Year 2 Pure Chapter 4 – Binomial Expansion

Expansion A

Expansion for $(a+b)^n$

use when n is positive

Examples:

$(2+x)^5$

$(1+3x)^4$

$(4-2x)^6$

Expansion B

Expansion for $(1+x)^n$

use when n is positive, negative or fractional

Examples

$(1+3x)^{-2}$

$(1+2x)^{\frac{1}{2}}$

$(1+4x)^{-\frac{1}{2}}$

Watch out for when you don't have a 1 in the bracket

$(2+4x)^{-3}$

becomes

$2^{-3}(1+2x)^{-3}$

becomes

$\frac{1}{8}(1+2x)^{-3}$

Now expand and multiple all terms by $\frac{1}{8}$

Identify n and x

$\sqrt{1-3x} = (1-3x)^{\frac{1}{2}}$

$n = \frac{1}{2} \qquad x = -3x$

$\frac{1}{(1+2x)^3} = (1+2x)^{-3}$

$n = -3 \qquad x = 2x$

$(2+4x)^{-2} = 2^{-3}(1+2x)^{-2}$

$= \frac{1}{4}(1+2x)^{-2}$

$n = -2 \qquad x = 2x$

$(1+x)^n$ Terms

1st Term = 1

2nd Term = nx

3rd Term = $\frac{n(n-1)}{2 \text{ x } 1}x^2$

4th Term = $\frac{n(n-1)(n-2)}{3 \text{ x } 2 \text{ x } 1}x^3$

Combining Expansions

$$\frac{(1+x)^{\frac{1}{2}}}{(1-x)^{-\frac{1}{2}}} = (1+x)^{\frac{1}{2}}(1-x)^{-\frac{1}{2}}$$

Expand individually.

$$= (1+\frac{1}{2}x-\frac{1}{8}x^2)(1+\frac{1}{2}x+\frac{3}{8}x^2)$$

Multiply out the brackets.

$$= 1+x+\frac{1}{2}x^2+\cdots$$

Partial Fractions and Expansions

$$\frac{4-5x}{(1+x)(2-x)}$$

$$= \frac{A}{(1+x)} + \frac{B}{(2-x)}$$

Find A and B

(In the normal way)

$$= \frac{3}{(1+x)} - \frac{2}{(2-x)}$$

Then rewrite

$$= 3(1+x)^{-1} - 2(2-x)^{-1}$$

Now expand

$$= 3 - 3x + 3x^2 - 1 - \frac{x}{2} - \frac{x^2}{4}$$

Simplify

$$= 2 - \frac{7x}{2} - \frac{11x^2}{4}$$

Expansion Valid or Not

$$(1+x)^n$$

If n is negative or a fraction

The expansion is only VALID if

$$(1+x) \neq 0$$

$$\therefore |x| < 1$$

$(1+x)^{\frac{1}{2}}$ valid if

$$|x| < 1$$

$(1-2x)^{-2}$ valid if

$$|-2x| < 1$$

$$|x| < -\frac{1}{2}$$

$(1-\frac{x}{4})^{-\frac{1}{3}}$ valid if

$$\left|\frac{x}{4}\right| < 1$$

$$|x| < -4$$