# 2020 YEAR 13 MOCK PAPER 2

①

(a) $$2\log(3-x) - \log(21-2x) = 0$$

$$\log \frac{(3-x)^2}{(21-2x)} = 0$$

$$10^0 = \frac{(3-x)^2}{(21-2x)}$$

$$21-2x = (3-x)^2$$

$$21-2x = x^2-6x+9$$

$$0 = x^2-4x-12$$

(b) $$(x-6)(x+2)=0$$

$$x = 6 \quad x = -2$$

$x = 6$ is not a solution

as $\log(3-6) = \log(-3)$

cant log negatives

(2)

(a) $f(x) = 3x^4 + 2x^2 - 12x + 8$

$f'(x) = 12x^3 + 4x - 12$

at turning point $f'(x) = 0$

$0 = 12x^3 + 4x - 12$

$12 - 4x = 12x^3$

$3 - x = 3x^3$

$1 - \frac{x}{3} = x^3$

$\sqrt{1 - \frac{x}{3}} = x$

(b) $x_1 = 1$

$x_2 = \sqrt[3]{1 - \frac{1}{3}} = 0.8736$

$x_5 = 0.8894$

(c) $f'(0.8885) = -0.029$

$f'(0.8895) = 0.003$

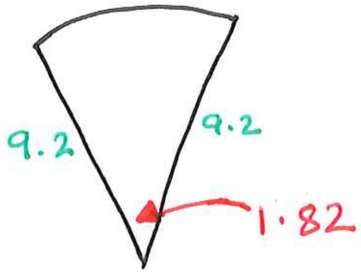
change of sign

$f'(x)$ is continous

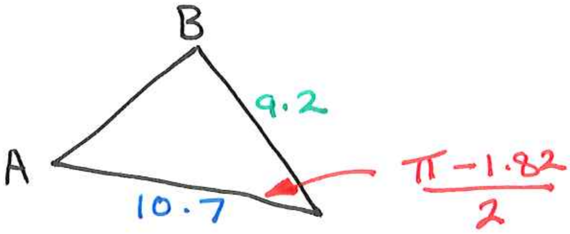
$\therefore \alpha = 0.889$ (3dp)

(3)

(a)




$$\text{Arc} = r\theta$$
$$= (9.2)(1.82)$$
$$= 16.744$$



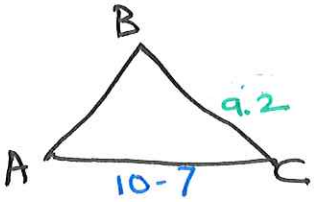

$$AB^2 = 9.2^2 + 10.7^2 - 2(9.2)(10.7)\cos\left(\frac{\pi - 1.82}{2}\right)$$

$$AB = 6.61$$

$$\text{Perimeter} = 16.744 + 2(6.61) + 2(10.7) = 51.4$$

(b)

$$\text{Sector Area} = \frac{1}{2} r^2 \theta$$
$$= \frac{1}{2}(9.2)^2(1.82) = 77.0$$

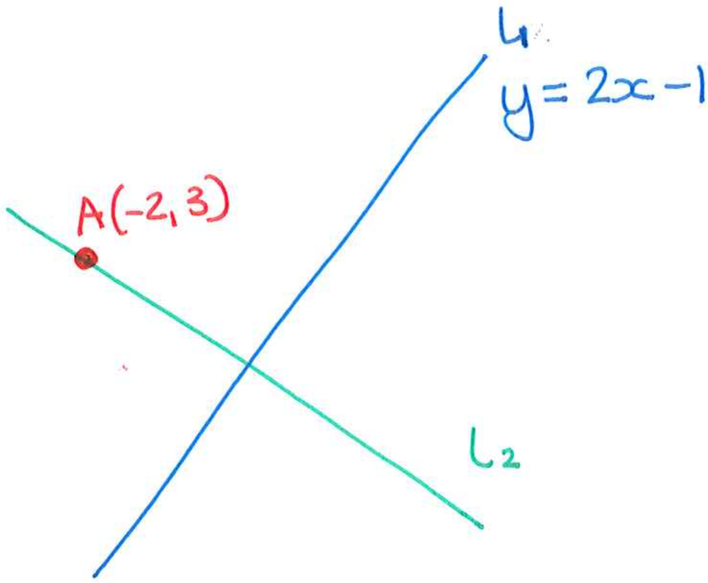



$$\text{Area} = \frac{1}{2} ab \sin C$$
$$= \frac{1}{2}(9.2)(10.7)\sin\left(\frac{\pi - 1.82}{2}\right) = 60.4$$

$$\text{Total Area} = 77.0 + 60.4$$
$$= 137.4 \text{ m}^2$$

4

(a)




