Write your name here

Surname	Other names

Pearson
Edexcel GCE

Centre Number | Candidate Number

AS and A level Further Mathematics
Core Pure Mathematics

Practice Paper
Complex numbers (part 2)

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 11 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

$$z = \frac{4}{1+\mathrm{i}}.$$

Find, in the form $a + \mathrm{i}b$ where $a, b \in \mathbb{R}$,

(a) z,

(2)

(b) z^2.

(2)

Given that z is a complex root of the quadratic equation $x^2 + px + q = 0$, where p and q are real integers,

(c) find the value of p and the value of q.

(3)

(Total 7 marks)

2.

$$\mathrm{f}(x) = (4x^2 + 9)(x^2 - 6x + 34).$$

(a) Find the four roots of $\mathrm{f}(\mathrm{x}) = 0$.

Give your answers in the form $x = p + \mathrm{i}q$, where p and q are real.

(5)

(b) Show these four roots on a single Argand diagram.

(2)

(Total 7 marks)

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3. The roots of the equation

$$z^3 - 8z^2 + 22z - 20 = 0$$

are z_1, z_2 and z_3.

(*a*) Given that $z_1 = 3 + \mathrm{i}$, find z_2 and z_3.

(4)

(*b*) Show, on a single Argand diagram, the points representing z_1, z_2 and z_3.

(2)

(Total 6 marks)

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4. Given that 4 and $2\mathrm{i} - 3$ are roots of the equation

$$x^3 + ax^2 + bx - 52 = 0$$

where *a* and *b* are real constants,

(*a*) write down the third root of the equation,

(1)

(*b*) find the value of *a* and the value of *b*.

(5)

(Total 6 marks)

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5. Given that $z = x + \mathrm{i}y$, find the value of x and the value of y such that

$$z + 3\mathrm{i}z^* = -1 + 13\mathrm{i}$$

where z^* is the complex conjugate of z.

(Total 7 marks)

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6. A complex number z is given by $z = a + 2\text{i}$,

where a is a non-zero real number.

(a) Find $z^2 + 2z$ in the form $x + \text{i}y$ where x and y are real expressions in terms of a.

(4)

Given that $z^2 + 2z$ is real,

(b) find the value of a.

(1)

Using this value for a,

(c) find the values of the modulus and argument of z, giving the argument in radians, and giving your answers to 3 significant figures.

(3)

(d) Show the points P, Q and R, representing the complex numbers z, z^2 and $z^2 + 2z$ respectively, on a single Argand diagram with origin O.

(3)

(e) Describe fully the geometrical relationship between the line segments OP and QR.

(2)

(Total 13 marks)

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7. $z_1 = 2 + 3\text{i}$, $\quad z_2 = 3 + 2\text{i}$, $\quad z_3 = a + b\text{i}$, $\quad a, b \in \mathbb{R}$

(a) Find the exact value of $|z_1 + z_2|$.

(2)

Given that $w = \dfrac{z_1 z_3}{z_2}$,

(b) find w in terms of a and b, giving your answer in the form $x + \text{i}y$, $\quad x, y \in \mathbb{R}$

(4)

Given also that $w = \dfrac{17}{13} - \dfrac{7}{13}\text{i}$,

(c) find the value of a and the value of b,

(3)

(d) find arg w, giving your answer in radians to 3 decimal places.

(2)

(Total 11 marks)

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8. $$z = 2 - \mathrm{i}\sqrt{3}.$$

(*a*) Calculate arg z, giving your answer in radians to 2 decimal places.

(2)

Use algebra to express

(*b*) $z + z^2$ in the form $a + b\mathrm{i}\sqrt{3}$, where a and b are integers,

(3)

(*c*) $\dfrac{z+7}{z-1}$ in the form $c + d\mathrm{i}\sqrt{3}$, where c and d are integers.

(4)

Given that $$w = \lambda - 3\mathrm{i},$$

where λ is a real constant, and $\arg(4 - 5\mathrm{i} + 3w) = -\dfrac{\pi}{2}$,

(*d*) find the value of λ.

(2)

(Total 11 marks)

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9. $$z = -24 - 7\mathrm{i}$$

(*a*) Show z on an Argand diagram.

(1)

(*b*) Calculate arg z, giving your answer in radians to 2 decimal places.

(2)

It is given that $$w = a + b\mathrm{i}, \quad a \in \mathbb{R}, \quad b \in \mathbb{R}.$$

Given also that $|w| = 4$ and $\arg w = \dfrac{5\pi}{6}$,

(*c*) find the values of a and b,

(3)

(*d*) find the value of $|zw|$.

(3)

(Total 9 marks)

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10. The point P represents a complex number z on an Argand diagram such that

$$|z - 6\mathrm{i}| = 2|z - 3|.$$

(*a*) Show that, as z varies, the locus of P is a circle, stating the radius and the coordinates of the centre of this circle.

(6)

The point Q represents a complex number z on an Argand diagram such that

$$\arg(z - 6) = -\frac{3\pi}{4}.$$

(*b*) Sketch, on the same Argand diagram, the locus of P and the locus of Q as z varies.

(4)

(*c*) Find the complex number for which both $|z - 6\mathrm{i}| = 2|z - 3|$ and $\arg(z - 6) = -\dfrac{3\pi}{4}$.

(4)

(Total 14 marks)

11. The complex number w is given by

$$w = 10 - 5\mathrm{i}$$

(*a*) Find $|w|$.

(1)

(*b*) Find arg w, giving your answer in radians to 2 decimal places

(2)

The complex numbers z and w satisfy the equation

$$(2 + \mathrm{i})(z + 3\mathrm{i}) = w$$

(*c*) Use algebra to find z, giving your answer in the form $a + b\mathrm{i}$, where a and b are real numbers.

(4)

Given that

$$\arg(\lambda + 9\mathrm{i} + w) = \frac{\pi}{4}$$

where λ is a real constant,

(*d*) find the value of λ.

(2)

(Total 9 marks)

TOTAL FOR PAPER: 100 MARKS