Year 1 Pure Chapter 12 - Differentiation

First Principles

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$f(x) = x^3$ then $f'(x) = \lim_{h \to 0} \frac{(x+h)^3 - x^3}{h}$

expand + simplify $= \lim_{h \to 0} 3x^2 + 3xh + h^2$

(as $h \to 0$ $3xh = 0$, $h^2 = 0$)

$f'(x) = 3x^2$

Notation

$f(x) \xrightarrow[\text{differentiate}]{} f'(x) \xrightarrow[\text{differentiate}]{} f''(x)$

$y \longrightarrow \frac{dy}{dx} \longrightarrow \frac{d^2y}{dx^2}$

$4x^3 \longrightarrow 12x^2 \longrightarrow 24x$

multiply by power then subtract 1 from power

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Differentiation

$y = \sqrt{x} = x^{1/2} \quad \frac{dy}{dx} = \frac{1}{2}x^{-1/2}$

$y = \frac{1}{3x^3} = \frac{1}{3}x^{-3} \quad \frac{dy}{dx} = -\frac{3}{3}x^{-4}$

$y = \frac{2x^3 + 3x}{x} = 2x^2 + 3 \quad \frac{dy}{dx} = 4x$

both terms get divided by x

Gradient of a Tangent

Find $\frac{dy}{dx}$ then sub in x

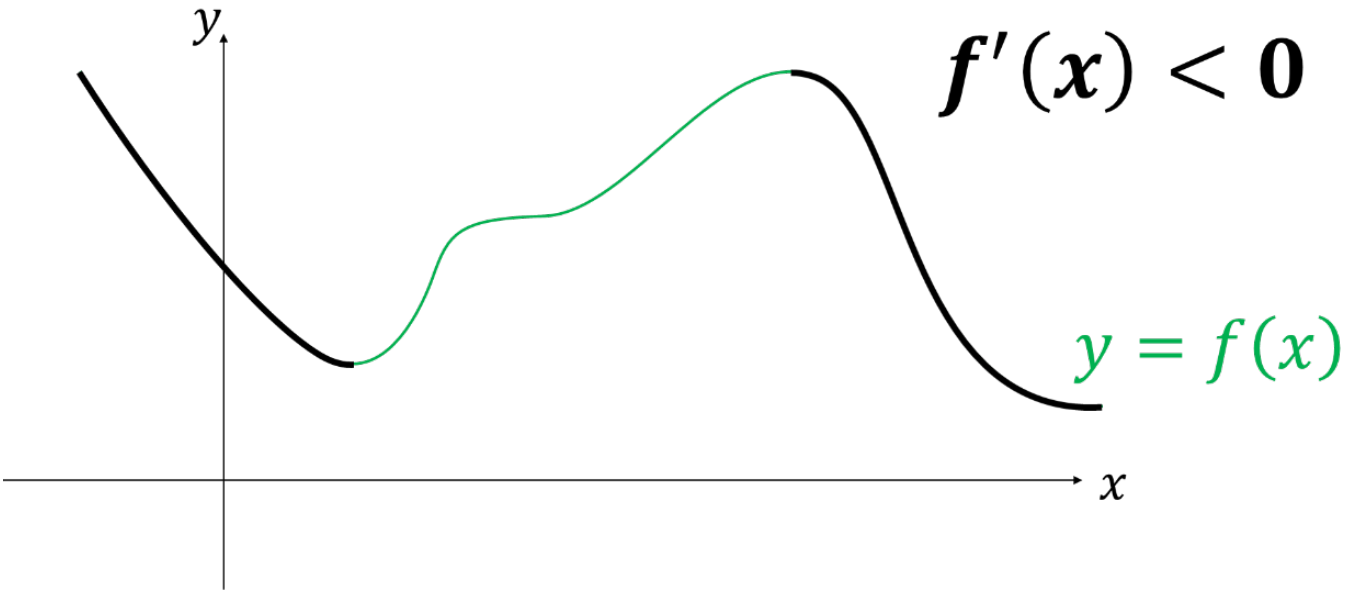
Max and Min Points

max point $\quad \frac{d^2y}{dx^2} < 0$

min point $\quad \frac{d^2y}{dx^2} > 0$

Increasing and Decreasing Functions

Decreasing



A function is decreasing if the gradient is less than zero

Example

Find the interval in which the function $f(x) = x^3 + 3x^2 - 9x$ is decreasing.

