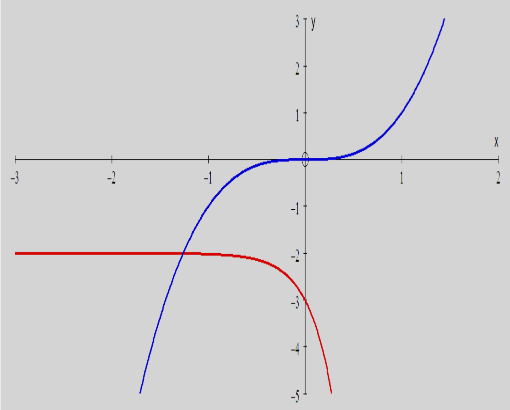
A level Mathematics Practice Paper – Numerical methods – Mark scheme

Question	Scheme		Marks
1(a)	$\frac{dy}{dx} = 4e^{4x} + 4x^3 + 8$		M1 A1
	Puts $\frac{dy}{dx} = 0$ to give $x^3 = -2 - e^{4x}$		A1 *
			(3)
1(b)		$y = x^3$	B1
		Shape of $y = -2 - e^{4x}$	B1
		$y = -2 - e^{4x}$ cuts y axis at (0, –3)	B1
		$y = -2 - e^{4x}$ has asymptote at $y = -2$	B1
			(4)
1(c)	Only one crossing point		B1
			(1)
1(d)	–1.26376, -1.26126 Accept answers which round to these answers to 5dp		M1 A1
			(2)
1(e)	$\alpha = -1.26$ and so turning point is at (-1.26, -2.55)		M1 A1cao
			(2)
			(12 marks)

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Question	Scheme	Marks
2(a)	$x^3+3x^2+4x-12=0 \Rightarrow x^3+3x^2=12-4x$	
	$\Rightarrow x^2(x+3)=12-4x$	M1
	$\Rightarrow x^2=\dfrac{12-4x}{(x+3)} \Rightarrow x=\sqrt{\dfrac{4(3-x)}{(x+3)}}$	dM1 A1*
		(3)
2(b)	$x_1=1.41, \quad awrt\ x_2=1.20 \quad x_3=1.31$	M1 A1 A1
		(3)
2(c)	Choosing (1.2715,1.2725)	
	or tighter containing root 1.271998323	M1
	$\text{f}(1.2725)=(+)0.00827... \quad \text{f}(1.2715)=-0.00821....$	M1
	Change of sign ⇒ α = 1.272	A1
		(3)
		(9 marks)
3(a)	$0=e^{x-1}+x-6 \Rightarrow x=\ln(6-x)+1$	M1 A1*
		(2)
3(b)	Sub $x_0=2$ into $x_{n+1}=\ln(6-x_n)+1 \Rightarrow x_1=2.3863$	M1 A1
	AWRT 4 dp. $x_2=2.2847$ $x_3=2.3125$	A1
		(3)
3(c)	Chooses interval [2.3065,2.3075]	M1
	g(2.3065)=-0.0002(7), g(2.3075)=0.004(4)	dM1
	Sign change , hence root (correct to 3dp)	A1
		(3)
		(8 marks)

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Question	Scheme	Marks
4(a)	$f'(x) = 50x^2e^{2x} + 50xe^{2x}$ oe.	M1A1
	Puts $f'(x) = 0$ to give x = -1 and x = 0 or one coordinate	dM1A1
	Obtains $(0,-16)$ and $(-1, 25e^{-2}-16)$ CSO	A1
		(5)
4(b)	Puts $25x^2e^{2x} - 16 = 0 \Rightarrow x^2 = \frac{16}{25}e^{-2x} \Rightarrow x = \pm\frac{4}{5}e^{-x}$	B1*
		(1)
4(c)	Subs $x_0 = 0.5$ into $x = \frac{4}{5}e^{-x} \Rightarrow x_1 = \text{awrt } 0.485$	M1A1
	$\Rightarrow x_2 = \text{awrt } 0.492,\ x_3 = \text{awrt } 0.489$	A1
		(3)
4(d)	$\alpha = 0.49$ $f(0.485) = -\ 0.487,\ f(0.495) = (+)0.485$, sign change and deduction	B1 B1
		(2)
		(11 marks)
5(a)	(i) 21	B1
	(ii) $4e^{2x} - 25 = 0 \Rightarrow e^{2x} = \frac{25}{4} \Rightarrow 2x = \ln\left(\frac{25}{4}\right) \Rightarrow x = \frac{1}{2}\ln\left(\frac{25}{4}\right), \Rightarrow x = \ln\left(\frac{5}{2}\right)$	M1 A1 A1
	(iii) 25	B1
		(5)
5(b)	$4e^{2x} - 25 = 2x + 43 \Rightarrow e^{2x} = \frac{1}{2}x + 17$	M1
	$\Rightarrow 2x = \ln\left(\frac{1}{2}x + 17\right) \Rightarrow x = \frac{1}{2}\ln\left(\frac{1}{2}x + 17\right)$	A1*
		(2)
5(c)	$x_1 = \frac{1}{2}\ln\left(\frac{1}{2} \times 1.4 + 17\right) = \textit{awrt } 1.44$	M1
	awrt $x_1 = 1.4368, x_2 = 1.4373$	A1
		(2)
5(d)	Defines a suitable interval 1.4365 and 1.4375	M1
	...and substitutes into a suitable function, e.g. $4e^{2x} - 2x - 68$, obtains correct values with both a reason and conclusion	A1
		(2)
		(11 marks)

