A level Mathematics Practice Paper – Differentiation (part 2) – Mark scheme

Question	Scheme	Marks
1(a)	$\mathrm{f}(x)=\dfrac{4x+1}{x-2}, \quad x>2$	
	Applies $\dfrac{vu'-uv'}{v^2}$ to get $\dfrac{(x-2)\times 4-(4x+1)\times 1}{(x-2)^2}$	M1A1
	$=\dfrac{-9}{(x-2)^2}$	A1*
		(3)
1(b)	$\dfrac{-9}{(x-2)^2}=-1 \Rightarrow x=..$	M1
	(5,7)	A1 A1
		(3)
		(6 marks)
2(a)	$-32=(2w-3)^5 \Rightarrow w=\dfrac{1}{2}$ oe	M1A1
		(2)
2(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x}=5\times(2x-3)^4\times 2 \quad$ or $\quad 10(2x-3)^4$	M1A1
	When $x=\dfrac{1}{2}$, Gradient = 160	M1
	Equation of tangent is $'160'=\dfrac{y-(-32)}{x-'\frac{1}{2}'}$ oe	dM1
	$y=160x-112$ cso	A1
		(5)
		(7 marks)

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3(a)	$x = 8\frac{\pi}{8}\tan\left(2\times\frac{\pi}{8}\right) = \pi$	B1*
		(1)
3(b)	$\frac{\mathrm{d}x}{\mathrm{d}y} = 8\tan 2y + 16y\sec^2(2y)$	M1A1A1
	At P $\frac{\mathrm{d}x}{\mathrm{d}y} = 8\tan 2\frac{\pi}{8} + 16\frac{\pi}{8}\sec^2\left(2\times\frac{\pi}{8}\right) = \{8+4\pi\}$	M1
	$\frac{y-\frac{\pi}{8}}{x-\pi} = \frac{1}{8+4\pi}$, accept $y-\frac{\pi}{8} = 0.049(x-\pi)$	M1A1
	$\Rightarrow (8+4\pi)y = x + \frac{\pi^2}{2}$	A1
		(7)
		(8 marks)
4(a)	$\frac{\mathrm{d}x}{\mathrm{d}y} = 4\sec^2 2y\tan 2y$	B1
	Use $\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{\frac{\mathrm{d}x}{\mathrm{d}y}}$	M1
	Uses $\tan^2 2y = \sec^2 2y - 1$ and $\sec 2y = \sqrt{x}$ to get $\frac{\mathrm{d}x}{\mathrm{d}y}$ or $\frac{\mathrm{d}y}{\mathrm{d}x}$ in terms of just x	M1
	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{1}{4x(x-1)^{\frac{1}{2}}}$ (conclusion stated with no errors previously)	A1*
		(4)
4(b)	$\frac{\mathrm{d}y}{\mathrm{d}x} = (x^2+x^3)\times\frac{2}{2x} + (2x+3x^2)\ln 2x$	M1 A1 A1
	When $x = \frac{\mathrm{e}}{2}$, $\frac{\mathrm{d}y}{\mathrm{d}x} = 3(\frac{\mathrm{e}}{2}) + 4(\frac{\mathrm{e}}{2})^2 = 3(\frac{\mathrm{e}}{2}) + \mathrm{e}^2$	dM1 A1
		(5)
4(c)	$\mathrm{f}'(x) = \frac{(x+1)^{\frac{1}{3}}(-3\sin x) - 3\cos x(\frac{1}{3}(x+1)^{-\frac{2}{3}})}{(x+1)^{\frac{2}{3}}}$	M1 A1
	$\mathrm{f}'(x) = \frac{-3(x+1)(\sin x) - \cos x}{(x+1)^{\frac{4}{3}}}$	A1
		(3)
		(12 marks)

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Question	Scheme	Marks
5(a)	$p = 4\pi^2$ or $(2\pi)^2$	B1
		(1)
5(b)	$x = (4y - \sin 2y)^2 \Rightarrow \dfrac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$	M1A1
	Sub $y = \dfrac{\pi}{2}$ into $\dfrac{dx}{dy} = 2(4y - \sin 2y)(4 - 2\cos 2y)$	
	$\Rightarrow \dfrac{dx}{dy} = 24\pi \quad (= 75.4)$ / $\dfrac{dy}{dx} = \dfrac{1}{24\pi}(= 0.013)$	M1
	Equation of tangent $y - \dfrac{\pi}{2} = \dfrac{1}{24\pi}(x - 4\pi^2)$	M1
	Using $y - \dfrac{\pi}{2} = \dfrac{1}{24\pi}(x - 4\pi^2)$ with $x = 0 \Rightarrow y = \dfrac{\pi}{3}$ cso	M1 A1
		(6)
		(7 marks)
6(a)	$y = e^{3x}\cos 4x \Rightarrow \left(\dfrac{dy}{dx}\right) = \cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x$	M1 A1
	Sets $\cos 4x \times 3e^{3x} + e^{3x} \times -4\sin 4x = 0 \Rightarrow 3\cos 4x - 4\sin 4x = 0$	M1
	$\Rightarrow x = \dfrac{1}{4}\arctan\dfrac{3}{4}$	M1
	$\Rightarrow x =$ awrt 0.9463 4dp	A1
		(5)
6(b)	$x = \sin^2 2y \Rightarrow \dfrac{dx}{dy} = 2\sin 2y \times 2\cos 2y$	M1 A1
	Uses $\sin 4y = 2\sin 2y\cos 2y$ in their expression	M1
	$\dfrac{dx}{dy} = 2\sin 4y \Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2\sin 4y} = \dfrac{1}{2}\operatorname{cosec} 4y$	M1 A1
		(5)
		(10 marks)

