

Topic Test Summer 2022

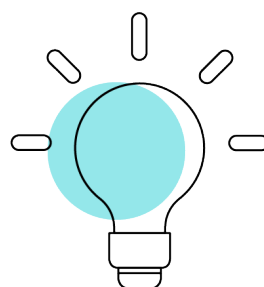
Pearson Edexcel GCE Mathematics (9MA0)

Paper 1 and Paper 2

Topic 1: Proof

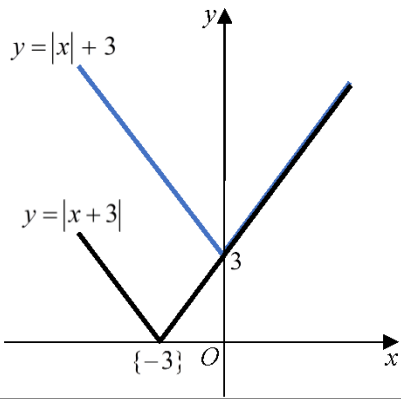
Contents

<u>General guidance to Topic Tests</u>	<u>3</u>
<u>Revise Revision Guide content coverage</u>	<u>4</u>
<u>Questions</u>	<u>5</u>
<u>Mark Scheme</u>	<u>20</u>



Mark Scheme

Question T1_Q1

Question	Scheme		Marks	AOs
3	Statement: "If m and n are irrational numbers, where $m \neq n$, then mn is also irrational."			
(a)	E.g. $m = \sqrt{3}$, $n = \sqrt{12}$		M1	1.1b
	$\{mn = \} \quad (\sqrt{3})(\sqrt{12}) = 6$ $\Rightarrow$ statement untrue or 6 is not irrational or 6 is rational		A1	2.4
			(2)	
(b)(i), (ii) Way 1		V shaped graph {reasonably} symmetrical about the y -axis with vertical intercept (0, 3) or 3 stated or marked on the positive y -axis	B1	1.1b
		Superimposes the graph of $y = x + 3 $ on top of the graph of $y = x + 3$	M1	3.1a
	the graph of $y = x + 3$ is either the same or above the graph of $y = x + 3 $ {for corresponding values of x } or when $x \geq 0$, both graphs are equal (or the same) when $x < 0$, the graph of $y = x + 3$ is above the graph of $y = x + 3 $		A1	2.4
			(3)	
(b)(ii) Way 2	<u>Reason 1</u> When $x \geq 0$, $ x + 3 = x + 3 $	Any one of Reason 1 or Reason 2	M1	3.1a
	<u>Reason 2</u> When $x < 0$, $ x + 3 > x + 3 $	Both Reason 1 and Reason 2	A1	2.4
(5 marks)				
Notes for Question 3				
(a)				
M1:	States or uses any pair of different numbers that will disprove the statement. E.g. $\sqrt{3}$, $\sqrt{12}$; $\sqrt{2}$, $\sqrt{8}$; $\sqrt{5}$, $-\sqrt{5}$; $\frac{1}{\pi}$, 2π ; $3e$, $\frac{4}{5e}$;			
A1:	Uses correct reasoning to disprove the given statement, with a correct conclusion			
Note:	Writing $(3e)\left(\frac{4}{5e}\right) = \frac{12}{5} \Rightarrow$ untrue is sufficient for M1A1			
(b)(i)				
B1:	See scheme			
(b)(ii)				
M1:	For constructing a method of comparing $ x + 3$ with $ x + 3 $. See scheme.			
A1:	Explains fully why $ x + 3 \geq x + 3 $. See scheme.			
Note:	Do not allow either $x > 0$, $ x + 3 \geq x + 3 $ or $x \geq 0$, $ x + 3 \geq x + 3 $ as a valid reason			
Note:	$x = 0$ (or where necessary $x = -3$) need to be considered in their solutions for A1			
Note:	Do not allow an incorrect statement such as $x \leq 0$, $ x + 3 > x + 3 $ for A1			

