

Completing the Square - Section Test

(Answers)

1)

$$\begin{aligned}x^2 + 4x + 5 &= (x+2)^2 - 4 + 5 \\&= (x+2)^2 + 1\end{aligned}$$

2)

$$\begin{aligned}x^2 - 6x + 5 &= (x-3)^2 - 9 + 5 \\&= (x-3)^2 - 4\end{aligned}$$

3)

$$\begin{aligned}4 + 2x - x^2 &= -(x^2 - 2x) + 4 \\&= -((x-1)^2 - 1) + 4 \\&= -(x-1)^2 + 1 + 4 \\&= -(x-1)^2 + 5\end{aligned}$$

4)

$$\begin{aligned}3x^2 + 6x + 1 &= 3(x^2 + 2x) + 1 \\&= 3((x+1)^2 - 1) + 1 \\&= 3(x+1)^2 - 3 + 1 \\&= 3(x+1)^2 - 2\end{aligned}$$

5)

$$\begin{aligned}4 - 8x - 2x^2 &= -2(x^2 + 4x) + 4 \\&= -2((x+2)^2 - 4) + 4 \\&= -2(x+2)^2 + 8 + 4 \\&= -2(x+2)^2 + 12\end{aligned}$$

6)

$$\begin{aligned}
 2x^2 + 6x + 1 &= 2(x^2 + 3x) + 1 \\
 &= 2\left(\left(x + \frac{3}{2}\right)^2 - \frac{9}{4}\right) + 1 \\
 &= 2\left(x + \frac{3}{2}\right)^2 - \frac{9}{2} + 1 \\
 &= 2\left(x + \frac{3}{2}\right)^2 - \frac{7}{2}
 \end{aligned}$$

7)

$$\begin{aligned}
 x^2 + ax + 1 &= \left(x + \frac{a}{2}\right)^2 - \frac{a^2}{4} + 1 \\
 &= \left(x + \frac{a}{2}\right)^2 - \frac{a^2 - 4}{4}
 \end{aligned}$$

$$\frac{a}{2} = 2 \text{ so } a = 4$$

$$\text{So, } b = \frac{a^2 - 4}{4} = \frac{16 - 4}{4} = \frac{12}{4} = 3$$

8)

$$\begin{aligned}
 a(x+3)^2 - c &= a(x^2 + 6x + 9) - c \\
 &= ax^2 + 6ax + 9a - c
 \end{aligned}$$

Equating coefficients,

$$a = 4$$

$$b = 6a = 6(4) = 24$$

$$9a - c = 30$$

$$c = 9a - 30 = 9(4) - 30 = 36 - 30 = 6$$

9)

First, complete the square:

$$y = x^2 + 6x + 1$$

$$= (x+3)^2 - 9 + 1$$

$$= (x+3)^2 - 8$$

Then rearrange to make x the subject of the formula

$$(x+3)^2 - 8 = y$$

$$(x+3)^2 = y + 8$$

$$x+3 = \pm\sqrt{y+8}$$

$$x = \pm\sqrt{y+8} - 3$$