$L_1$ gradient $= 2$

$L_2$ gradient $= -\frac{1}{2}$

$$y = -\frac{1}{2}x + c$$

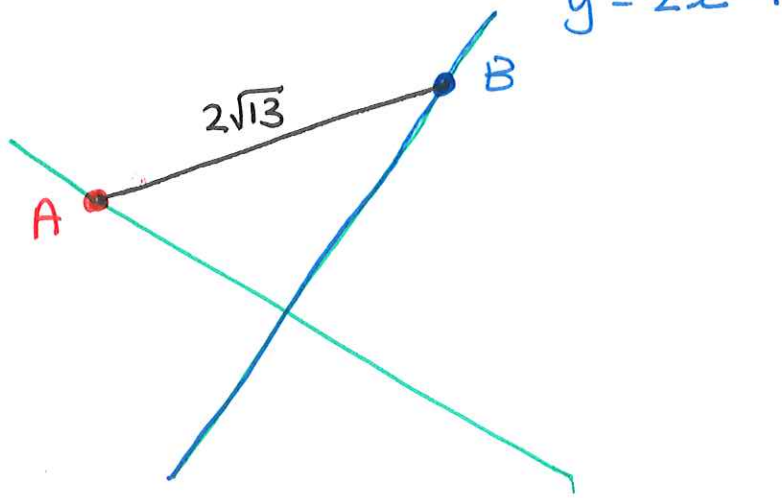
$$3 = -\frac{1}{2}(-2) + c$$

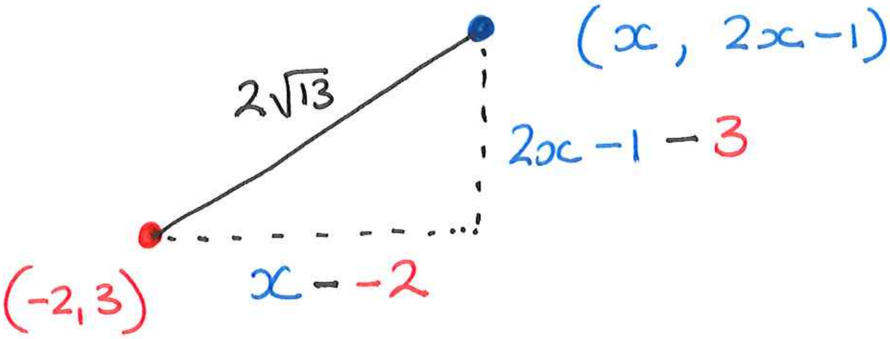
$$c = 2$$

$$y = -\frac{1}{2}x + 2$$

(4)

(b)




$$(2\sqrt{13})^2 = (x+2)^2 + (2x-4)^2$$

$$52 = x^2 + 4x + 4 + 4x^2 - 16x + 16$$

$$52 = 5x^2 - 12x + 20$$

$$0 = 5x^2 - 12x - 32$$

(c) Solutions $x = -\frac{8}{5}, 4$

$$y = 2\left(-\frac{8}{5}\right) - 1$$

$$y = -\frac{21}{5}$$

$$B\left(-\frac{8}{5}, -\frac{21}{5}\right)$$

(5)

$$AB + BC = AC$$

$$\begin{pmatrix} 2p \\ q \\ 4 \end{pmatrix} + \begin{pmatrix} q \\ -3p \\ 2 \end{pmatrix} = \begin{pmatrix} 2p+q \\ q-3p \\ 6 \end{pmatrix}$$

parallel $\begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix}$ but need a multipler as could be different lengths

$$k\begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} 2p+q \\ q-3p \\ 6 \end{pmatrix} = k\begin{pmatrix} 3 \\ -4 \\ 3 \end{pmatrix}$$

$$6 = k3 \qquad \therefore k = 2$$

$$\begin{aligned} 2p + q &= 3k \\ q - 3p &= -4k \end{aligned} \quad \Rightarrow \quad \begin{aligned} 2p + q &= 6 \\ -3p + q &= -8 \\ \hline 5p &= 14 \\ p &= \frac{14}{5} \\ q &= \frac{2}{5} \end{aligned}$$

