1. FM1_2019a Q3.

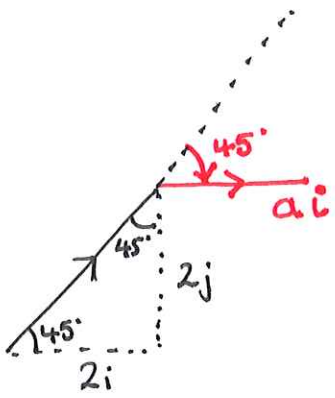
$u = \begin{pmatrix} 4 \\ 4 \end{pmatrix}$ $m = 0.5$ $I = \begin{pmatrix} \lambda \\ \mu \end{pmatrix}$ $|I| = 2.5$

$v = \begin{pmatrix} x \\ y \end{pmatrix}$

momentum before $= 0.5 \begin{pmatrix} 4 \\ 4 \end{pmatrix} = 2i + 2j$

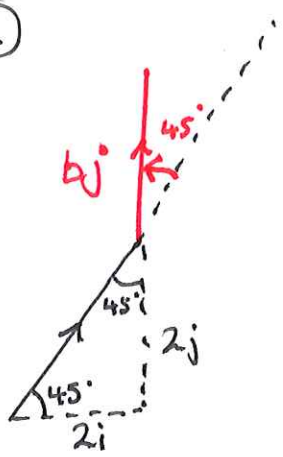
Possible momentum vectors (with 45° deflection)

(A)



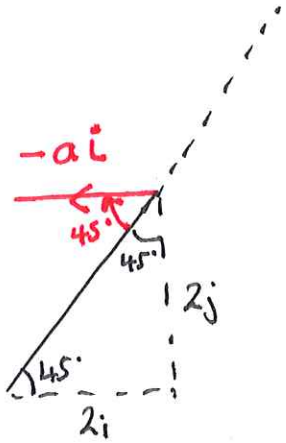
or

(C)



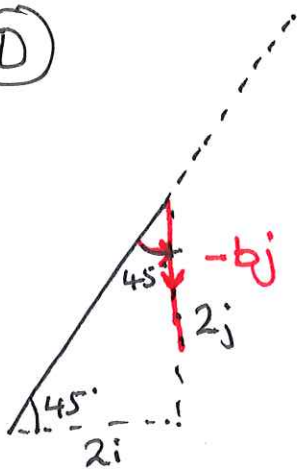
or

(B)

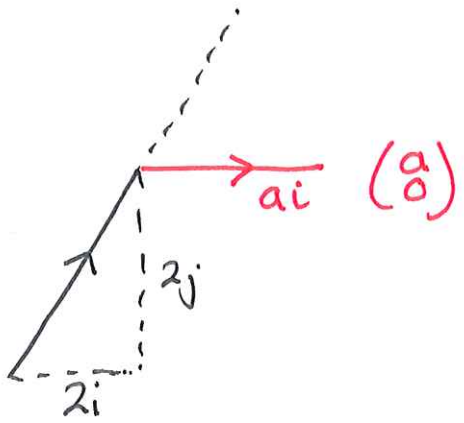


or

(D)



(A)



Momentum after = mv

$\begin{pmatrix} a \\ 0 \end{pmatrix} = 0.5v$

$v = \begin{pmatrix} 2a \\ 0 \end{pmatrix}$

$I = mv - mu$

$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0.5\begin{pmatrix} 2a \\ 0 \end{pmatrix} - 0.5\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$

$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} a-2 \\ -2 \end{pmatrix}$

$\lambda = (a-2) \quad \mu = -2$

$|I| = \sqrt{\lambda^2 + \mu^2}$

$2.5 = \sqrt{(a-2)^2 + (-2)^2}$

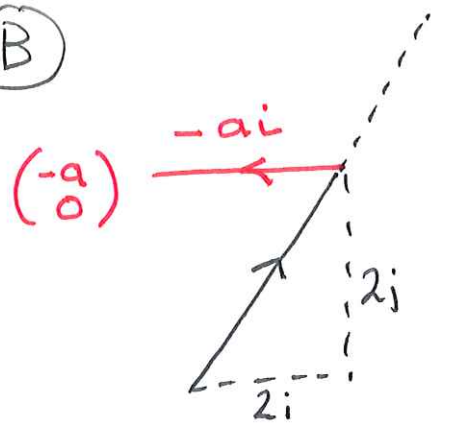
$6.25 = a^2 - 4a + 4 + 4$

$0 = a^2 - 4a + \frac{7}{4}$

$a = \frac{7}{2}$

$a = \frac{1}{2}$

(B)



