

Differentiation Chain, Product and Quotient Exam Questions

Q1.

Differentiate with respect to x

(a) $\ln(x^2 + 3x + 5)$

(2)

(b) $\frac{\cos x}{x^2}$

(3)

(Total 5 marks)

Q2.

Differentiate with respect to x , giving your answer in its simplest form,

(a) $x^2 \ln(3x)$

(4)

(b) $\frac{\sin 4x}{x^3}$

(5)

(Total 9 marks)

Q3.

(a) Find the value of $\frac{dy}{dx}$ at the point where $x = 2$ on the curve with equation

$$y = x^2 \sqrt{5x - 1}.$$

(6)

(b) Differentiate $\frac{\sin 2x}{x^2}$ with respect to x .

(4)

(Total 10 marks)

Q4.

Given $y = x(2x + 1)^4$, show that

$$\frac{dy}{dx} = (2x + 1)^n (Ax + B)$$

where n , A and B are constants to be found.

(4)

(Total for question = 4 marks)

Q5.

The curve C has equation

$$y = (2x - 3)^5$$

The point P lies on C and has coordinates $(w, -32)$.

Find

(a) the value of w ,

(2)

(b) the equation of the tangent to C at the point P in the form $y = mx + c$, where m and c are constants.

(5)

(Total 7 marks)

Q6.

The curve C has equation

$$y = \frac{3 + \sin 2x}{2 + \cos 2x}$$

(a) Show that

$$\frac{dy}{dx} = \frac{6 \sin 2x + 4 \cos 2x + 2}{(2 + \cos 2x)^2}$$

(4)

(b) Find an equation of the tangent to C at the point on C where $x = \frac{\pi}{2}$.

Write your answer in the form $y = ax + b$, where a and b are exact constants.

(4)

(Total 8 marks)

Q7.

A curve C has equation

$$y = x^2 e^x.$$

(a) Find $\frac{dy}{dx}$, using the product rule for differentiation.

(3)

(b) Hence find the coordinates of the turning points of C.

(3)

(c) Find $\frac{d^2y}{dx^2}$.

(2)

(d) Determine the nature of each turning point of the curve C.

(2)

(Total for question = 10 marks)

Q8.

(a) Differentiate with respect to x ,

(i) $e^{3x}(\sin x + 2 \cos x)$,

(3)

(ii) $x^3 \ln (5x + 2)$.

(3)

Given that $y = \frac{3x^2 + 6x - 7}{(x+1)^2}$, $x \neq -1$,

(b) show that $\frac{dy}{dx} = \frac{20}{(x+1)^3}$.

(5)

(c) Hence find $\frac{d^2y}{dx^2}$ and the real values of x for which $\frac{d^2y}{dx^2} = -\frac{15}{4}$.

(3)

(Total 14 marks)