

Year 2 Pure Chapter 7: Double Angle - Exam Questions (Answers)

1.

$$5(1 - 2\sin^2 x) + 3 \sin x = 4$$

$$5 - 10\sin^2 x + 3 \sin x = 4$$

$$10 \sin^2 x + 3 \sin x = 4$$

$$10\sin^2 x - 3\sin x - 1 = 0$$

$$\sin x = 0.5$$

$$x = \pi/6$$

$$x = \pi - \pi/6 = 5/6\pi$$

and

$$\sin x = -1/5$$

$$x = -0.201$$

$$x = \pi - -0.201 = 3.34$$

$$x = -0.201 + 2\pi = 6.08$$

Final Solution: $\pi/6$, $5/6\pi$, 3.34, 6.08

$$2. \quad 3(2\cos^2\theta - 1) - \cos\theta + 1 = 0 \quad \text{B1}$$

$$6c^2 - c - 2 = 0$$

$$\cos \theta = \frac{2}{3} \text{ and } -\frac{1}{2} \quad \text{M1}$$

$$\cos \theta = \frac{2}{3}, \theta = 48^\circ \quad \text{A1}$$

$$360^\circ - 48^\circ = 312^\circ \quad \text{A1ft}$$

$$\cos \theta = -\frac{1}{2}, \theta = 120^\circ \quad \text{A1}$$

$$360^\circ - 120^\circ = 240^\circ \quad \text{A1ft}$$

3. (a) $\sec x = 2$

$$\cos x = \frac{1}{2} \quad \text{M1}$$

$$= \frac{\pi}{3}, \frac{5\pi}{3} \quad \text{A1}$$

(b) $\cos 2x = \cos(x + x)$

$$= \cos x \times \cos x - \sin x \times \sin x$$

$$= \cos^2 x - \sin^2 x \quad \text{M1}$$

$$= \cos^2 x - (1 - \cos^2 x)$$

$$= 2 \cos^2 x - 1 \quad \text{A1}$$

(c) $(2 \cos^2 x - 1) + 3 \cos x - 1 = 0 \quad \text{M1}$

$$2 \cos^2 x + 3 \cos x - 2 = 0$$

$$(2 \cos x - 1)(\cos x + 2) = 0 \quad \text{M1}$$

$$\cos x = -2$$

no solutions B1

$$\cos x = \frac{1}{2} \quad \text{M1}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3} \quad \text{A1 ft}$$

$$4. \quad (\mathbf{a}) \quad (\mathbf{i}) \quad \sin 2x = 2 \sin x \cos x \quad \text{B1}$$

$$(\mathbf{ii}) \quad \cos 2x = 2 \cos^2 x - 1 \quad \text{B1}$$

$$(\mathbf{b}) \quad \sin 2x - \tan x = 2 \sin x \cos x - \frac{\sin x}{\cos x} \quad \text{M1}$$

$$= \sin x \left(2 \cos x - \frac{1}{\cos x} \right) \quad \text{M1}$$

$$= \sin x \left(\frac{2 \cos^2 x - 1}{\cos x} \right) = \tan x \cos 2x \quad \text{A1}$$

$$(\mathbf{c}) \quad \tan x \cos 2x = 0 \quad x = 180 \quad \text{B1}$$

$$\cos 2x = 0 \quad \text{or} \quad \cos^2 x = \frac{1}{2} \left(\text{or} \sin^2 x = \frac{1}{2} \right) \quad \text{M1}$$

$$x = 45 \quad \text{A1}$$

$$x = 135, 225, 315 \quad \text{B1}$$

5.

$$6 \sin^2 \theta \cos \theta = 2 \cos^3 \theta$$

B1

$$\cos \theta = 0, \theta = \frac{\pi}{2}, \frac{3\pi}{2}$$

M1A1

$$\tan^2 \theta = \frac{1}{3} \quad \left(\text{or } \cos^2 \theta = \frac{3}{4}; \sin^2 \theta = \frac{1}{4} \right)$$

M1

$$\theta = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

A1 (any two)

A1 (all rest +
no others)