$\begin{pmatrix} -a \\ 0 \end{pmatrix} = 0.5v$

$v = \begin{pmatrix} -2a \\ 0 \end{pmatrix}$

$I = mv - mu$

$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = 0.5\begin{pmatrix} -2a \\ 0 \end{pmatrix} - 0.5\begin{pmatrix} 4 \\ 4 \end{pmatrix}$

$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} -a-2 \\ -2 \end{pmatrix}$

$\lambda = (-a-2) \quad \mu = -2$

$2.5 = \sqrt{(-a-2)^2 + (-2)^2}$

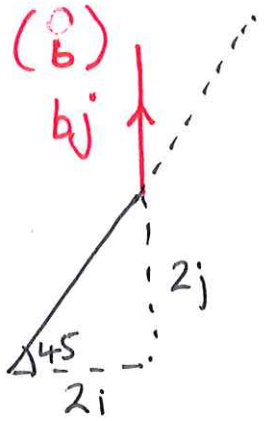
$6.25 = a^2 + 4a + 4 + 4$

$0 = a^2 + 4a + \frac{7}{4}$

$a = -\frac{7}{2}$

$a = -\frac{1}{2}$

(C)



momentum after = mv

$$\begin{pmatrix}0\\b\end{pmatrix} = 0.5\,v$$

$$v = \begin{pmatrix}0\\2b\end{pmatrix}$$

$$I = mv - mu$$

$$\begin{pmatrix}\lambda\\\mu\end{pmatrix} = 0.5\begin{pmatrix}0\\2b\end{pmatrix} - 0.5\begin{pmatrix}4\\4\end{pmatrix}$$

$$\begin{pmatrix}\lambda\\\mu\end{pmatrix} = \begin{pmatrix}0\\b\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix}$$

$$\begin{pmatrix}\lambda\\\mu\end{pmatrix} = \begin{pmatrix}-2\\b-2\end{pmatrix}$$

$\lambda = -2 \qquad \mu = (b-2)$

$$|I| = \sqrt{\lambda^2 + \mu^2}$$

$$2.5 = \sqrt{(-2)^2 + (b-2)^2}$$

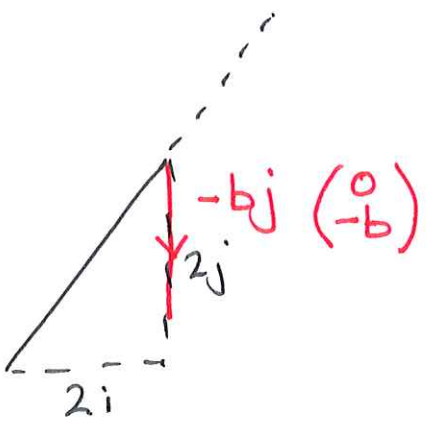
$$6.25 = 4 + b^2 - 4b + 4$$

$$0 = b^2 - 4b + \frac{7}{4}$$

$$b = \frac{7}{2}$$

$$b = \frac{1}{2}$$

(D)



$$\begin{pmatrix}0\\-b\end{pmatrix} = 0.5\,v$$

$$v = \begin{pmatrix}0\\-2b\end{pmatrix}$$

$$\begin{pmatrix}\lambda\\\mu\end{pmatrix} = 0.5\begin{pmatrix}0\\-2b\end{pmatrix} - 0.5\begin{pmatrix}4\\4\end{pmatrix}$$

$$\begin{pmatrix}\lambda\\\mu\end{pmatrix} = \begin{pmatrix}0\\-b\end{pmatrix} - \begin{pmatrix}2\\2\end{pmatrix}$$

$$\begin{pmatrix}\lambda\\\mu\end{pmatrix} = \begin{pmatrix}-2\\-b-2\end{pmatrix}$$

