

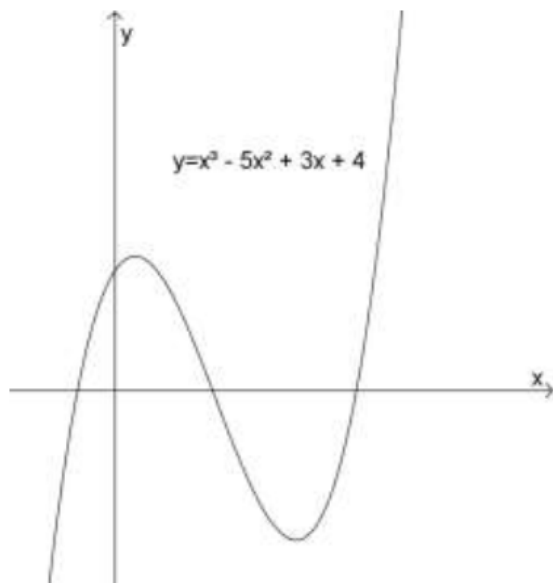
Further Differentiation – Section Test

1. A curve has equation $y = x^3 + 6x^2 + 9x$.
 - (i) Differentiate the function to obtain $\frac{dy}{dx}$.
 - (ii) Find the x co-ordinates of the points where $\frac{dy}{dx} = 0$ and hence the co-ordinates of the stationary points on the curve.
 - (iii) By considering the sign of $\frac{dy}{dx}$ on either side of the stationary points, determine whether the stationary points are maximum or minimum points.
 - (iv) Sketch the curve showing the stationary points and points of intersection with the axes clearly.
2. The equation of a curve is given by $y = 2x + x^2 - 4x^3$.
 - (i) Find the co-ordinates of the stationary points on the curve, and distinguish between them by considering the gradient on either side of the stationary points.
 - (ii) Sketch the curve marking the stationary points and points of intersection with the axes clearly.
3. A curve has equation $y = x^3 - 3x^2 + 6$. Find the co-ordinates of any stationary points and determine their nature showing clearly how your decisions were made. Sketch the curve.
4. The equation of a curve is $y = (x + 1)(x - 3)^3$.
 - (i) Write the equation of the curve in the form $y = ax^4 + bx^3 + cx^2 + dx + e$.
 - (ii) Find the co-ordinates of the points where $\frac{dy}{dx} = 0$.
 - (iii) Classify the stationary points.
 - (iv) Sketch the curve.
5. Find the stationary points on the curve $y = x^4 - 2x^3$ and distinguish between them, showing all of the relevant working clearly. Sketch the curve.
6. The curve $y = x^3 + px^2 + q$ has a minimum point at $(4, -11)$. Find the co-ordinates of the maximum point on the curve.
7. The curve $y = x^3 + ax^2 + bx + c$ passes through the point $(1, 1)$.
 - (i) Construct an equation connecting a , b and c .

The curve also has stationary points when $x = -1$ and when $x = 3$.

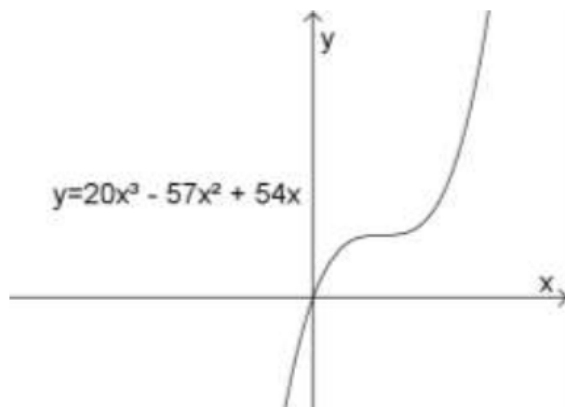
- (ii) Construct two further equations connecting a , b and c .
- (iii) Solve the three equations simultaneously to obtain values for a , b and c .

8. A sketch of the curve $y = x^3 - 5x^2 + 3x + 4$ is shown below.



Find the set of values of x for which the value of y is less than that at the minimum stationary point.

9. A sketch of the curve $y = 20x^3 - 57x^2 + 54x$ is shown below.



Decide whether the curve has one or two stationary points. You must show all your reasoning.

10. Work out $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$ for the following functions:

- (i) $y = 10 - 3x - x^2$
- (ii) $y = 3x(x^2 - 2x)$
- (iii) $y = (2x + 5)(x^2 - 3x)$

11. For the curve $y = -2x^3 + 6x^2 + x - 7$

- (i) Write down expressions for $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$
- (ii) Find the gradient of the curve at the points $(1, -2)$, $(3, -4)$ and $(-1, 0)$
- (iii) Work out the rate of change of the gradient at each of these points.