A level Mathematics Practice Paper – Differentiation (part 2) – Mark scheme

Question	Scheme	Marks
7(a)	$y = 2x\left(x^2-1\right)^5 \Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \left(x^2-1\right)^5 \times 2 + 2x \times 10x\left(x^2-1\right)^4$	M1A1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \left(x^2-1\right)^4\left(2x^2-2+20x^2\right) = \left(x^2-1\right)^4\left(22x^2-2\right)$	M1 A1
		(4)
7(b)	$\dfrac{\mathrm{d}y}{\mathrm{d}x} \geqslant 0 \Rightarrow \left(22x^2-2\right) \geqslant 0 \Rightarrow$ critical values of $\pm\dfrac{1}{\sqrt{11}}$	M1
	$x \geqslant \dfrac{1}{\sqrt{11}} \quad x \leqslant -\dfrac{1}{\sqrt{11}}$	A1
		(2)
7(c)	$x = \ln(\sec 2y) \Rightarrow \dfrac{\mathrm{d}x}{\mathrm{d}y} = \dfrac{1}{\sec 2y} \times 2\sec 2y \tan 2y$	B1
	$\Rightarrow \dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{1}{2\tan 2y} = \dfrac{1}{2\sqrt{\sec^2 2y - 1}} = \dfrac{1}{2\sqrt{\mathrm{e}^{2x}-1}}$	M1 M1 A1
		(4)
		(10 marks)

A level Mathematics Practice Paper – Differentiation (part 2) – Mark scheme

Question	Scheme	Marks
8(a)	$x^2 - 9 = (x+3)(x-3)$	B1
	$\dfrac{4x-5}{(2x+1)(x-3)} - \dfrac{2x}{(x+3)(x-3)}$	
	$\dfrac{(4x-5)(x+3)}{(2x+1)(x-3)(x+3)} - \dfrac{2x(2x+1)}{(2x+1)(x+3)(x-3)}$	M1
	$\dfrac{5x-15}{(2x+1)(x-3)(x+3)}$	M1A1
	$\dfrac{5(x-3)}{(2x+1)(x-3)(x+3)} = \dfrac{5}{(2x+1)(x+3)}$	A1*
		(5)
8(b)	$\mathrm{f}(x) = \dfrac{5}{2x^2+7x+3}$	
	$\mathrm{f}'(x) = \dfrac{-5(4x+7)}{(2x^2+7x+3)^2}$	M1 M1 A1
	$\mathrm{f}'(-1) = -\dfrac{15}{4}$	M1 A1
	Uses $m_1m_2 = -1$ to give gradient of normal = $\dfrac{4}{15}$	M1
	$\dfrac{y - \left(-\frac{5}{2}\right)}{(x - -1)} = \text{their } \dfrac{4}{15}$	M1
	$y + \dfrac{5}{2} = \dfrac{4}{15}(x+1)$ or any equivalent form	A1
		(8)
		(13 marks)

A level Mathematics Practice Paper – Differentiation (part 2) – Mark scheme

Question	Scheme	Marks
9(a)	Applies $vu'+uv'$ to $\left(x^2-x^3\right)\mathrm{e}^{-2x}$	
	$\mathrm{g}'(x)=\left(x^2-x^3\right)\times\text{-}2\mathrm{e}^{-2x}+\left(2x-3x^2\right)\times\mathrm{e}^{-2x}$	M1 A1
	$\mathrm{g}'(x)=\left(2x^3-5x^2+2x\right)\mathrm{e}^{-2x}$	A1
		(3)
9(b)	Sets $\left(2x^3-5x^2+2x\right)\mathrm{e}^{-2x}=0\Rightarrow 2x^3-5x^2+2x=0$	M1
	$x\left(2x^2-5x+2\right)=0\Rightarrow x=(0),\frac{1}{2},2$	M1 A1
	Sub $x=\frac{1}{2},2$ into $g(x)=\left(x^2-x^3\right)\mathrm{e}^{-2x}\Rightarrow g\left(\frac{1}{2}\right)=\frac{1}{8\mathrm{e}},\quad g(2)=-\frac{4}{\mathrm{e}^4}$	dM1 A1
	Range $-\frac{4}{\mathrm{e}^4}\leqslant g(x)\leqslant\frac{1}{8\mathrm{e}}$	A1
		(6)
9(c)	Accept $\mathrm{g}(x)$ is NOT a ONE to ONE function Accept $\mathrm{g}(x)$ is a MANY to ONE function Accept $\mathrm{g}^{-1}(x)$ would be ONE to MANY	B1
		(1)
		(10 marks)