$\lambda = -2 \qquad \mu = (-b-2)$

$$2.5 = \sqrt{(-2)^2 + (-b-2)^2}$$

$$6.25 = 4 + b^2 + 4b + 4$$

$$0 = b^2 + 4b + \frac{7}{4}$$

$$b = -\frac{7}{2}$$

$$b = -\frac{1}{2}$$

(A) $a = \frac{7}{2}$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 7/2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \frac{3}{2}i - 2j$$

(B) $a = \frac{1}{2}$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = -\frac{3}{2}i - 2j$$

(C) $a = \frac{7}{2}$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 0 \\ b \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 0 \\ 7/2 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = -2i + \frac{3}{2}j$$

(D) $a = \frac{1}{2}$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 0 \\ b \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$\begin{pmatrix} \lambda \\ \mu \end{pmatrix} = \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = -2i - \frac{3}{2}j$$

1. continued (ii)

$a = 7/2$

$$\text{Impulse} = \begin{pmatrix} a \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 7/2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = \frac{3}{2}i - 2j$$

or

$a = 1/2$

$$\text{Impulse} = \begin{pmatrix} 1/2 \\ 0 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = -\frac{3}{2}i - 2j$$

or

$a = 7/2$

$$\text{Impulse} = \begin{pmatrix} 0 \\ b \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 7/2 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = -2i + \frac{3}{2}j$$

or

$a = \frac{1}{2}$

$$\text{Impulse} = \begin{pmatrix} 0 \\ b \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix}$$

$$= \begin{pmatrix} 0 \\ 1/2 \end{pmatrix} - \begin{pmatrix} 2 \\ 2 \end{pmatrix} = -2i - \frac{3}{2}j$$

2. FM1 – 2020 Q1

(a) $I = mv - mu$

$$\begin{pmatrix} x \\ y \end{pmatrix} = 0.5 \begin{pmatrix} -1 \\ 6 \end{pmatrix} - 0.5 \begin{pmatrix} 4 \\ 3 \end{pmatrix}$$

$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -0.5 \\ 3 \end{pmatrix} - \begin{pmatrix} 2 \\ 1.5 \end{pmatrix}$$

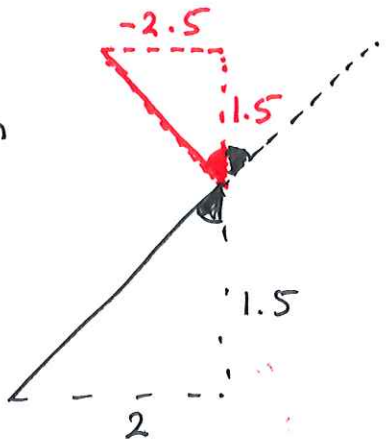
$$\begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} -2.5 \\ 1.5 \end{pmatrix}$$

$$I = -2.5\,\mathbf{i} + 1.5\,\mathbf{j}$$

$$|I| = \sqrt{(-2.5)^2 + (1.5)^2}$$

$$|I| = \frac{\sqrt{34}}{2}$$

(b) momentum vectors



$\tan^{-1}\left(\frac{2.5}{1.5}\right) = 59°$

$\tan^{-1}\left(\frac{2}{1.5}\right) = 53°$

$53 + 59 = \underline{\underline{112°}}$

Q3. FM1 – 2021 Q4

$$I = \begin{pmatrix} -\lambda \\ \lambda \end{pmatrix}$$

$$m = 0.5$$

$$u = \begin{pmatrix} 8 \\ 0 \end{pmatrix} \quad v = \begin{pmatrix} x \\ y \end{pmatrix} \quad |v| = 4\sqrt{10}$$

$$I = mv - mu$$

$$\begin{pmatrix} -\lambda \\ \lambda \end{pmatrix} = 0.5 \begin{pmatrix} x \\ y \end{pmatrix} - 0.5 \begin{pmatrix} 8 \\ 0 \end{pmatrix}$$