Notes for Question 3 Continued			
(b)(ii)			
Note:	Allow M1A1 for $x > 0$, $ x + 3 = x + 3 $ and for $x \leq 0$, $ x + 3 \geq x + 3 \geq$		
Note:	Allow M1 for any of x is positive, $x + 3 = x + 3$ x is negative, $x + 3 > x + 3$ $x > 0$, $x + 3 = x + 3$ $x \leq 0$, $x + 3 \geq x + 3$ $x > 0$, $x + 3$ and $x + 3$ are equal $x \geq 0$, $x + 3$ and $x + 3$ are equal - when $x \geq 0$, both graphs are equal - for positive values $x + 3$ and $x + 3$ are the same Condone for M1 $x \leq 0$, $x + 3 > x + 3$ $x < 0$, $x + 3 \geq x + 3$		
(b)(ii) Way 3	- For $x > 0$, $x + 3 = x + 3$ - For $-3 < x < 0$, as $x + 3 > 3$ and $\{0 < \} x + 3 < 3$, then $x + 3 > x + 3$ - For $x \leq -3$, as $x + 3 = -x + 3$ and $x + 3 = -x - 3$, then $x + 3 > x + 3$	M1	3.1a
		A1	2.4

Question T1_Q2

Question 10

General points for marking question 10 (i):

- Students who just try random numbers in part (i) are not going to score any marks.
- Students can mix and match methods. Eg you may see odd numbers via logic and even via algebra
- Students who state $4m^2 + 2$ **cannot be divided** by (instead of is not divisible by) cannot be awarded credit for the accuracy/explanation marks, unless they state correctly that $4m^2 + 2$ **cannot be divided by 4 to give an integer**.
- Students who write $n^2 + 2 = 4k \Rightarrow k = \frac{1}{4}n^2 + \frac{1}{2}$ which is not a whole number gains no credit unless they then start to look at odd and even numbers for instance
- Proofs via induction usually tend to go nowhere unless they proceed as in the main scheme
- Watch for unusual methods that are worthy of credit (See below)
- If the final conclusion is $n \in \mathbb{R}$ then the final mark is withheld. $n \in \mathbb{Z}^+$ is correct

Watch for methods that may not be in the scheme that you feel may deserve credit.

If you are uncertain of a method please refer these up to your team leader.

Eg 1. Solving part (i) by modulo arithmetic.

All $n \in \mathbb{N} \pmod{4}$	0	1	2	3
All $n^2 \in \mathbb{N} \pmod{4}$	0	1	0	1
All $n^2 + 2 \in \mathbb{N} \pmod{4}$	2	3	2	3

Hence for all n , $n^2 + 2$ is not divisible by 4.

Question 10 (i)	Scheme	Marks	AOs
-----------------	--------	-------	-----

Notes: Note that M0 A0 M1 A1 and M0 A0 M1 A0 are not possible due to the way the scheme is set up (i)

M1: Awarded for setting up the proof for either the even or odd numbers.

A1: Concludes correctly with a reason why $n^2 + 2$ cannot be divisible by 4 for either n odd or even.

dM1: Awarded for setting up the proof for both even and odd numbers

A1: Fully correct proof with valid explanation and conclusion for all n

Example of an algebraic proof

For $n = 2m$, $n^2 + 2 = 4m^2 + 2$	M1	2.1
Concludes that this number is not divisible by 4 (as the explanation is trivial)	A1	1.1b
For $n = 2m+1$, $n^2 + 2 = (2m+1)^2 + 2 = \dots$ FYI $(4m^2 + 4m + 3)$	dM1	2.1
Correct working and concludes that this is a number in the 4 times table add 3 so cannot be divisible by 4 or writes $4(m^2 + m) + 3$AND stateshence true for all	A1*	2.4
	(4)	

Example of a very similar algebraic proof

For $n = 2m$, $\frac{4m^2 + 2}{4} = m^2 + \frac{1}{2}$	M1	2.1
Concludes that this is not divisible by 4 due to the $\frac{1}{2}$ (A suitable reason is required)	A1	1.1b
For $n = 2m + 1$, $\frac{n^2 + 2}{4} = \frac{4m^2 + 4m + 3}{4} = m^2 + m + \frac{3}{4}$	dM1	2.1
Concludes that this is not divisible by 4 due to the $\frac{3}{4}$...AND states hence for all n , $n^2 + 2$ is not divisible by 4	A1*	2.4
	(4)	

