Exam Questions – Simultaneous Equations (55mins)

Q1.

Solve the simultaneous equations

$$x + y = 2$$
$$4y^2 - x^2 = 11$$

$y = 2 - x$

$4(2-x)^2 - x^2 = 11$

$4(2-x)(2-x) - x^2 = 11$

$4x^2 - 16x + 16 - x^2 = 11$

$3x^2 - 16x + 16 - 11 = 0$

$3x^2 - 16x + 5 = 0$

$(3x - 1)(x - 5) = 0$

$x = \frac{1}{3}$ $x = 5$

$y = 2 - \frac{1}{3} = 1\frac{2}{3}$

$y = 2 - 5 = -3$

$x = 1/2$ $y = 1\frac{2}{3}$

and

$x = 5$ $y = -3$

(7)

(Total 7 marks)

Q2.

Solve the simultaneous equations

$$y - 3x + 2 = 0 \qquad y = 3x - 2$$

$$y^2 - x - 6x^2 = 0$$

(7)
(Total 7 marks)

$(3x-2)^2 - x - 6x^2 = 0$

$(3x-2)(3x-2) - x - 6x^2 = 0$

$9x^2 - 12x + 4 - x - 6x^2 = 0$

$3x^2 - 13x + 4 = 0$

$(3x-1)(x-4) = 0$

$x = \frac{1}{3} \quad x = 4$

$y = 3(\frac{1}{3}) - 2 = -1$

$y = 3(4) - 2 = 10$

$x = \frac{1}{3} \quad y = -1$

and

$x = 4 \quad y = 10$

Q3.

Given the simultaneous equations

$$2x + y = 1 \qquad y = 1 - 2x$$

$$x^2 - 4ky + 5k = 0$$

where k is a non zero constant,

(a) show that

$$x^2 + 8kx + k = 0$$

(2)

Given that $x^2 + 8kx + k = 0$ has equal roots,

(b) find the value of k.

(3)

(c) For this value of k, find the solution of the simultaneous equations.

(3)

(Total 8 marks)

a) $x^2 - 4k(1-2x) + 5k = 0$

$x^2 - 4k + 8kx + 5k = 0$

$x^2 + 8kx + k = 0$

b) $b^2 - 4ac = 0$

$a = 1 \quad b = 8k \quad c = k$

$(8k)^2 - 4(1)(k) = 0$

$64k^2 - 4k = 0$

$4k(16k - 1) = 0 \qquad \therefore \quad k = \frac{1}{16}$

c) $x^2 + 8\left(\frac{1}{16}\right)x + \frac{1}{16} = 0$

$x^2 + \frac{1}{2}x + \frac{1}{16} = 0$

(x16)

$16x^2 + 8x + 1 = 0$

$(4x+1)(4x+1) = 0$

$x = -\frac{1}{4} \qquad \therefore \quad y = 1\frac{1}{2}$

Q4.

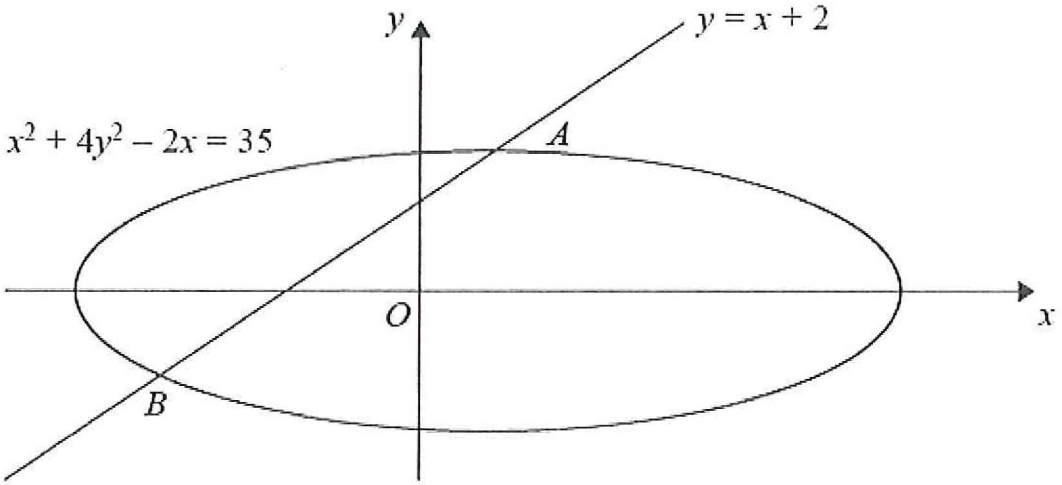

Figure 2

The line $y = x + 2$ meets the curve $x^2 + 4y^2 - 2x = 35$ at the points A and B as shown in Figure 2.

