AS and A level Mathematics Practice Paper – Implicit differentiation – Mark scheme

Question	Scheme	Marks
1 (a)	$x^3 + 2xy - x - y^3 - 20 = 0$	
	$\left\{\frac{d}{dx}\right\}\quad 3x^2 + \left(2y + 2x\frac{dy}{dx}\right) - 1 - 3y^2\frac{dy}{dx} = 0$	M1 A1 B1
	$3x^2 + 2y - 1 + \left(2x - 3y^2\right)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{3x^2 + 2y - 1}{3y^2 - 2x}$ or $\frac{1 - 3x^2 - 2y}{2x - 3y^2}$	A1 **cso**
		(5)
1 (b)	At $P(3,-2)$, $\mathrm{m}(\mathbf{T}) = \frac{dy}{dx} = \frac{3(3)^2 + 2(-2) - 1}{3(-2)^2 - 2(3)}; = \frac{22}{6}$ or $\frac{11}{3}$	M1
	and either T: $y - -2 = "\frac{11}{3}"(x - 3)$	
	or $(-2) = \left(\frac{11}{3}\right)(3) + c \Rightarrow c = \ldots,$	
	T: $11x - 3y - 39 = 0$ or $K(11x - 3y - 39) = 0$	A1 **cso**
		(2)
		(7 marks)
2(a)	$\left\{\frac{d}{dx}\right\}\quad 2 + 6y\frac{dy}{dx} + \left(6xy + 3x^2\frac{dy}{dx}\right) = 8x$	M1 A1 B1
	$\left\{\frac{dy}{dx} = \frac{8x - 2 - 6xy}{6y + 3x^2}\right\}$	
	At $P(-1,1)$, $\mathrm{m}(\mathbf{T}) = \frac{dy}{dx} = \frac{8(-1) - 2 - 6(-1)(1)}{6(1) + 3(-1)^2} = -\frac{4}{9}$	dM1 A1 **cso**
		(5)
2(b)	So, $\mathrm{m}(\mathbf{N}) = \frac{-1}{-\frac{4}{9}}\left\{= \frac{9}{4}\right\}$	M1
	N: $y - 1 = \frac{9}{4}(x + 1)$	M1
	N: $9x - 4y + 13 = 0$	A1
		(3)
		(8 marks)

AS and A level Mathematics Practice Paper – Implicit differentiation – Mark scheme

Question	Scheme	Marks
3(a)	$x^2 - 3xy - 4y^2 + 64 = 0$	
	$\left\{\frac{dy}{dx} =\right\}\ 2x - \left(3y + 3x\frac{dy}{dx}\right) - 8y\frac{dy}{dx} = 0$	**M1A1** **M1**
	$2x - 3y + (-3x - 8y)\frac{dy}{dx} = 0$	dM1
	$\frac{dy}{dx} = \frac{2x-3y}{3x+8y}$ **or** $\frac{3y-2x}{-3x-8y}$	A1 **cso**
		(5)
3(b)	$\left\{\frac{dy}{dx} = 0 \Rightarrow\right\}\ 2x - 3y = 0$	M1
	$y = \frac{2}{3}x$	A1ft
	$x^2 - 3x\left(\frac{2}{3}x\right) - 4\left(\frac{2}{3}x\right)^2 + 64 = 0$	dM1
	$x^2 - 2x^2 - \frac{16}{9}x^2 + 64 = 0 \Rightarrow -\frac{25}{9}x^2 + 64 = 0$	
	$\left\{\Rightarrow x^2 = \frac{576}{25} \Rightarrow\right\}\ x = \frac{24}{5}$ **or** $-\frac{24}{5}$	A1 **cso**
	When $x = \pm\frac{24}{5}$, $y = \frac{2}{3}\left(\frac{24}{5}\right)$ **and** $-\frac{2}{3}\left(\frac{24}{5}\right)$	
	$\left(\frac{24}{5}, \frac{16}{5}\right)$ **and** $\left(-\frac{24}{5}, -\frac{16}{5}\right)$ **or** $x = \frac{24}{5}$, $y = \frac{16}{5}$ **and** $x = -\frac{24}{5}$, $y = -\frac{16}{5}$	ddM1 A1
		(6)
		(11 marks)