$$-\lambda = 0.5x - 4 \qquad \lambda = 0.5y$$

$$x = -2\lambda + 8 \qquad y = 2\lambda$$

$$|v| = \sqrt{x^2 + y^2}$$

$$4\sqrt{10} = \sqrt{(-2\lambda + 8)^2 + (2\lambda)^2}$$

$$160 = 4\lambda^2 - 32\lambda + 64 + 4\lambda^2$$

$$8\lambda^2 - 32\lambda + 64 = 160$$

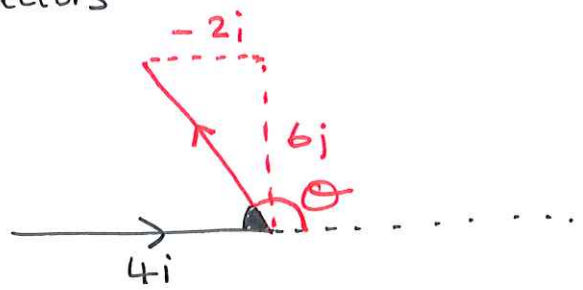
$$8\lambda^2 - 32\lambda - 96 = 0$$

$$\lambda = 6$$

Q3. continued

$$v = \begin{pmatrix} -2\lambda + 8 \\ 2\lambda \end{pmatrix} = \begin{pmatrix} -4 \\ 12 \end{pmatrix}$$

momentum vectors



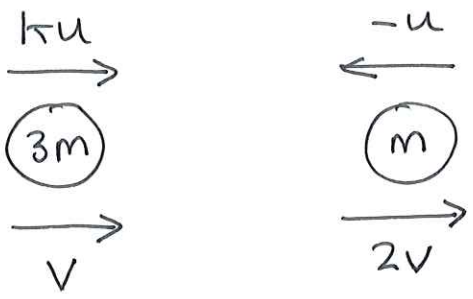
$$\tan^{-1}\left(\frac{6}{2}\right) = 72^\circ$$

$$\theta = 180^\circ - 72^\circ$$

$$= \underline{108^\circ}$$

Q4. FM1_2022 Q1

$|I| = \frac{3}{2}mu$



(a) Impulse $= mv - mu$

$$\frac{3mu}{2} = m(2v) - m(-u)$$

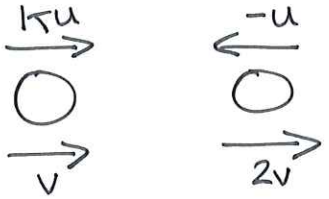
$$\frac{3mu}{2} = 2vm + mu$$

$$3u = 4v + 2u$$

$$u = 4v$$

$$v = \frac{u}{4}$$

Q4 continued



Com

$$ku(3m) + m(-u) = 3m(v) + (2v)m$$

$\div m$

$$3ku - u = 3v + 2v$$

$$3ku - u = 5v$$

sub $v = \frac{u}{4}$

$$3ku - u = \frac{5u}{4}$$

$\div u$

$$3k - 1 = \frac{5}{4}$$

$$3k = \frac{9}{4}$$

$$k = \frac{3}{4}$$

---

or

Com

$$ku(3m) + m(-u) = 3m(-v) + (2v)m$$

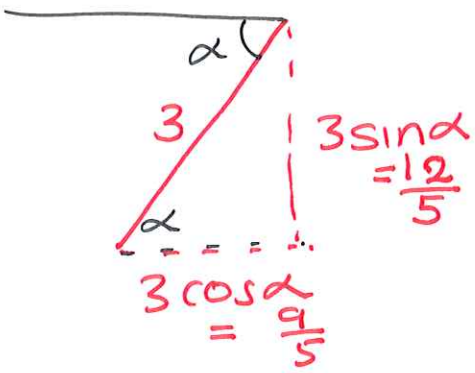
$$3ku - u = -v$$

$$3ku - u = \frac{-u}{4}$$

$$3ku = \frac{3u}{4}$$

$$k = \frac{1}{4}$$

Q5. FM1_2022 Q3



$\tan\alpha = \frac{4}{3}$

$\sin\alpha = \frac{4}{5}$

$\cos\alpha = \frac{3}{5}$

$I = mv - mu$

$$\begin{pmatrix} \frac{9}{5} \\ \frac{12}{5} \end{pmatrix} = 0.5\begin{pmatrix} x \\ y \end{pmatrix} - 0.5\begin{pmatrix} 2.8 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} 9/5 \\ 12/5 \end{pmatrix} = \begin{pmatrix} 0.5x \\ 0.5y \end{pmatrix} - \begin{pmatrix} 1.4 \\ 0 \end{pmatrix}$$