(6)

(a) Arithetic sequence

$u_1$ $u_{10}$ $u_n$

2600 12000 $a + (n-1)d$

$a = 2600$

$n = 2$

$d = \frac{12000 - 2600}{9} = 1044.\dot{4}$

$u_2 = 2600 + (2-1)1044.\dot{4} = 3644$

(b) Geometric Sequence

$u_1$ $u_{10}$ $u_n$

2600 12000 $ar^{n}-1$

$a = 2600$

$n = 2$

$r \Rightarrow 12000 = 2600 \times r^9$

$r = 1.185$

$u_2 = 2600(1.185)^1$

$= 3081$

(6)

(c) Arithmetic

$$S_n = \frac{n}{2}[a + l]$$

$$= \frac{10}{2}[2600 + 12000]$$

$$= 73\ 000$$

Geometric

$$S_n = \frac{a(1 - r^n)}{1 - r}$$

$$= \frac{2600(1 - 1.185^{10})}{1 - 1.185}$$

$$= 62679 = 62700 \text{ nearest } 100$$

Difference $= 10\ 300$

(7)

(a) $g(x) = 4x^3 + ax^3 + 4x + b$

factor $(2x+1)$ means when $x = -\frac{1}{2}$
$g(x) = 0$

$$0 = 4\left(-\tfrac{1}{2}\right)^3 + a\left(-\tfrac{1}{2}\right)^3 + 4\left(-\tfrac{1}{2}\right) + b$$

