AS and A level Mathematics Practice Paper – Differentiation in context – Mark scheme

Question	Scheme	Marks
1(a)	$\frac{1}{2}(9x+6x)4x$ or $2x\times 15x$ or $\left(\frac{1}{2}4x\times(9x-6x)+6x\times 4x\right)$ or $6x^2+24x^2$ or $\left(9x\times 4x-\frac{1}{2}4x\times(9x-6x)\right)$ or $36x^2-6x^2$	M1A1**cso**
	$\Rightarrow 30x^2y=9600 \Rightarrow y=\frac{9600}{30x^2} \Rightarrow y=\frac{320}{x^2}$ *	**(2)**
1(b)	$(S=)\frac{1}{2}(9x+6x)4x+\frac{1}{2}(9x+6x)4x+6xy+9xy+5xy+4xy$	M1A1
	$y=\frac{320}{x^2} \Rightarrow (S=)30x^2+30x^2+24x\left(\frac{320}{x^2}\right)$	M1
	So, $(S=)60x^2+\frac{7680}{x}$ *	A1* **cso**
		(4)
1(c)	$\frac{dS}{dx}=120x-7680x^{-2}$ $\left\{=120x-\frac{7680}{x^2}\right\}$	M1
		A1 aef
	$120x-\frac{7680}{x^2}=0$ $\Rightarrow x^3=\frac{7680}{120};=64 \Rightarrow x=4$	M1A1**cso**
	$\{x=4,\}$ $S=60(4)^2+\frac{7680}{4}=2880$ (cm^2)	**dd**M1 A1 **cao and cso**
		(6)
1(d)	$\frac{d^2S}{dx^2}=120+\frac{15360}{x^3}>0$ $\Rightarrow$ Minimum	M1A1ft
	Note parts (c) and (d) can be marked together.	**(2)**
		(14 marks)

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2(a)	$\text{Area}(FEA) = \frac{1}{2}x^2\left(\frac{2\pi}{3}\right); = \frac{\pi x^2}{3}$	M1 A1
		(2)
2(b)	$\{A =\} \frac{1}{2}x^2 \sin 60^\circ + \frac{1}{3}\pi x^2 + 2xy$	M1 A1
	$1000 = \frac{\sqrt{3}x^2}{4} + \frac{\pi x^2}{3} + 2xy \Rightarrow y = \frac{500}{x} - \frac{\sqrt{3}x}{8} - \frac{\pi x}{6}$ $\Rightarrow \underline{y = \frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)} \quad *$	A1 *
		(3)
2(c)	$\{P =\} x + x\theta + y + 2x + y \left\{= 3x + \frac{2\pi x}{3} + 2y\right\}$	B1ft
	$\ldots 2y = \quad + 2\left(\frac{500}{x} - \frac{x}{24}\left(4\pi + 3\sqrt{3}\right)\right)$	M1
	$P = 3x + \frac{2\pi x}{3} + \frac{1000}{x} - \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x \Rightarrow P = \frac{1000}{x} + 3x + \frac{\pi x}{3} - \frac{\sqrt{3}}{4}x$	
	$\Rightarrow \underline{P = \frac{1000}{x} + \frac{x}{12}\left(4\pi + 36 - 3\sqrt{3}\right)} \quad *$	A1 *
		(3)
2(d)	$\frac{\mathrm{d}P}{\mathrm{d}x} = -1000x^{-2} + \frac{4\pi + 36 - 3\sqrt{3}}{12}; = 0$	M1 A1 M1
	$\Rightarrow x = \sqrt{\frac{1000(12)}{4\pi + 36 - 3\sqrt{3}}} \ (= 16.63392808\ldots)$	A1
	$\left\{P = \frac{1000}{(16.63\ldots)} + \frac{(16.63\ldots)}{12}\left(4\pi + 36 - 3\sqrt{3}\right)\right\} \Rightarrow P = 120.236..\ (\text{m})$	A1
		(5)
2(e)	$\frac{\mathrm{d}^2P}{\mathrm{d}x^2} = \frac{2000}{x^3} > 0 \Rightarrow$ Minimum	M1 A1ft
		(2)
		(15 marks)

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Question	Scheme		Marks
3(a)	$\{V =\}\ 2x^2y = 81$	$2x^2y = 81$	B1 oe
	$\{L = 2(2x + x + 2x + x) + 4y \Rightarrow L = 12x + 4y\}$		
	$y = \dfrac{81}{2x^2} \Rightarrow L = 12x + 4\left(\dfrac{81}{2x^2}\right)$	Making y the subject of their expression and substitute this into the correct L formula.	M1
	So, $L = 12x + \dfrac{162}{x^2}$ **AG**	Correct solution only.	A1 **cso**
			(3)
3(b)	$\dfrac{\mathrm{d}L}{\mathrm{d}x} = 12 - \dfrac{324}{x^3}\ \{= 12 - 324x^{-3}\}$	Either $12x \to 12$ or $\dfrac{162}{x^2} \to \dfrac{\pm\lambda}{x^3}$	M1
		Correct differentiation (need not be simplified).	A1 aef
	$\left\{\dfrac{\mathrm{d}L}{\mathrm{d}x} =\right\} 12 - \dfrac{324}{x^3} = 0 \Rightarrow x^3 = \dfrac{324}{12};=$	$L' = 0$ and "their $x^3 = \pm$ value" or "their $x^{-3} = \pm$ value"	M1;
		$x = \sqrt[3]{27}$ or $x = 3$	A1 **cso**
	$\{x = 3,\}\ \ L = 12(3) + \dfrac{162}{3^2} = 54$ (cm)	Substitute candidate's value of x $(\neq 0)$ into a formula for L.	ddM1
		54	A1 **cao**
			(6)
3(c)	$\{\text{For } x = 3\}$, $\dfrac{\mathrm{d}^2L}{\mathrm{d}x^2} = \dfrac{972}{x^4} > 0 \Rightarrow$ Mini	Correct ft L'' and considering sign.	M1
		$\dfrac{972}{x^4}$ and > 0 and conclusion.	A1
			(2)
			(11 marks)