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Question	Scheme	Marks
6(a)	At P $x=-2 \Rightarrow y=3$	B1
	$\dfrac{dy}{dx}=\dfrac{4}{2x+5}-\dfrac{3}{2}$	M1 A1
	$\left.\dfrac{dy}{dx}\right\vert_{x=-2}=\dfrac{5}{2} \Rightarrow$ Equation of normal is $y-'3'=-\dfrac{2}{5}\left(x-(-2)\right)$	M1
	$\Rightarrow 2x+5y=11$	A1
		(5)
6(b)	Combines $5y+2x=11$ and $y=2\ln(2x+5)-\dfrac{3x}{2}$ to form equation in x	
	$5\left(2\ln(2x+5)-\dfrac{3x}{2}\right)+2x=11$	M1
	$\Rightarrow x=\dfrac{20}{11}\ln(2x+5)-2$	dM1 A1*
		(3)
6(c)	Substitutes $x_1=2 \Rightarrow x_2=\dfrac{20}{11}\ln 9-2$	M1
	Awrt $x_2=1.9950$ and $x_3=1.9929$.	A1
		(2)
		(10 marks)

Question	Scheme		Marks
7(a)	Crosses x-axis $\Rightarrow \mathrm{f}(x) = 0 \Rightarrow (8 - x)\ln x = 0$		
	Either $(8 - x) = 0$ or $\ln x = 0 \Rightarrow x = 8, 1$	Either one of $\{x\} = 1$ OR $x = \{8\}$	B1
	Coordinates are $A(1, 0)$ and $B(8, 0)$.	Both $A(1, \{0\})$ and $B(8, \{0\})$	B1
			(2)
7(b)	Apply product rule: $\left\{\begin{array}{ll} u = (8 - x) & v = \ln x \\ \frac{\mathrm{d}u}{\mathrm{d}x} = -1 & \frac{\mathrm{d}v}{\mathrm{d}x} = \frac{1}{x} \end{array}\right\}$	$vu' + uv'$	M1
	$\mathrm{f}'(x) = -\ln x + \frac{8 - x}{x}$	Any one term correct	A1
		Both terms correct	A1
			(3)
7(c)	$\mathrm{f}'(3.5) = 0.032951317...$ $\mathrm{f}'(3.6) = -0.058711623...$ Sign change (and as $\mathrm{f}'(x)$ is continuous) therefore the x-coordinate of Q lies between 3.5 and 3.6.	Attempts to evaluate **both** $\mathrm{f}'(3.5)$ and $\mathrm{f}'(3.6)$	M1
		both values correct to at least 1 sf, sign change and conclusion	A1
			(2)
7(d)	At Q, $\mathrm{f}'(x) = 0 \Rightarrow -\ln x + \frac{8 - x}{x} = 0$	Setting $\mathrm{f}'(x) = 0$	M1
	$\Rightarrow -\ln x + \frac{8}{x} - 1 = 0$	Splitting up the numerator and proceeding to $x =$	M1
	$\Rightarrow \frac{8}{x} = \ln x + 1 \Rightarrow 8 = x(\ln x + 1)$		
	$\Rightarrow x = \frac{8}{\ln x + 1}$ (as required)	For correct proof. No errors seen in working.	A1
			(3)