$$0 = -\frac{1}{2} + \frac{a}{4} - 2 + b$$

$\times 4$

$$0 = -2 + a - 8 + 4b$$

$$10 = a + 4b$$

Inflection point means $g''(x) = 0$ when $x = \frac{1}{6}$

$$g'(x) = 12x^2 + 2ax + 4$$

$$g''(x) = 24x + 2a$$

$$0 = 24\left(\tfrac{1}{6}\right) + 2a$$

$$0 = 4 + 2a$$

$$a = -2$$

$$\therefore \quad b = 3$$

⑦

(b) $g'(x) = 12x^2 - 4x + 4$

stationary points mean $g'(x) = 0$

$$0 = 12x^2 - 4x + 4$$

$a = 12$
$b = -4$
$c = 4$

$b^2 - 4ac$

$= 16 - 4(12)(4)$

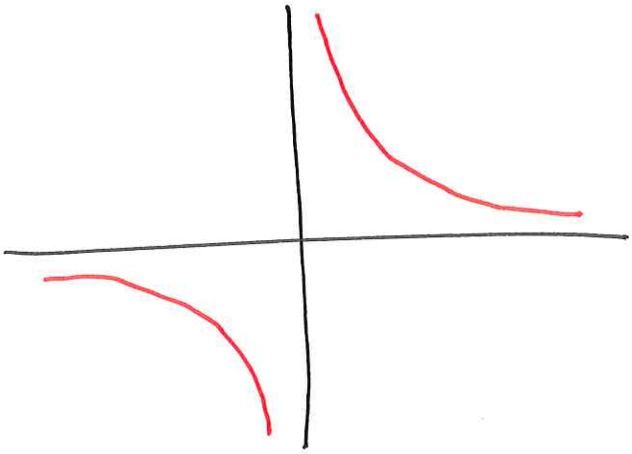
$= -176$

discriminate $< 0$ $\therefore$ no solutions

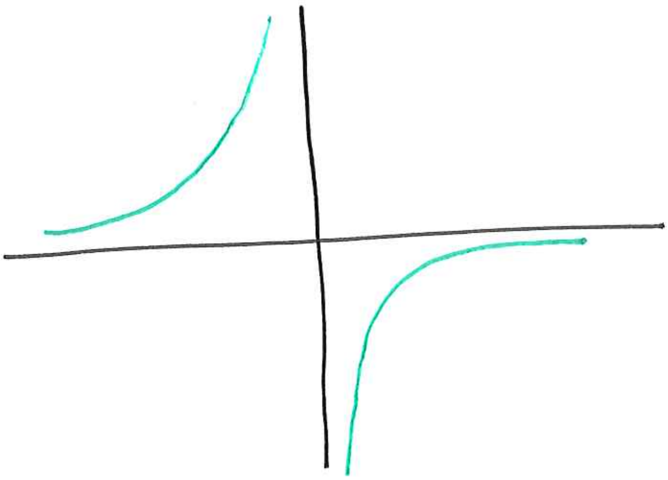
hence $g'(x) > 0$

8

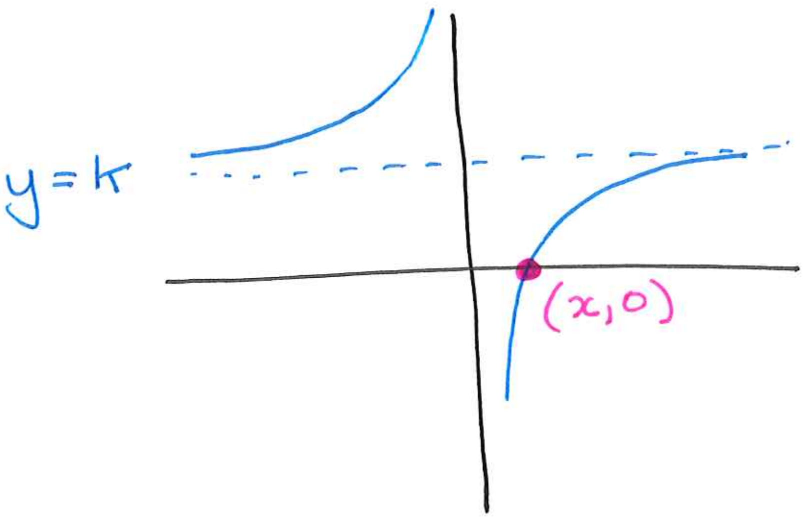
(a) $$y = k - \frac{1}{2x}$$



$$y = \frac{1}{2x}$$



$$y = -\frac{1}{2x}$$




$$y = k - \frac{1}{2x}$$

$$0 = k - \frac{1}{2x}$$

$$\frac{1}{2x} = k$$

$$\frac{1}{2k} = x$$

$$\left(x, \frac{1}{2k}\right)$$

8

(b)

$$2x + 3 = k - \frac{1}{2x}$$

$\times 2x$

$$4x^2 + 6x = 2xk - 1$$

$$4x^2 + 6x - 2xk + 1 = 0$$

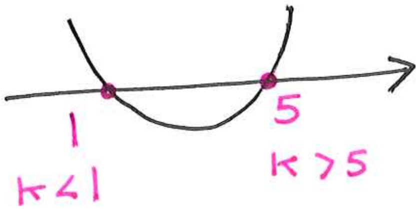
$$4x^2 + (6-2k)x + 1 = 0$$

Two distinct solution means $b^2 - 4ac > 0$

$$(6-2k)^2 - 4(4)(1) > 0$$

$$4k^2 - 24k + 36 \quad -16 > 0$$

$$4k^2 - 24k + 20 > 0$$

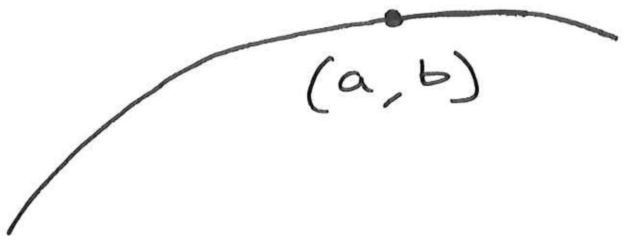



$$\{k : k < 1\} \cup \{k : k > 5\}$$

(9)

turning point $\frac{dy}{dx} = 0$

(a)




