Write your name here

Surname	Other names

Pearson Edexcel GCE

Centre Number | Candidate Number

A level Mathematics

Practice Paper
Pure Mathematics - Differentiation in context

You must have:
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 82.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

1.

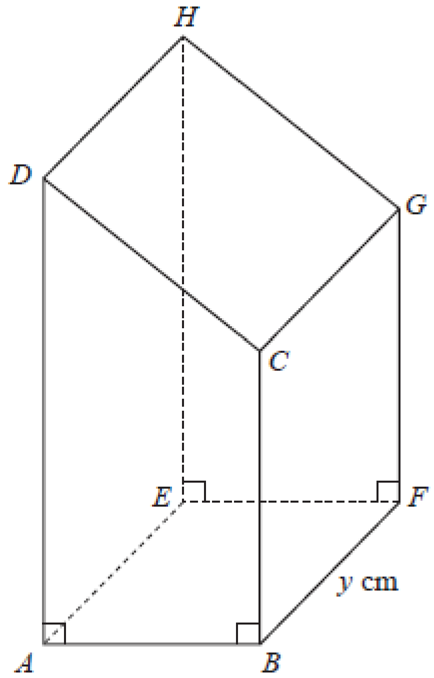

Figure 1

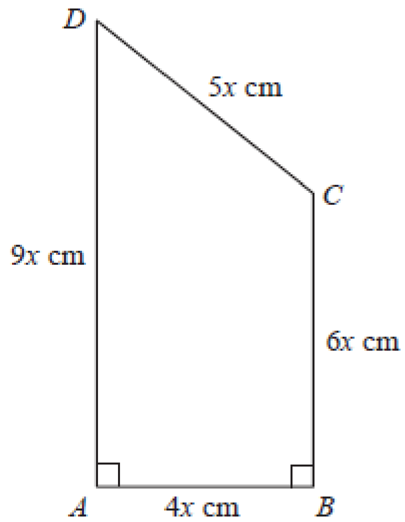

Figure 2

Figure 1 shows a closed letter box *ABFEHGCD*, which is made to be attached to a wall of a house.

The letter box is a right prism of length y cm as shown in Figure 1. The base *ABFE* of the prism is a rectangle. The total surface area of the six faces of the prism is S cm^2.

The cross section *ABCD* of the letter box is a trapezium with edges of lengths $DA = 9x$ cm, $AB = 4x$ cm, $BC = 6x$ cm and $CD = 5x$ cm as shown in Figure 2.

The angle $DAB = 90°$ and the angle $ABC = 90°$. The volume of the letter box is 9600 cm^3.

(a) Show that $y = \dfrac{320}{x^2}$.

(2)

(b) Hence show that the surface area of the letter box, S cm^2, is given by $S = 60x^2 + \dfrac{7680}{x}$.

(4)

(c) Use calculus to find the minimum value of S.

(6)

(d) Justify, by further differentiation, that the value of S you have found is a minimum.

(2)

(Total 14 marks)

2.

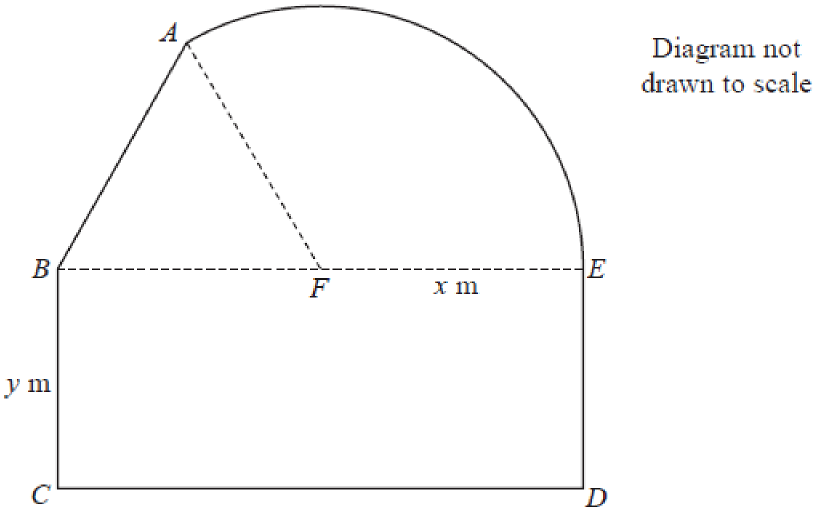

Figure 3

Figure 3 shows a plan view of a sheep enclosure.

The enclosure *ABCDEA*, as shown in Figure 4, consists of a rectangle *BCDE* joined to an equilateral triangle *BFA* and a sector *FEA* of a circle with radius x metres and centre *F*.

