pmatrix} \right]$	M1A1 **(2)**
3(c)	$\frac{1}{5}\begin{pmatrix} 2 & 1 \\ -3 & 1 \end{pmatrix}\begin{pmatrix} 1-\lambda \\ 7\lambda-2 \end{pmatrix} = \frac{1}{5}\begin{pmatrix} 2-2\lambda+7\lambda-2 \\ -3+3\lambda+7\lambda-2 \end{pmatrix} = \begin{pmatrix} \lambda \\ 2\lambda-1 \end{pmatrix}$	M1dep M1A1A1 **(4)**
		(8 marks)
4(i)	$5k(k+1)$ - $-3(3k-1)=0$ $5k^2+5k+9k-3=0$ $(5k-1)(k+3)=0$ so $k =$ $k = \frac{1}{5}$ or -3	M1 A1 M1 A1 **(4)**
4(ii)(a)	$\mathbf{B}^{-1} = \frac{1}{45}\begin{pmatrix} 3 & -5 \\ 3 & 10 \end{pmatrix}$	M1 A1 **(2)**
4(ii)(b)	$\frac{1}{45}\begin{pmatrix} 3 & -5 \\ 3 & 10 \end{pmatrix}\begin{pmatrix} 0 & -20 & 10c \\ 0 & 6 & 6c \end{pmatrix} =$ $\frac{1}{45}\begin{pmatrix} 0 & -90 & 0 \\ 0 & 0 & 90c \end{pmatrix}$ Vertices at (0, 0) (-2, 0) (0, $2c$)	M1 A1 A1 **(3)**
4(ii)(c)	Area of T is $\frac{1}{2}\times 2\times 2c = 2c$ Area of $T\times$determinant $=135$ So $c = \frac{3}{2}$	B1 M1 A1 **(3)**
		(12 marks)

Question	Scheme	Marks
5(i)	$\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix}$	B1
	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	B1
	$\begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}$	M1A1
		(4)
5(ii)	Area triangle T = $\frac{1}{2} \times (11-3) \times k = 4k$	M1A1
	$\det\begin{pmatrix} 6 & -2 \\ 1 & 2 \end{pmatrix} = 6 \times 2 - 1 \times (-2) (= 14)$	M1A1
	Area triangle $T = \frac{364}{\text{"14"}} (= 26) \Rightarrow 4k = 26$	M1
	$k = \frac{26}{4} \left(= \frac{13}{2}\right)$	A1
		(6)
		(10 marks)

AS and A level Further Mathematics Practice Paper – Matrix algebra (part 2) – Mark scheme

Question	Scheme	Marks
6(a)	$\mathbf{A}^2 = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$ $7\mathbf{A}+2\mathbf{I} = \begin{pmatrix} 42 & -14 \\ -28 & 7 \end{pmatrix} + \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = \begin{pmatrix} 44 & -14 \\ -28 & 9 \end{pmatrix}$ OR $\mathbf{A}^2 - 7\mathbf{A} = \mathbf{A}(\mathbf{A}-7\mathbf{I})$ $\mathbf{A}(\mathbf{A}-7\mathbf{I}) = \begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix} = \begin{pmatrix} 2 & 0 \\ 0 & 2 \end{pmatrix} = 2\mathbf{I}$	M1A1 **(2)**
6(b)	$\mathbf{A}^2 = 7\mathbf{A}+2\mathbf{I} \Rightarrow \mathbf{A} = 7\mathbf{I}+2\mathbf{A}^{-1}$ $\mathbf{A}^{-1} = \frac{1}{2}(\mathbf{A}-7\mathbf{I})$* Numerical approach award 0/2.	M1 A1* cso **(2)**
6(c)	$\mathbf{A}^{-1} = \frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}$ $\frac{1}{2}\begin{pmatrix} -1 & -2 \\ -4 & -6 \end{pmatrix}\begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} -2k-8+4k+10 \\ -8k-32+12k+30 \end{pmatrix}$ $\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$ **OR:** $\begin{pmatrix} 6 & -2 \\ -4 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 2k+8 \\ -2k-5 \end{pmatrix}$ $6x-2y=2k+8$ $-4x+y=-2k-5 \Rightarrow x = \ldots or\ y = \ldots$ $\begin{pmatrix} k+1 \\ 2k-1 \end{pmatrix}$ or $(k+1,\ 2k-1)$	B1 M1 A1 A1 B1 M1 A1 A1 **(4)**
		(8 marks)

AS and A level Further Mathematics Practice Paper – Matrix algebra (part 2) – Mark scheme