AS and A level Mathematics Practice Paper – Implicit differentiation – Mark scheme

Question	Scheme	Marks
4(a)	$x^2 + y^2 + 10x + 2y - 4xy = 10$	
	$\left\{\frac{\mathrm{d}}{\mathrm{d}x}\right\}$ $2x + 2y\frac{\mathrm{d}y}{\mathrm{d}x} + 10 + 2\frac{\mathrm{d}y}{\mathrm{d}x} - \left(4y + 4x\frac{\mathrm{d}y}{\mathrm{d}x}\right) = 0$	M1 A1 M1
	$2x + 10 - 4y + (2y + 2 - 4x)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	dM1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{2x + 10 - 4y}{4x - 2y - 2}$	
	Simplifying gives $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{x + 5 - 2y}{2x - y - 1}$ $\left\{= \frac{-x - 5 + 2y}{-2x + y + 1}\right\}$	A1 **cso oe**
		(5)
4(b)	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} = 0 \Rightarrow\right\}$ $x + 5 - 2y = 0$	M1
	So $x = 2y - 5$,	
	$(2y - 5)^2 + y^2 + 10(2y - 5) + 2y - 4(2y - 5)y = 10$	M1
	$4y^2 - 20y + 25 + y^2 + 20y - 50 + 2y - 8y^2 + 20y = 10$	
	gives $-3y^2 + 22y - 35 = 0$ or $3y^2 - 22y + 35 = 0$	A1 oe
	$(3y - 7)(y - 5) = 0$ and $y = ...$	ddM1
	$y = \frac{7}{3}, 5$	A1 **cao**
		(5)
		(10 marks)

AS and A level Mathematics Practice Paper – Implicit differentiation – Mark scheme

Question	Scheme	Marks
5(a)	$\left\{\cancel{\frac{\mathrm{d}x}{\mathrm{d}x}} \times\right\} \left(4xy + 2x^2\frac{\mathrm{d}y}{\mathrm{d}x}\right) + 2 + 4\frac{\mathrm{d}y}{\mathrm{d}x} + \pi\sin(\pi y)\frac{\mathrm{d}y}{\mathrm{d}x} = 0$	M1 A1 B1
	$\frac{\mathrm{d}y}{\mathrm{d}x}\left(2x^2 + 4 + \pi\sin(\pi y)\right) + 4xy + 2 = 0$	dM1
	$\left\{\frac{\mathrm{d}y}{\mathrm{d}x} =\right\} \frac{-4xy - 2}{2x^2 + 4 + \pi\sin(\pi y)}$ or $\frac{4xy + 2}{-2x^2 - 4 - \pi\sin(\pi y)}$	A1 **cso**
		(5)
5(b)	At $\left(3, \frac{1}{2}\right)$, $m_{\mathrm{T}} = \frac{\mathrm{d}y}{\mathrm{d}x} = \frac{-4(3)(\frac{1}{2}) - 2}{2(3)^2 + 4 + \pi\sin\left(\frac{1}{2}\pi\right)} \left\{= \frac{-8}{22 + \pi}\right\}$	M1
	$m_{\mathrm{N}} = \frac{22 + \pi}{8}$	M1
	• $y - \frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$ • $\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(3) + c \Rightarrow c = \frac{1}{2} - \frac{66 + 3\pi}{8}$ $\Rightarrow y = \left(\frac{22 + \pi}{8}\right)x + \frac{1}{2} - \frac{66 + 3\pi}{8}$	dM1
	Cuts x-axis $\Rightarrow y = 0 \Rightarrow -\frac{1}{2} = \left(\frac{22 + \pi}{8}\right)(x - 3)$	
	So, $\left\{x = \frac{-4}{22 + \pi} + 3 \Rightarrow\right\} x = \frac{3\pi + 62}{\pi + 22}$	A1 o.e.
		(4)
		(9 marks)

AS and A level Mathematics Practice Paper – Implicit differentiation – Mark scheme