The points *B*, *F* and *E* lie on a straight line with $FE = x$ metres and $10 \leq x \leq 25$.

(*a*) Find, in m^2, the exact area of the sector *FEA*, giving your answer in terms of x, in its simplest form.

(2)

Given that $BC = y$ metres, where $y > 0$, and the area of the enclosure is 1000 m^2,

(*b*) show that

$$y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right).$$

(3)

(*c*) Hence show that the perimeter P metres of the enclosure is given by

$$P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right).$$

(3)

(*d*) Use calculus to find the minimum value of P, giving your answer to the nearest metre.

(5)

(*e*) Justify, by further differentiation, that the value of P you have found is a minimum.

(2)

(Total 15 marks)

3.

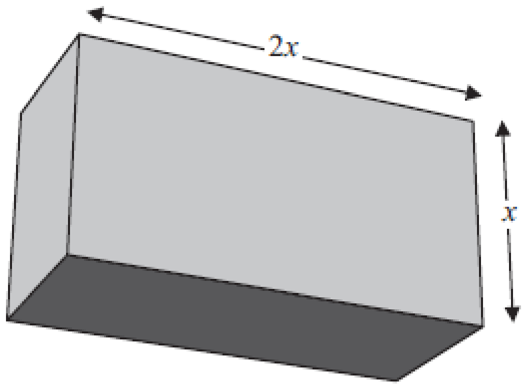

Figure 4

A cuboid has a rectangular cross-section where the length of the rectangle is equal to twice its width, x cm, as shown in Figure 4.

The volume of the cuboid is 81 cubic centimetres.

(*a*) Show that the total length, L cm, of the twelve edges of the cuboid is given by

$$L = 12x + \frac{162}{x^2}.$$

(3)

(*b*) Use calculus to find the minimum value of L.

(6)

(*c*) Justify, by further differentiation, that the value of L that you have found is a minimum.

(2)

(Total 11 marks)

4.

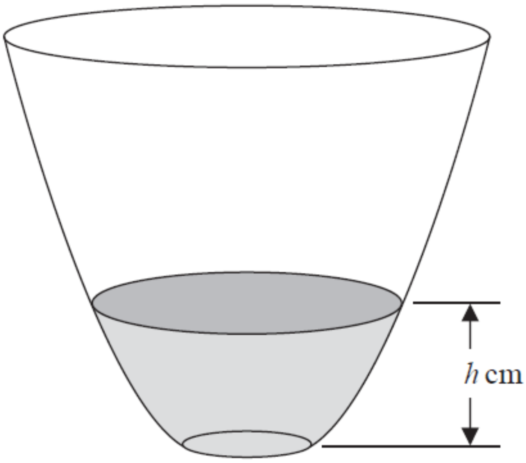

Figure 5

A vase with a circular cross-section is shown in Figure 5. Water is flowing into the vase.

When the depth of the water is h cm, the volume of water V cm^3 is given by

$$V = 4\pi h(h + 4), \qquad 0 \leq h \leq 25$$

Water flows into the vase at a constant rate of 80π cm^3 s^{-1}.

Find the rate of change of the depth of the water, in cm s^{-1}, when $h = 6$.

(Total 5 marks)

5. A rare species of primrose is being studied. The population, P, of primroses at time t years after the study started is modelled by the equation

$$P = \frac{800e^{0.1t}}{1+3e^{0.1t}}, \qquad t \geq 0, \quad t \in \mathbb{R}$$

(a) Calculate the number of primroses at the start of the study.

(2)

(b) Find the exact value of t when $P = 250$, giving your answer in the form $a \ln(b)$ where a and b are integers.

(4)

(c) Find the exact value of $\frac{dP}{dt}$ when $t = 10$. Give your answer in its simplest form.

(4)

(d) Explain why the population of primroses can never be 270.

(1)

(Total 11 marks)

6.