Question	Scheme		Marks
7(a)	$\det(\mathbf{A}) = 3\times 0 - 2\times 1 (= -2)$	Correct attempt at the determinant	M1
	$\det(\mathbf{A}) \neq 0$ (so **A** is non singular)	det(A) = -2 **and** some reference to zero	A1
	$\frac{1}{\det(\mathbf{A})}$ **scores M0**		**(2)**
7(b)	$\mathbf{BA}^2 = \mathbf{A} \Rightarrow \mathbf{BA} = \mathbf{I} \Rightarrow \mathbf{B} = \mathbf{A}^{-1}$	Recognising that $\mathbf{A}^{-1}$ is required	M1
		At least 3 correct terms in $\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	M1
	$\mathbf{B} = -\frac{1}{2}\begin{pmatrix} 3 & -1 \\ -2 & 0 \end{pmatrix}$	$\frac{1}{\text{their}\det(\text{A})}\begin{pmatrix} * & * \\ * & * \end{pmatrix}$	B1ft
		Fully correct answer	A1
	Correct answer only score 4/4		**(4)**
	Ignore poor matrix algebra notation if the intention is clear		**(6 marks)**

AS and A level Further Mathematics Practice Paper – Matrix algebra (part 2) – Mark scheme

Question	Scheme		Marks
8(a)	$\mathbf{A} = \begin{pmatrix} 2 & -2 \\ -1 & 3 \end{pmatrix}$		
	$\det \mathbf{A} = 2(3) - (-1)(-2) = 6 - 2 = \underline{4}$	$\underline{4}$	$\underline{\text{B1}}$
			(1)
8(b)	$\mathbf{A}^{-1} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	$\frac{1}{\det \mathbf{A}}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	M1
		$\frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}$	A1
			(2)
8(c)	$\text{Area}(R) = \frac{72}{4} = \underline{18}\ (\text{units})^2$	$\frac{72}{\text{their }\det \mathbf{A}}$ or $72(\text{their }\det \mathbf{A})$	M1
		$\underline{18}$ or ft answer.	A1
			(2)
8(d)	$\mathbf{AR} = \mathbf{S} \Rightarrow \mathbf{A}^{-1}\mathbf{AR} = \mathbf{A}^{-1}\mathbf{S} \Rightarrow \mathbf{R} = \mathbf{A}^{-1}\mathbf{S}$		
	$\mathbf{R} = \frac{1}{4}\begin{pmatrix} 3 & 2 \\ 1 & 2 \end{pmatrix}\begin{pmatrix} 0 & 8 & 12 \\ 4 & 16 & 4 \end{pmatrix}$	At least one attempt to apply $\mathbf{A}^{-1}$by any of the three vertices in $\mathbf{S}$.	M1
	$= \frac{1}{4}\begin{pmatrix} 8 & 56 & 44 \\ 8 & 40 & 20 \end{pmatrix}$		
	$= \begin{pmatrix} 2 & 14 & 11 \\ 2 & 10 & 5 \end{pmatrix}$	At least one correct **column** o.e.	A1
		At least two correct **columns** o.e.	A1
	Vertices are $(2, 2)$, $(14, 10)$ and $(11, 5)$.	All three **coordinates** correct.	A1
			(4)
			(9 marks)

AS and A level Further Mathematics Practice Paper – Matrix algebra (part 2) – Mark scheme

Question	Scheme	Marks
9(a)	$\det \mathbf{M} = 3(-5) - (4)(2) = -15 - 8 = \underline{-23}$	B1
		(1)
9(b)	Therefore, $\begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 2a-7 \\ a-1 \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	M1
	Either, $3(2a-7)+4(a-1)=25$ or $2(2a-7)-5(a-1)=-14$ or $\begin{pmatrix} 3(2a-7)+4(a-1) \\ 2(2a-7)-5(a-1) \end{pmatrix} = \begin{pmatrix} 25 \\ -14 \end{pmatrix}$	A1
	giving $a = 5$	A1
		(3)
9(c)	$\text{Area}(ORS) = \frac{1}{2}(6)(4); = \underline{12}\ (\text{units})^2$	M1 A1
		(2)
9(d)	$\text{Area}(OR'S') = \pm 23 \times (12)$	M1 A1
		(2)
9(e)	Rotation; 90° anti-clockwise (or 270° clockwise) about $(0, 0)$.	B1 B1
		(2)
9(f)	$\mathbf{M} = \mathbf{BA}$	M1
	$\mathbf{A}^{-1} = \frac{1}{(0)(0)-(1)(-1)}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}; = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	A1
	$\mathbf{B} = \begin{pmatrix} 3 & 4 \\ 2 & -5 \end{pmatrix}\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$	M1
	$\mathbf{B} = \begin{pmatrix} -4 & 3 \\ 5 & 2 \end{pmatrix}$	A1
		(4)
		(14 marks)

AS and A level Further Mathematics Practice Paper – Matrix algebra (part 2) – Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	FP1 2014R	3		Matrix algebra	1.1b, 3.1a
2	FP1 2013R	6		Matrix algebra	1.1b, 2.1, 3.1a
3	FP1 Jan 2013	6		Matrix algebra	1.1b, 2.1, 3.1a
4	FP1 2015	7		Matrix algebra	1.1b, 2.1, 3.1a
5	FP1 2014	7		Matrix algebra	1.1b, 1.2
6	FP1 2013	8		Matrix algebra	1.1b, 2.1, 3.1a
7	FP1 Jan 2012	8		Matrix algebra	1.1b, 2.1, 3.1a
8	FP1 2011	8		Matrix algebra	1.1b, 3.1a
9	FP1 2012	9		Matrix algebra	1.1b, 2.1