Question	Scheme	Marks
6(a)	$4x^2 - y^3 - 4xy + 2^y = 0$	
	$\left\{\frac{d}{dx}\right\} \; 8x - 3y^2\frac{dy}{dx} \; - 4y - 4x\frac{dy}{dx} \; + 2^y \ln 2\frac{dy}{dx} = 0$	M1 A1 M1 B1
	$8(-2) - 3(4)^2\frac{dy}{dx} - 4(4) - 4(-2)\frac{dy}{dx} + 2^4 \ln 2\frac{dy}{dx} = 0$	dM1
	$-16 - 48\frac{dy}{dx} - 16 + 8\frac{dy}{dx} + 16\ln 2\frac{dy}{dx} = 0$	
	$\frac{dy}{dx} = \frac{32}{-40+16\ln 2}$ or $\frac{-32}{40-16\ln 2}$ or $\frac{4}{-5+2\ln 2}$ or $\frac{4}{-5+\ln 4}$ or exact equivalent	A1 **cso**
		(6)
6(b)	e.g. $m_N = \frac{-40+16\ln 2}{-32}$ or $\frac{40-16\ln 2}{32}$	M1
	• $y - 4 = \left(\frac{40-16\ln 2}{32}\right)(x - -2)$ Cuts y-axis $\Rightarrow x = 0 \Rightarrow y - 4 = \left(\frac{40-16\ln 2}{32}\right)(2)$ $4 = \left(\frac{40-16\ln 2}{32}\right)(-2) + c$	M1
	$\left\{\Rightarrow c = 4 + \frac{40-16\ln 2}{16}, \text{ so } y = \frac{104-16\ln 2}{16} \Rightarrow\right\}$	
	y (or c) $= \frac{13}{2} - \ln 2$	A1 **cso isw**
		(3)
		(9 marks)

AS and A level Mathematics Practice Paper – Implicit differentiation – Mark scheme

Question	Scheme	Marks
7(a)	Differentiating implicitly to obtain $\pm ay^2 \frac{\mathrm{d}y}{\mathrm{d}x}$ and/or $\pm bx^2 \frac{\mathrm{d}y}{\mathrm{d}x}$	M1
	$48y^2 \frac{\mathrm{d}y}{\mathrm{d}x} + \ ... \ - 54 \ ...$	A1
	$9x^2 y \to 9x^2 \frac{\mathrm{d}y}{\mathrm{d}x} + 18xy$ or equivalent	B1
	$\left(48y^2 + 9x^2\right)\frac{\mathrm{d}y}{\mathrm{d}x} + 18xy - 54 = 0$	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{54 - 18xy}{48y^2 + 9x^2} \quad \left(= \frac{18 - 6xy}{16y^2 + 3x^2}\right)$	A1
		(5)
7(b)	$18 - 6xy = 0$	M1
	Using $x = \frac{3}{y}$ or $y = \frac{3}{x}$	
	$16y^3 + 9\left(\frac{3}{y}\right)^2 y - 54\left(\frac{3}{y}\right) = 0$ or $16\left(\frac{3}{x}\right)^3 + 9x^2\left(\frac{3}{x}\right) - 54x = 0$	M1
	Leading to	
	$16y^4 + 81 - 162 = 0$ or $16 + x^4 - 2x^4 = 0$	M1
	$y^4 = \frac{81}{16}$ or $x^4 = 16$	
	$y = \frac{3}{2}, -\frac{3}{2}$ or $x = 2, -2$	A1 A1
	Substituting either of their values into $xy = 3$ to obtain a value of the other variable.	M1
	$\left(2, \frac{3}{2}\right), \left(-2, -\frac{3}{2}\right)$ both	A1
		(7)
		(9 marks)

AS and A level Mathematics Practice Paper – Implicit differentiation – Mark scheme

	Source paper	Question number	New spec references	Question description	New AOs
1	C4 June 2014	1	7.5, 7.3	Implicit differentiation	1.1b
2	C4 Jan 2012	1	7.3, 7.5	Implicit differentiation	1.1b
3	C4 2015	2	7.5,	Implicit differentiation	1.1b,3.1a
4	C4 June 2014R	3	7.5, 2.4	Implicit differentiation	1.1b, 3.1a
5	C4 2016	3	7.2, 7.3, 7.4, 7.5	Implicit differentiation	1.1b, 2.1, 3.1a
6	C4 2017	4	6.4, 7.2, 7.3, 7.5	Implicit differentiation	1.1b, 2.1, 3.1a
7	C4 2012	5	7.5	Implicit differentiation	1.1b