Find the coordinates of A and the coordinates of B.

(6)

(Total 6 marks)

$$x^2+4(x+2)^2-2x = 35$$

$$x^2+4x^2+16x+16-2x=35$$

$$5x^2+14x+16-35=0$$

$$5x^2+14x-19=0$$

$$(5x+19)(x-1)=0$$

$$x=-\frac{19}{5} \quad x=1$$

$$y=-\frac{9}{5} \quad y=3$$

$$A\ (1,3)$$

$$B\ \left(-\frac{19}{5},-\frac{9}{5}\right)$$

Q5.

(a) By eliminating y from the equations

$$y = x - 4$$

$$2x^2 - xy = 8,$$

show that

$$x^2 + 4x - 8 = 0$$

(2)

(b) Hence, or otherwise, solve the simultaneous equations

$$y = x - 4,$$

$$2x^2 - xy = 8,$$

giving your answers in the form $a \pm b\sqrt{3}$, where a and b are integers.

(5)

(Total 7 marks)

a) $2x^2 - x(x-4) = 8$

$2x^2 - x^2 + 4x = 8$

$x^2 + 4x - 8 = 0$

b) $a = 1 \quad b = 4 \quad c = -8$

$$x = \frac{-4 \pm \sqrt{16 - 4(1)(-8)}}{2(1)}$$

$$x = \frac{-4 \pm \sqrt{48}}{2}$$

$$x = \frac{-4 \pm \sqrt{16}\sqrt{3}}{2}$$

$$x = \frac{-4 \pm 4\sqrt{3}}{2}$$

$$x = -2 \pm 2\sqrt{3}$$

$\therefore \quad y = -2 + 2\sqrt{3} - 4 = -6 + 2\sqrt{3}$

$y = -2 - 2\sqrt{3} - 4 = -6 - 2\sqrt{3}$

Q6.

Given that

$$f(x) = 2x^2 + 8x + 3$$

(a) find the value of the discriminant of $f(x)$.

(2)

(b) Express $f(x)$ in the form $p(x + q)^2 + r$ where p, q and r are integers to be found.

(3)

The line $y = 4x + c$, where c is a constant, is a tangent to the curve with equation $y = f(x)$.

(c) Calculate the value of c.

(5)

(Total 10 marks)

a) $a = 2 \quad b = 8 \quad c = 3$

$b^2 - 4ac$

$= 64 - 4(2)(3)$

$= 64 - 24 = 40$

b) $2[x^2 + 4x] + 3$

$= 2[(x+2)^2 - 4] + 3$

$= 2(x+2)^2 - 8 + 3 = 2(x+2)^2 - 5$

c) $2x^2 + 8x + 3 = 4x + c$

$= 2x^2 + 4x + 3 = c$

$= 2x^2 + 4x + (3-c) = 0$

$a = 2 \quad b = 4 \quad c = 3 - c$

$b^2 - 4ac = 0$

$16 - 4(2)(3-c) = 0$

$16 - 24 + 8c = 0$

$8c = 8$

$c = 1$

Q7.

Solve the simultaneous equations

$$y - 2x - 4 = 0$$
$$4x^2 + y^2 + 20x = 0$$

$y = 2x + 4$

(7)

(Total for question = 7 marks)

$4x^2 + y^2 + 20x = 0$

$4x^2 + (2x+4)^2 + 20x = 0$

$4x^2 + 4x^2 + 16x + 16 + 20x = 0$

$8x^2 + 36x + 16 = 0$

$4(2x^2 + 9x + 4) = 0$

$4(2x+1)(x+4) = 0$

$x = -\frac{1}{2}$ or -4

$y = 3$ or -4