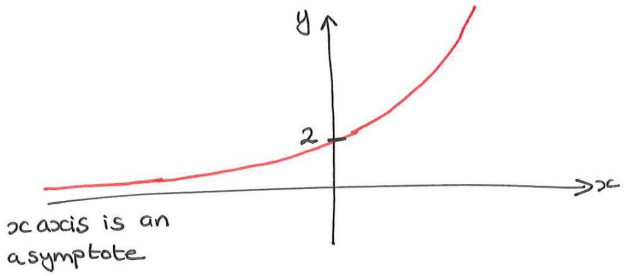
Year 1 Pure Chapter 14 - Exponential and Logs

Exponentials

$y = 2^x$ Index is a variable



Special Exponential

$y = e^x$ $e \approx 2.71828$

used for modelling

$f(x) = e^{kx}$

$f'(x) = ke^{kx}$

Logarithms

$3^2 = 9$

$\log_3 9 = 2$

Laws of Logarithms

multiplication law: $\log x + \log y = \log xy$

division law: $\log x - \log y = \log \frac{x}{y}$

power law: $k \log x = \log x^k$

WATCH OUT FOR:

Only apply multiplication + division law after applying power law.

$5 \log 2 + \log 3$

$= \log 2^5 + \log 3$

$= \log 32 + \log 3 = \log 96$

Solving Equations Using Logs

$$3^x = 2^{x+1}$$

$$\log 3^x = \log 2^{x+1}$$

$$x \log 3 = (x+1) \log 2$$

$$x \log 3 = x \log 2 + \log 2$$

$$x \log 3 - x \log 2 = \log 2$$

$$x(\log 3 - \log 2) = \log 2$$

$$x = \frac{\log 2}{\log 3 - \log 2}$$

Quadratic Equations Using Logs

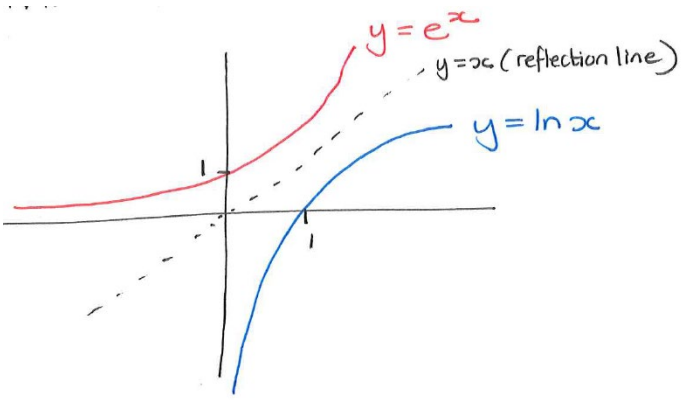
$$2^{2x} - 6(2^x) + 5 = 0$$

$y = 2^x$

$$(2^x)^2 - 6(2^x) + 5 = 0$$

$$y^2 - 6y + 5 = 0$$

Natural Logarithms



$$e^x = 5$$

$$\ln e^x = \ln 5$$

$$x = \ln 5$$

Natural Logarithms and Quadratics

$$e^{2x} + 5e^x - 14 = 0$$

$$(e^x + 7)(e^x - 2) = 0$$

$e^x = -7$ or $e^x = 2$

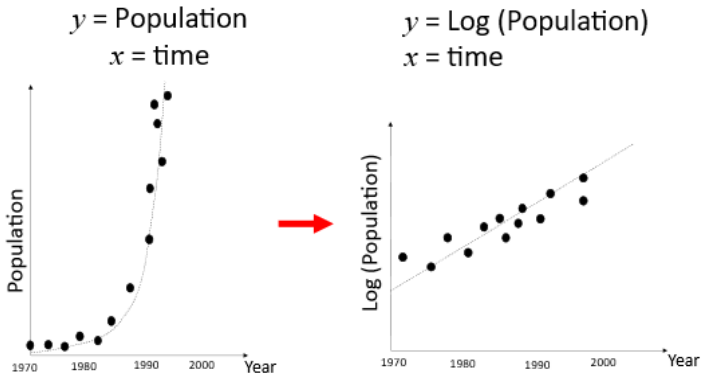
no solutions

e^x is always positive

$x = \ln 2$

Non-Linear Modelling

$y = ab^x$



Exponential Model to a Linear Model

$$y = ab^x$$

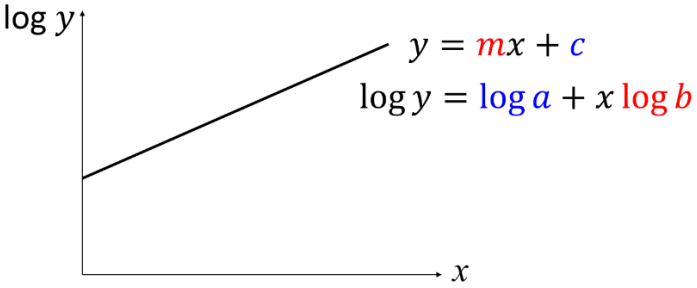
$$\log y = \log ab^x$$

$$\log y = \log a + \log b^x$$

$$\log y = \log a + x \log b$$

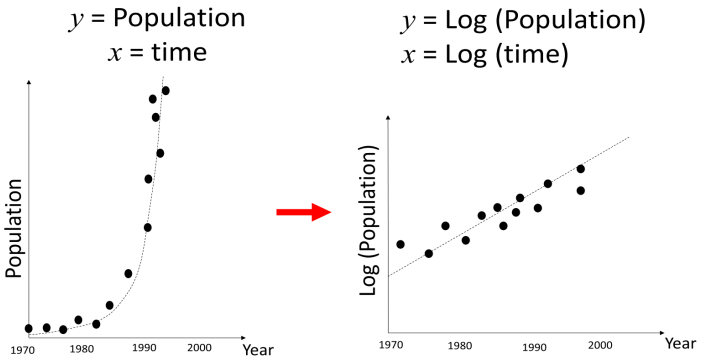
$$y = mx + c$$

If $y = ab^x$,
then the graph of $\log y$ against x
will be a straight line with
gradient $\log b$ and vertical intercept $\log a$.



Non-Linear Modelling

$y = ax^n$



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