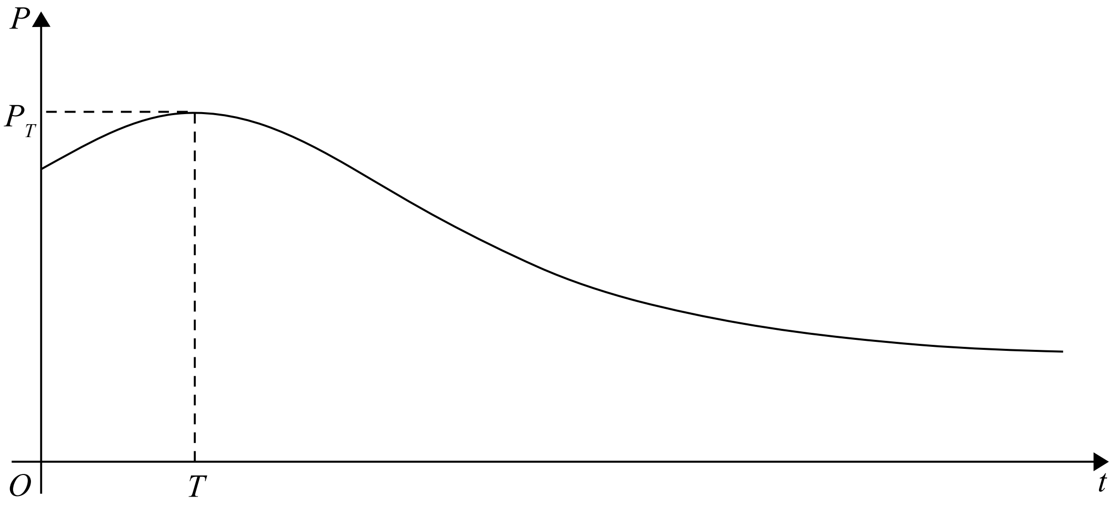

Figure 6

The number of rabbits on an island is modelled by the equation

$$P = \frac{100e^{-0.1t}}{1+3e^{-0.9t}} + 40, \qquad t \in \mathbb{R}, t \geqslant 0$$

where P is the number of rabbits, t years after they were introduced onto the island.

A sketch of the graph of P against t is shown in Figure 6.

(*a*) Calculate the number of rabbits that were introduced onto the island.

(1)

(*b*) Find $\dfrac{dP}{dt}$

(3)

The number of rabbits initially increases, reaching a maximum value P_T when $t = T$

(*c*) Using your answer from part (*b*), calculate

(i) the value of T to 2 decimal places,

(ii) the value of P_T to the nearest integer.

(*Solutions based entirely on graphical or numerical methods are not acceptable*.)

(4)

For $t > T$, the number of rabbits decreases, as shown in Figure 6, but never falls below k, where k is a positive constant.

(*d*) Use the model to state the maximum value of k.

(1)

(Total 9 marks)

7. The current, I amps, in an electric circuit at time t seconds is given by

$$I = 16 - 16(0.5)^t, \qquad t \geq 0.$$

Use differentiation to find the value of $\frac{\mathrm{d}I}{\mathrm{d}t}$ when $t = 3$.

Give your answer in the form $\ln a$, where a is a constant.

(Total 5 marks)

8.

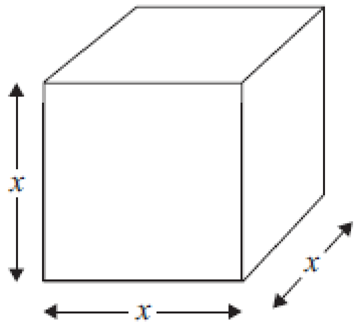

Figure 7

Figure 7 shows a metal cube which is expanding uniformly as it is heated.

At time t seconds, the length of each edge of the cube is x cm, and the volume of the cube is V cm^3.

(a) Show that $\frac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$.

(1)

Given that the volume, V cm^3, increases at a constant rate of 0.048 cm^3 s^{-1},

(b) find $\frac{\mathrm{d}x}{\mathrm{d}t}$ when $x = 8$,

(2)

(c) find the rate of increase of the total surface area of the cube, in cm^2 s^{-1}, when $x = 8$.

(3)

(Total 6 marks)

9.

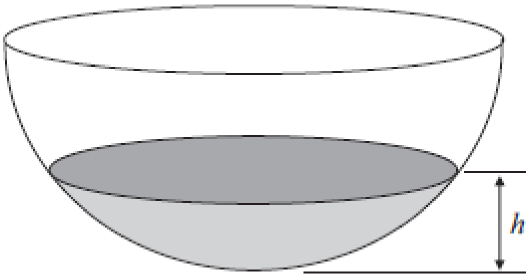

Figure 8

A hollow hemispherical bowl is shown in Figure 8. Water is flowing into the bowl.

When the depth of the water is h m, the volume V m^3 is given by

$$V = \frac{1}{12}\pi h^2(3 - 4h), \qquad 0 \leq h \leq 0.25.$$

(a) Find, in terms of π, $\frac{\mathrm{d}V}{\mathrm{d}h}$ when $h = 0.1$.

(4)

Water flows into the bowl at a rate of $\frac{\pi}{800}$ m^3 s^{-1}.

(b) Find the rate of change of h, in m s^{-1}, when $h = 0.1$.

(2)

(Total 6 marks)

TOTAL FOR PAPER: 82 MARKS