A level Mathematics Practice Paper – Differentiation (part 2) – Mark scheme

Question	Scheme	Marks
10(a)	$x^2 - 3kx + 2k^2 = (x-2k)(x-k)$	B1
	$2 - \frac{(x-5k)(x-k)}{(x-2k)(x-k)} = 2 - \frac{(x-5k)}{(x-2k)} = \frac{2(x-2k)-(x-5k)}{(x-2k)}$	M1
	$= \frac{x+k}{(x-2k)}$	A1*
		(3)
10(b)		
	Applies $\frac{vu'-uv'}{v^2}$ to $y = \frac{x+k}{x-2k}$ with $u = x+k$ and $v = x-2k$	
	$\Rightarrow \mathrm{f}'(x) = \frac{(x-2k)\times 1-(x+k)\times 1}{(x-2k)^2}$	M1 A1
	$\Rightarrow \mathrm{f}'(x) = \frac{-3k}{(x-2k)^2}$	A1
		(3)
10(c)		
	If $\mathrm{f}'(x) = \frac{-Ck}{(x-2k)^2} \Rightarrow \mathrm{f}(x)$ is an increasing function as $\mathrm{f}'(x) > 0$,	M1
	$\mathrm{f}'(x) = \frac{-3k}{(x-2k)^2} > 0$ for all values of x as $\frac{\text{negative}\times\text{negative}}{\text{positive}} = \text{positive}$	A1
		(2)
		(8 marks)

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Question	Scheme	Marks
11(a)	$$\begin{array}{r} x^2+3 \\ x^2+x-6\overline{\big)\,x^4+x^3-3x^2+7x-6} \\ \underline{x^4+x^3-6x^2} \\ 3x^2+7x-6 \\ \underline{3x^2+3x-18} \\ 4x+12 \end{array}$$	M1 A1
	$\dfrac{x^4+x^3-3x^2+7x-6}{x^2+x-6} \equiv x^2+3+\dfrac{4(x+3)}{(x+3)(x-2)}$	M1
	$\equiv x^2+3+\dfrac{4}{(x-2)}$	A1
		(4)
11(b)	$\mathrm{f}'(x)=2x-\dfrac{4}{(x-2)^2}$	M1A1ft
	Substitutes $x=3$ into $\mathrm{f}'(x=3)=2\times3-\dfrac{4}{(3-2)^2}=(2)$	M1
	Uses $m=-\dfrac{1}{\mathrm{f}'(3)}=\left(-\dfrac{1}{2}\right)$ with $(3,\mathrm{f}(3))=(3,16)$ to form equation of normal	
	$y-16=-\dfrac{1}{2}(x-3)$ or equivalent cso	M1 A1
		(5)
		(9 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C3 June 2014	1	7.4,	Differentiation using quotient rule	1.1b, 2.1
2	C3 Jan 2013	1	7.3, 7.4	Differentiation	1.1b, 2.1
3	C3 June 2014	3	7.3, 7.4, 7.5	Finding equation of tangent from $x = \mathrm{f}(y)$	1.1b
4	C3 June 2014R	4	7.2, 7.4, 6.3, 2.6, 5.5	Differentiation	1.1b, 2.1, 3.1a
5	C3 2015	5	7.2, 7.3, 7.4	Differentiation, including chain rule, diff of $\sin 2x$ and tangent	1.1b
6	C3 2016	5	7.2, 7.3, 7.4, 7.5, 5.6	Differentiation, Trigonometry	1.1b, 2.1, 3.1a
7	C3 2017	7	5.5, 7.2, 7.4	Differentiation of product + $x = \mathrm{f}(y)$	1.1b, 2.1, 3.1a
8	C3 2011	7	2.6, 7.3, 7.4	Partial fractions, Differentiation	1.1b, 3.1a
9	C3 2015	7	7.4, 2.8	Differentiation using product rule, turning points leading to range	1.1b, 2.1, 2.4
10	C3 2015	9	2.6, 7.3, 7.4	Rational expression followed by quotient rule	1.1b, 2.1, 2.4
11	C3 2016	6	2.6, 7.3, 7.4	Algebraic division, equation of normal	1.1b, 3.1a