Example of a proof via logic

When n is odd, "odd $\times$ odd" = odd	M1	2.1
so $n^2 + 2$ is odd, so (when n is odd) $n^2 + 2$ cannot be divisible by 4	A1	1.1b
When n is even, it is a multiple of 2, so "even $\times$ even" is a multiple of 4	dM1	2.1
Concludes that when n is even $n^2 + 2$ cannot be divisible by 4 because n^2 is divisible by 4.....AND STATEStrue for all n .	A1*	2.4
	(4)	

Example of proof via contradiction

Sets up the contradiction 'Assume that $n^2 + 2$ is divisible by 4 $\Rightarrow n^2 + 2 = 4k$ '	M1	2.1
$\Rightarrow n^2 = 4k - 2 = 2(2k - 1)$ and concludes even Note that the M mark (for setting up the contradiction must have been awarded)	A1	1.1b
States that n^2 is even, then n is even and hence n^2 is a multiple of 4	dM1	2.1
Explains that if n^2 is a multiple of 4 then $n^2 + 2$ cannot be a multiple of 4 and hence divisible by 4 Hence there is a contradiction and concludes Hence true for all n .	A1*	2.4
	(4)	

A similar proof exists via contradiction where

A1: $n^2 = 2(2k - 1) \Rightarrow n = \sqrt{2} \times \sqrt{2k - 1}$

dM1: States that $2k - 1$ is odd, so does not have a factor of 2, meaning that n is irrational

Question 10 (ii)	Scheme	Marks	AOs
------------------	--------	-------	-----

(ii)

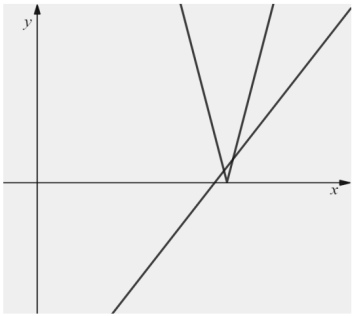
M1: States or implies ‘sometimes true’ or ‘not always true’ and gives an example where it is not true.

A1: and gives an example where it is true,

Proof using numerical values

SOMETIMES TRUE and chooses any number $x : 9.25 < x < 9.5$ and shows false Eg $x = 9.4 \quad 3x - 28 = 0.2 \quad \text{and} \quad x - 9 = 0.4 \quad \times$	M1	2.3
Then chooses a number where it is true Eg $x = 12 \quad 3x - 28 = 8 \quad x - 9 = 3 \quad \checkmark$	A1	2.4
	(2)	

Graphical Proof

 States or implies “sometimes true” Sketches both graphs on the same axes. Expect shapes and relative positions to be correct. V shape on +ve x-axis Linear graph with +ve gradient intersecting twice	M1	2.3
Graphs accurate and explains that as there are points where $ 3x - 28 < x - 9$ and points where $ 3x - 28 > x - 9$ or in words like ‘above’ and ‘below’ or ‘dips below at one point’	A1	2.4
	(2)	

Proof via algebra

States sometimes true and attempts to solve both $3x - 28 < x - 9$ and $-3x + 28 < x - 9$ or one of these with the bound $9.\dot{3}$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Alt: It is possible to find where it is always true

States sometimes true and attempts to solve where it is just true Solves both $3x - 28 \geq x - 9$ and $-3x + 28 \geq x - 9$	M1	2.3
States that it is false when $9.25 < x < 9.5$ or $9.25 < x < 9.\dot{3}$ or $9.\dot{3} < x < 9.5$	A1	2.4
	(2)	

Question T1_Q3

Question	Scheme	Marks	AOs
16	Sets up the contradiction and factorises: There are positive integers p and q such that $(2p+q)(2p-q)=25$	M1	2.1
	If true then $\begin{array}{ccc} 2p+q=25 & & 2p+q=5 \\ 2p-q=1 & \text{or} & 2p-q=5 \end{array}$ Award for deducing either of the above statements	M1	2.2a
	Solutions are $p=6.5, q=12$ or $p=2.5, q=0$ Award for one of these	A1	1.1b
	This is a contradiction as there are no integer solutions hence there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	
	(4 marks)		
Notes:			

M1: For the key step in setting up the contradiction and factorising

M1: For deducing that for p and q to be integers then either
$$\begin{array}{cc} 2p+q=25 & 2p+q=5 \\ 2p-q=1 & \text{or} \\ & 2p-q=5 \end{array}$$
 must be true.