$y = xe^{-2x}$

$$y = xe^{-2x}$$

product rule

| | |
|---|---|
| $x$ | $e^{-2x}$ |
| $1$ | $-2e^{-2x}$ |

$$\frac{dy}{dx} = -2xe^{-2x} + 1e^{-2x}$$

$$0 = -2xe^{-2x} + e^{-2x}$$

$$2xe^{-2x} = e^{-2x}$$

$$2x = 1$$

$$x = \frac{1}{2}$$

$$y = xe^{-2x}$$

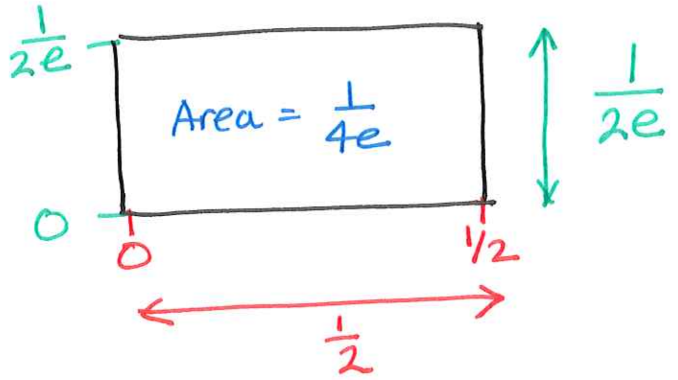
$$y = \left(\frac{1}{2}\right)e^{-2\left(\frac{1}{2}\right)}$$

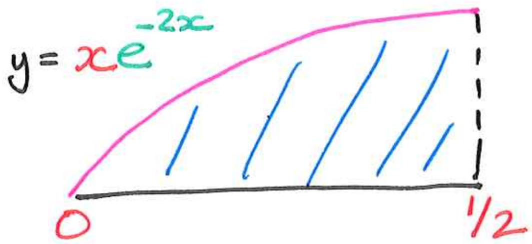
$$y = \frac{1}{2e}$$

$$\left(\frac{1}{2}, \frac{1}{2e}\right)$$

(9)

(b)




$$\int_0^{\frac{1}{2}} xe^{-2x}\, dx$$

Integrate by parts

| $u = x$ | $v = -\frac{1}{2}e^{-2x}$ |
|---|---|
| $\frac{du}{dx} = 1$ | $\frac{dv}{dx} = e^{-2x}$ |

$$uv - \int v \frac{du}{dx}\, dx$$

$$x\left(-\frac{1}{2}e^{-2x}\right) - \int -\frac{1}{2}e^{-2x} \times 1 \quad du$$

$$= -\frac{1}{2}xe^{-2x} - \int -\frac{1}{2}e^{-2x}\, du$$

$$= \left[-\frac{1}{2}xe^{-2x} - \frac{1}{4}e^{-2x}\right]_0^{\frac{1}{2}}$$

9

(b) continue

$$= \left[-\frac{1}{2}\left(\frac{1}{2}\right)e^{-2\left(\frac{1}{2}\right)} - \frac{1}{4}e^{-2\left(\frac{1}{2}\right)}\right] - \left[0 - \frac{1}{4}\right]$$

$$= \left[-\frac{1}{4e} - \frac{1}{4e}\right] + \left[\frac{1}{4}\right]$$

$$= \left[-\frac{2}{4e}\right] + \left[\frac{1}{4}\right]$$

$$= \frac{1}{4} - \frac{2}{4e}$$

$$R \text{ Area} = \frac{1}{4e} - \left[\frac{1}{4} - \frac{2}{4e}\right]$$

$$= \frac{1}{4e} - \frac{1}{4} + \frac{2}{4e}$$

$$= \frac{3}{4e} - \frac{1}{4}$$

(10)

(a) let $t = 0$ $H = 20$

$\sin 0 = 0$
$\cos 0 = 1$

$$20 = \frac{140}{A + 45(0) - 28(1)}$$

$$20 = \frac{140}{A - 28}$$

$$A = 35$$

(b) $45 \sin 2t - 28 \cos 2t$

addition formula

$53 \sin(2t - \alpha) = 53 \cos\alpha \sin 2t - 53 \sin\alpha \cos 2t$

$\therefore \; 53 \cos\alpha = 45$

$53 \sin\alpha = 28$

$$\frac{53 \sin\alpha}{53 \cos\alpha} = \tan\alpha = \frac{28}{45}$$

$$\alpha = 31.9°$$

(10)

(c) $$H_{min} = \frac{140}{\text{biggest value}}$$

$$= \frac{140}{35 + 53(\sin 2t - 31.9)}$$

$$= \frac{140}{35 + 53(1)}$$

$$= 1.59\,m$$

$$= 159\text{ cm}$$

(d) $$H = \frac{140}{35 + 45\sin 2t - 28\cos 2t} = \frac{14}{35 + 53\sin(2t - 31.9)}$$

$$35 + 53\sin(2t - 31.9) > 0$$

$$\sin(2t - 31.9) > \frac{-35}{53}$$

1ST solution = $-41.32$
2ND solution = $221.32$

$$2t - 31.9 > 221.32$$

$$t > 126.6$$

model only valid if t is greater than 126.6sec

(11)