Question	Scheme		Marks
7(e)	Iterative formula: $x_{n+1} = \dfrac{8}{\ln x_n + 1}$		
	$x_1 = \dfrac{8}{\ln(3.55)+1}$	An attempt to substitute $x_0 = 3.55$ into the iterative formula. Can be implied by $x_1 = 3.528(97)...$	M1
	$x_1 = 3.528974374...$	Both x_1 = awrt 3.529 and x_2 = awrt 3.538	A1
	$x_2 = 3.538246011...$		
	$x_3 = 3.534144722...$		
	$x_1 = 3.529,\ x_2 = 3.538,\ x_3 = 3.534$, to 3 dp.	x_1, x_2, x_3 all stated correctly to 3 dp	A1
			(3)
			(13 marks)
8(a)	$2^{x+1} - 3 = 17 - x \Rightarrow 2^{x+1} = 20 - x$		M1
	$(x+1)\ln 2 = \ln(20 - x) \Rightarrow x = \ldots$		dM1
	$x = \dfrac{\ln(20-x)}{\ln 2} - 1$		A1*
			(3)
8(b)	Sub $x_0 = 3$ into $x_{n+1} = \dfrac{\ln(20-x_n)}{\ln 2} - 1 \Rightarrow x_1 = 3.087$ (awrt)		M1 A1
	$x_2 = 3.080$, $x_3 = 3.081$ (awrt)		A1
			(3)
8(c)	$A = (3.1, 13.9)$ cao		M1 A1
			(2)
			(8 marks)

Question	Scheme	Marks
9(a)	$y_{2.1} = -0.224$, $y_{2.2} = (+)0.546$	M1
	Change of sign$\Rightarrow Q$ lies between	A1
		(2)
9(b)	At R $\dfrac{\mathrm{d}y}{\mathrm{d}x} = -2x\sin\left(\dfrac{1}{2}x^2\right) + 3x^2 - 3$	M1A1
	$-2x\sin\left(\dfrac{1}{2}x^2\right) + 3x^2 - 3 = 0 \Rightarrow$ $x = \sqrt{1 + \dfrac{2}{3}x\sin\left(\dfrac{1}{2}x^2\right)}$ cso	M1A1*
		(4)
9(c)	$x_1 = \sqrt{1 + \dfrac{2}{3}\times 1.3\sin\left(\dfrac{1}{2}\times 1.3^2\right)}$	M1
	$x_1 = \text{awrt } 1.284$ $x_2 = \text{awrt } 1.276$	A1
		(2)
		(8 marks)
10(a)	f(0.8) = 0.082, f(0.9)= –0.089	M1
	Change of sign $\Rightarrow$ root (0.8,0.9)	A1
		(2)
10(b)	$\mathrm{f}'(x) = 2x - 3 - \sin\left(\dfrac{1}{2}x\right)$	M1 A1
	Sets $\mathrm{f}'(x) = 0 \Rightarrow$ $x = \dfrac{3 + \sin\left(\dfrac{1}{2}x\right)}{2}$	M1 A1*
		(4)
10(c)	Sub $x_0 = 2$ into $x_{n+1} = \dfrac{3 + \sin\left(\dfrac{1}{2}x_n\right)}{2}$	M1
	x_1 = awrt 1.921, x_2 = awrt 1.91(0) and x_3 = awrt 1.908	A1 A1
		(3)
10(d)	[1.90775,1.90785]	M1
	f'(1.90775) = –0.00016... AND f'(1.90785) = 0.0000076...	M1
	Change of sign $\Rightarrow x = 1.9078$	A1
		(3)
		(12 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C3 June 2014R	2	7.2, 2.7, 6.1, 9.1, 9.2	Differentiation of exponential, graph of exponential, using iterative methods	1.1b, 2.2a, 2.4
2	C3 2012	2	9.1, 9.2	Numerical methods	1.1b, 2.1, 2.4
3	C3 Jan 2013	2	9.1, 9.2, 6.3	Numerical methods	1.1b, 2.1. 2.4
4	C3 2013	4	7.2, 7.4, 9.1, 9.2	Differentiation and numerical analysis	1.1b, 2.1, 2.2a, 2.4
5	C3 2016	4	2.9, 6.3, 9.1, 9.2	Modulus of exponential, iterative equation	1.1b, 2.1, 2.4
6	C3 2017	5	7.2, 7.3, 2.4, 9.2	Normal to a function, leading to iterative methods	1.1b, 2.1
7	C3 Jan 2011	5	6.3, 7.3, 7.4, 9.1, 9.2	Exponentials and logarithms, Differentiation, Numerical methods	1.1b, 2.1, 2.4
8	C3 2015	6	6.5, 9.2	Intersection of graphs leading to iterative equation	1.1b, 2.2a
9	C3 June 2014	6	7.2, 7.4, 9.1, 9.2	Numerical methods	1.1b, 2.1, 2.4
10	C3 Jan 2012	6	7.2, 9.1, 9.2	Numerical methods	1.1b, 2.1, 2.4