Award for deducing either of the above statements.

You can ignore any reference to
$$\begin{array}{c} 2p+q=1 \\ 2p-q=25 \end{array}$$
 as this could not occur for positive p and q .

A1: For correctly solving one of the given statements,

For
$$\begin{array}{c} 2p+q=25 \\ 2p-q=1 \end{array}$$
 candidates only really need to proceed as far as $p=6.5$ to show the contradiction.

For
$$\begin{array}{c} 2p+q=5 \\ 2p-q=5 \end{array}$$
 candidates only really need to find either p or q to show the contradiction.

Alt for
$$\begin{array}{c} 2p+q=5 \\ 2p-q=5 \end{array}$$
 candidates could state that $2p+q \neq 2p-q$ if p, q are positive integers.

A1: For a complete and rigorous argument with both possibilities and a correct conclusion.

Question	Scheme	Marks	AOs
16 Alt 1	Sets up the contradiction, attempts to make q^2 or $4p^2$ the subject and states that either $4p^2$ is even(*) , or that q^2 (or q) is odd (**) Either There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow q^2 = 4p^2 - 25$ with * or ** Or There are positive integers p and q such that $4p^2 - q^2 = 25 \Rightarrow 4p^2 = q^2 + 25$ with * or **	M1	2.1
	Sets $q = 2n \pm 1$ and expands $(2n \pm 1)^2 = 4p^2 - 25$	M1	2.2a
	Proceeds to an expression such as $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n + 6) + 2$ $4p^2 = 4n^2 + 4n + 26 = 4(n^2 + n) + \frac{13}{2}$ $p^2 = n^2 + n + \frac{13}{2}$	A1	1.1b
	States This is a contradiction as $4p^2$ must be a multiple of 4 Or p^2 must be an integer And concludes there are no positive integers p and q such that $4p^2 - q^2 = 25$	A1	2.1
		(4)	

Alt 2

An approach using odd and even numbers is unlikely to score marks.

To make this consistent with the Alt method, score

M1: Set up the contradiction and start to consider one of the cases below where q is odd, $m \neq n$.

Solutions using the same variable will score no marks.

M1: Set up the contradiction and start to consider BOTH cases below where q is odd, $m \neq n$.

No requirement for evens

A1: Correct work and deduction for one of the two scenarios where q is odd

A1: Correct work and deductions for both scenarios where q is odd with a final conclusion

Options	Example of Calculation	Deduction
p (even) q (odd)	$4p^2 - q^2 = 4 \times (2m)^2 - (2n+1)^2 = 16m^2 - 4n^2 - 4n - 1$	One less than a multiple of 4 so cannot equal 25
p (odd) q (odd)	$4p^2 - q^2 = 4 \times (2m+1)^2 - (2n+1)^2 = 16m^2 + 16m - 4n^2 - 4n + 3$	Three more than a multiple of 4 so cannot equal 25

Question T1_Q4

Question	Scheme	Marks	AOs
16	NB any natural number can be expressed in the form: $3k, 3k + 1, 3k + 2$ or equivalent e.g. $3k - 1, 3k, 3k + 1$		
	Attempts to square any two distinct cases of the above	M1	3.1a
	Achieves accurate results and makes a valid comment for any two of the possible three cases: E.g. $(3k)^2 = 9k^2 (= 3 \times 3k^2)$ is a multiple of 3	A1 M1 on EPEN	1.1b

	$(3k+1)^2 = 9k^2 + 6k + 1 = 3 \times (3k^2 + 2k) + 1$ is one more than a multiple of 3 $(3k+2)^2 = 9k^2 + 12k + 4 = 3 \times (3k^2 + 4k + 1) + 1$ (or $(3k-1)^2 = 9k^2 - 6k + 1 = 3 \times (3k^2 - 2k) + 1$) is one more than a multiple of 3		
	Attempts to square in all 3 distinct cases. E.g. attempts to square $3k, 3k + 1, 3k + 2$ or e.g. $3k - 1, 3k, 3k + 1$	M1 A1 on EPEN	2.1
	Achieves accurate results for all three cases and gives a minimal conclusion (allow tick, QED etc.)	A1	2.4
		(4)	
(4 marks)			

Notes:

M1: Makes the key step of attempting to write the natural numbers in any 2 of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square these expressions.