(a)

$$\int y \frac{dx}{dt} \, dt$$

$x = 3\cos 2t$

$\frac{dx}{dt} = -6\sin 2t$

$$\int 2\tan t\,(-6\sin 2t)\, dt$$

$$\int -12 \tan t \sin 2t \, dt$$

$$\int -12 \frac{\sin t}{\cos t} (2\sin t \cos t) \, dt$$

$$\int -24 \sin^2 t \, dt$$

Limits

$0 \leq t \leq \frac{\pi}{4}$

$t = \frac{\pi}{4}$ gives $x = 0$

$t = 0$ gives $x = 3$

$$\int_0^3 dx = \int_{\pi/4}^0 dt = -\int_0^{\pi/4} dt$$

$$\int_{\pi/4}^0 -24 \sin^2 t \, dt$$

$$\int_0^{\pi/4} 24 \sin^2 t \, dt$$

(11)

(b) $\displaystyle\int^{\pi/4} 24 \sin^2 t \, dt$

double angle

$\cos 2t = 1 - 2\sin^2 t$

$\sin^2 t = \frac{1}{2} - \frac{1}{2}\cos 2t$

$$\int_0^{\pi/4} 24\left(\frac{1}{2} - \frac{1}{2}\cos 2t\right) dt$$

$$= \Big[12t - 6\sin 2t\Big]_0^{\pi/4}$$

$$= \left[12\left(\frac{\pi}{4}\right) - 6\sin\left(\frac{\pi}{2}\right)\right] - \left[0 - 0\right]$$

$$= 3\pi - 6$$

(12)

(a)

$$y = 3(2^{2x})$$

$$(a, 96\sqrt{2})$$

$$96\sqrt{2} = 3(2^{2x})$$

$$32\sqrt{2} = 2^{2x}$$

$$32\sqrt{2} = 2^{2x}$$

$$2^5\, 2^{\frac{1}{2}} = 2^{2x}$$

$$2^{5+\frac{1}{2}} = 2^{2x}$$

$$\therefore \quad \frac{11}{2} = 2x$$

$$x = \frac{11}{4}$$

(12)

(b) $$6^{3-x} = 3(2^{2x})$$

$$\log_2 6^{3-x} = \log_2 3(2^{2x})$$

$$(3-x)\log_2 6 = \log_2 3 + \log_2 2^{2x}$$

$$\boxed{\begin{aligned} \text{Let } \log_2 6 = \log_2 (2)(3) &= \log_2 2 + \log_2 3 \\ &= 1 + \log_2 3 \end{aligned}}$$

$$(3-x)(1+\log_2 3) = \log_2 3 + 2x\log_2 2$$

$$(3-x)(1+\log_2 3) = \log_2 3 + 2x$$

$$3 + 3\log_2 3 - x - x\log_2 3 = \log_2 3 + 2x$$

$$3 + 3\log_2 3 - \log_2 3 = 2x + x + x\log_2 3$$

$$3 + 2\log_2 3 = 3x + x\log_2 3$$

$$3 + 2\log_2 3 = x(3 + \log_2 3)$$

$$\frac{3 + 2\log_2 3}{3 + \log_2 3} = x$$

(13)

## Statement

There are **No** positive integers $a$ and $b$ with $a$ odd such that $a + 2b = \sqrt{8ab}$

## Assumption

There are positive integers $a$ and $b$ with $a$ odd such that $a + 2b = \sqrt{8ab}$

## Assumption Prove

$$a + 2b = \sqrt{8ab}$$

$$(a + 2b)^2 = 8ab$$

$$a^2 + 4ab + 4b^2 = 8ab$$

$$a^2 - 4ab + 4b^2 = 0$$

$$(a-2b)(a-2b) = 0$$

$$(a-2b)^2 = 0$$

$$\therefore \quad a - 2b = 0$$

$$a = 2b$$

contradiction as $a = 2b$ which means $a$ is even

Hence assumption is false given statement is true.