$\frac{12}{5} = 0.5y$

$y = \frac{24}{5}$

$\frac{9}{5} = 0.5x - 1.4$

$\frac{18}{5} = x - 2.8$

$x = \frac{32}{5}$

$$V = \sqrt{\left(\frac{32}{5}\right)^2 + \left(\frac{24}{5}\right)^2}$$

$V = 8$

Q6. FM1 – 2019 b Q3

see Q1.

Q7. FM1 – Sample Q1

$u = \begin{pmatrix} 4 \\ 1 \end{pmatrix}$ $\quad m = 0.5$

$I = \begin{pmatrix} 2 \\ -1 \end{pmatrix}$

$I = mv - mu$

$\begin{pmatrix} 2 \\ -1 \end{pmatrix} = 0.5 \begin{pmatrix} x \\ y \end{pmatrix} - 0.5 \begin{pmatrix} 4 \\ 1 \end{pmatrix}$

$\begin{pmatrix} 2 \\ -1 \end{pmatrix} = \begin{pmatrix} 0.5x \\ 0.5y \end{pmatrix} - \begin{pmatrix} 2 \\ 0.5 \end{pmatrix}$

$2 = 0.5x - 2$ $\qquad -1 = 0.5y - 0.5$

$x = 8$ $\qquad y = -1$

$v = 8i - j$

$|v| = \sqrt{65}$

$|u| = \sqrt{17}$

$KE = \frac{1}{2}m|v|^2 - \frac{1}{2}m|u|^2$

$= \frac{1}{2}(0.5)(\sqrt{65})^2 - \frac{1}{2}(0.5)(\sqrt{17})^2$

$= 12$

Q8. FM1 – Specimen Q3

(a) $m = 3$

$u = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ $\quad v = \begin{pmatrix} -1 \\ \lambda \end{pmatrix}$

$I = \begin{pmatrix} x \\ y \end{pmatrix}$ $\quad |I| = \sqrt{130}$

$I = mv - mu$

$\begin{pmatrix} x \\ y \end{pmatrix} = 3\begin{pmatrix} -1 \\ \lambda \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$x = -3 - 6 \qquad y = 3\lambda - 3$

$x = -9$

$|I| = \sqrt{x^2 + y^2}$

$\sqrt{130} = \sqrt{(-9)^2 + (3\lambda - 3)^2}$

$130 = 81 + 9\lambda^2 - 18\lambda + 9$

$9\lambda^2 - 18\lambda - 40 = 0$

$\lambda = \frac{10}{3}$

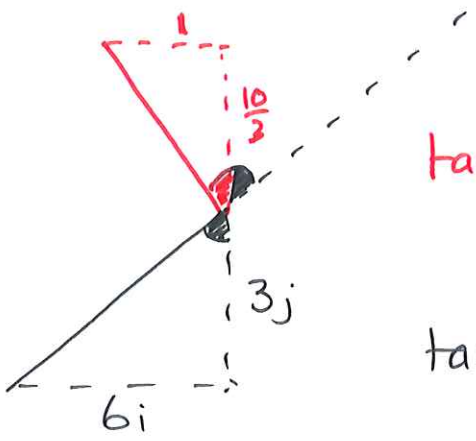
$I = 3\begin{pmatrix} -1 \\ 10/3 \end{pmatrix} - 3\begin{pmatrix} 2 \\ 1 \end{pmatrix}$

$I = \begin{pmatrix} -9 \\ 7 \end{pmatrix}$

$I = -9i + 7j$

Q8
(b)

momentum vectors



$$\tan^{-1}\left(\frac{1}{10/3}\right) + \tan^{-1}\left(\frac{6}{3}\right)$$

$$\theta = \underline{80.1^\circ}$$