A1(M1 on EPEN): Successfully shows for 2 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra. This must be made explicit e.g. reaches $3 \times (3k^2 + 2k) + 1$ and makes a statement that this is one more than a multiple of 3 but also allow other rigorous arguments that reason why $9k^2 + 6k + 1$ is one more than a multiple of 3 e.g. “ $9k^2$ is a multiple of 3 and $6k$ is a multiple of 3 so $9k^2 + 6k + 1$ is one more than a multiple of 3”

M1(A1 on EPEN): Recognises that all natural numbers can be written in one of the 3 distinct forms or equivalent expressions, as shown in the mark scheme, and attempts to square in all 3 cases.

A1: Successfully shows for all 3 cases that the squares are either a multiple of 3 or 1 more than a multiple of 3 using algebra and makes a conclusion

Question T1_Q5

Question	Scheme	Marks	AOs
15(i)	$n = 1, 2^3 = 8, 3^1 = 3, (8 > 3)$ $n = 2, 3^3 = 27, 3^2 = 9, (27 > 9)$ $n = 3, 4^3 = 64, 3^3 = 27, (64 > 27)$ $n = 4, 5^3 = 125, 3^4 = 81, (125 > 81)$	M1	2.1
	So if $n \leq 4, n \in \mathbb{N}$ then $(n + 1)^3 > 3^n$	A1	2.4
		(2)	
(ii)	Begins the proof by negating the statement. "Let m be odd " or "Assume m is not even"	M1	2.4
	Set $m = (2p \pm 1)$ and attempt $m^3 + 5 = (2p \pm 1)^3 + 5 = \dots$	M1	2.1
	$= 8p^3 + 12p^2 + 6p + 6$ AND deduces even	A1	2.2a
	Completes proof which requires reason and conclusion - reason for $8p^3 + 12p^2 + 6p + 6$ being even - acceptable statement such as "this is a contradiction so if $m^3 + 5$ is odd then m must be even"	A1	2.4
		(4)	
(6 marks)			
Notes			

(i)

M1: A full and rigorous argument that uses all of $n = 1, 2, 3$ and 4 in an attempt to prove the given result. Award for attempts at both $(n + 1)^3$ and 3^n for **ALL** values with at least 5 of the 8 values correct.

There is no requirement to compare their sizes, for example state that $27 > 9$

Extra values, say $n = 0$, may be ignored

A1: Completes the proof with no errors and an appropriate/allowable conclusion.

This requires

- all the values for $n = 1, 2, 3$ and 4 correct. Ignore other values
- all pairs compared correctly
- a minimal conclusion. Accept $\checkmark$ or hence proven for example

(ii)

M1: Begins the proof by negating the statement. See scheme

This cannot be scored if the candidate attempts m both odd and even

M1: For the key step in setting $m = 2p \pm 1$ and attempting to expand $(2p \pm 1)^3 + 5$

Award for a 4 term cubic expression.

A1: Correctly reaches $(2p + 1)^3 + 5 = 8p^3 + 12p^2 + 6p + 6$ and **states** even.

Alternatively reaches $(2p - 1)^3 + 5 = 8p^3 - 12p^2 + 6p + 4$ and **states** even.

A1: A full and complete argument that completes the contradiction proof. See scheme.

(1) A **reason** why the expression $8p^3 + 12p^2 + 6p + 6$ or $8p^3 - 12p^2 + 6p + 4$ is even

Acceptable reasons are

- all terms are even
- sight of a factorised expression E.g. $8p^3 - 12p^2 + 6p + 4 = 2(4p^3 - 6p^2 + 3p + 2)$

(2) Acceptable concluding statement

Acceptable concluding statements are

- "this is a contradiction, so if $m^3 + 5$ is odd then m is even"
- "this is contradiction, so proven."
- "So if $m^3 + 5$ is odd then m is even"

S.C If the candidate misinterprets the demand and does not use proof by contradiction but states a

counter example to the statement "if $m^3 + 5$ is odd then m must be even" such as when $m = \sqrt[3]{2}$ then they can score special case mark B1