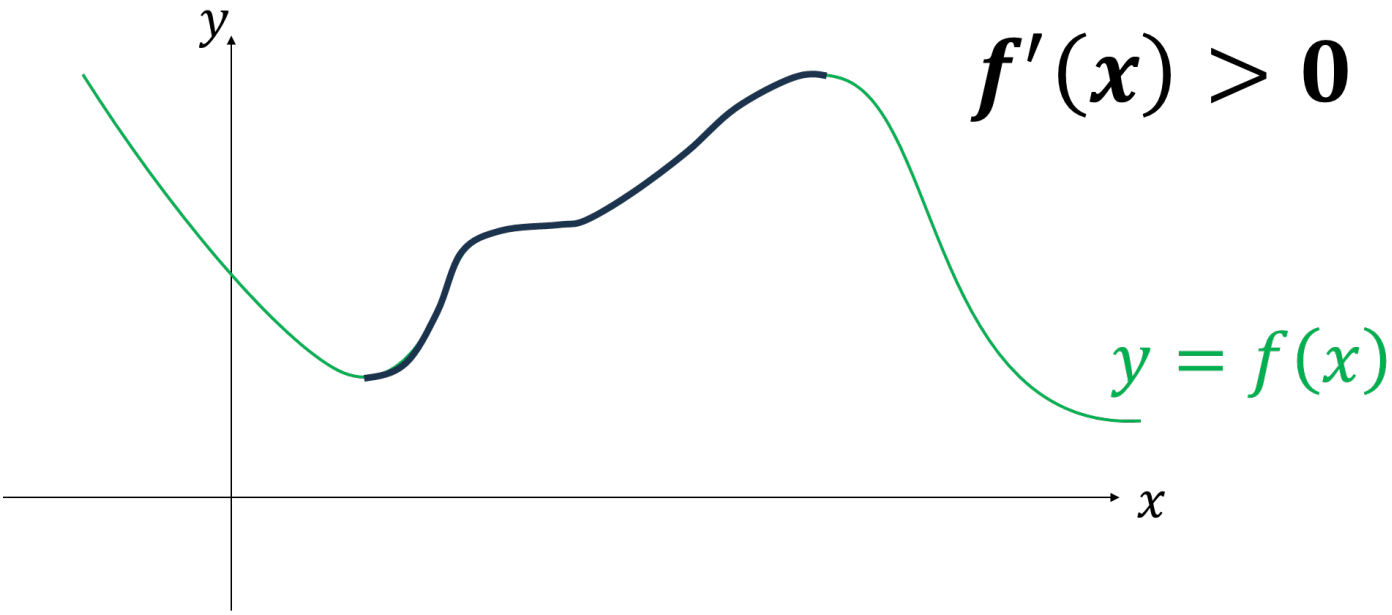
$f'(x) = 3x^2 + 6x - 9$

$3x^2 + 6x - 9 < 0$

$(x + 3)(x - 1) < 0$

$\boldsymbol{-3 < x < 1}$

Increasing



A function is decreasing if the gradient is more than zero

Example

Show that the function $f(x) = x^3 + 16x - 2$ is increasing for all real values of x.

$f'(x) = 3x^2 + 16$

$x^2 \geq 0$ for all real values of x

$\therefore 3x^2 + 16 > 0$ for all real x

Therefore $f(x)$ is an increasing function for all real values of x.