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Question	Scheme		Marks
4	$\frac{dV}{dt} = 80\pi$, $V = 4\pi h(h+4) = 4\pi h^2 + 16\pi h$,		
	$\frac{dV}{dh} = 8\pi h + 16\pi$	$\pm\alpha h \pm \beta,\ \alpha \neq 0, \beta \neq 0$	M1
		$8\pi h + 16\pi$	A1
	$\left\{\frac{dV}{dh}\times\frac{dh}{dt} = \frac{dV}{dt} \Rightarrow\right\}$ $(8\pi h + 16\pi)\frac{dh}{dt} = 80\pi$	$\left(\text{Candidate's } \frac{dV}{dh}\right)\times\frac{dh}{dt} = 80\pi$	M1 oe
	$\left\{\frac{dh}{dt} = \frac{dV}{dt} \div \frac{dV}{dh} \Rightarrow\right\}$ $\frac{dh}{dt} = 80\pi \times$	or $80\pi \div$ Candidate's $\frac{dV}{dh}$	
	When $h = 6$, $\left\{\frac{dh}{dt} =\right\} \frac{1}{8\pi(6)+16\pi} \times 80\pi \left\{= \frac{80\pi}{64\pi}\right\}$	**dependent on the previous M1**	dM1
	$\frac{dh}{dt} = \underline{1.25}$ (cm s^{-1})	1.25 or $\frac{5}{4}$ or $\frac{10}{8}$ or $\frac{80}{64}$	A1 oe
			(5 marks)
5(a)	$P = \frac{800e^0}{1+3e^0}, = \frac{800}{1+3} = 200$		M1 A1
			(2)
5(b)	$250 = \frac{800e^{0.1t}}{1+3e^{0.1t}}$		
	$250(1+3e^{0.1t}) = 800e^{0.1t} \Rightarrow 50e^{0.1t} = 250,\ \Rightarrow e^{0.1t} = 5$		M1 A1
	$t = \frac{1}{0.1}\ln(5)$		M1
	$t = 10\ln(5)$		A1
			(4)
5(c)	$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} \Rightarrow \frac{dP}{dt} = \frac{(1+3e^{0.1t})\times 800\times 0.1e^{0.1t} - 800e^{0.1t}\times 3\times 0.1e^{0.1t}}{(1+3e^{0.1t})^2}$		M1 A1
	At t=10		
	$\frac{dP}{dt} = \frac{(1+3e)\times 80e - 240e^2}{(1+3e)^2} = \frac{80e}{(1+3e)^2}$		M1 A1
			(4)
5(d)	$P = \frac{800e^{0.1t}}{1+3e^{0.1t}} = \frac{800}{e^{-0.1t}+3} \Rightarrow P_{\max} = \frac{800}{3} = 266$. Hence P cannot be 270		B1
			(1)
			(11 marks)

Question	Scheme	Marks
6(a)	$P_0 = \frac{100}{1+3} + 40 = 65$	B1
		(1)
6(b)	$\frac{\mathrm{d}}{\mathrm{d}t}\mathrm{e}^{kt} = C\mathrm{e}^{kt}$	M1
	$\frac{\mathrm{d}P}{\mathrm{d}t} = \frac{\left(1+3\mathrm{e}^{-0.9t}\right)\times -10\mathrm{e}^{-0.1t} - 100\mathrm{e}^{-0.1t} \times -2.7\mathrm{e}^{-0.9t}}{\left(1+3\mathrm{e}^{-0.9t}\right)^2}$	M1 A1
		(3)
6(c)	At maximum $\quad -10\mathrm{e}^{-0.1t} - 30\mathrm{e}^{-0.1t} \times \mathrm{e}^{-0.9t} + 270\mathrm{e}^{-0.1t} \times \mathrm{e}^{-0.9t} = 0$	
	$\mathrm{e}^{-0.1t}\left(-10 + 240\mathrm{e}^{-0.9t}\right) = 0$	
	$\mathrm{e}^{-0.9t} = \frac{10}{240}$ oe $\mathrm{e}^{0.9t} = 24$	M1
	$-0.9t = \ln\left(\frac{1}{24}\right) \Rightarrow t = \frac{10}{9}\ln(24) = 3.53$	M1,A1
	Sub $t = 3.53 \Rightarrow P_T = 102$	A1
		(4)
6(d)	40	B1
		(1)
		(9 marks)
7	$\frac{\mathrm{d}I}{\mathrm{d}t} = -16\ln(0.5)0.5^t$	M1 A1
	At $t = 3$ $\quad \frac{\mathrm{d}I}{\mathrm{d}t} = -16\ln(0.5)0.5^3$	M1
	$= -2\ln 0.5 = \ln 4$	M1 A1
		(5 marks)

Question	Scheme	Marks
8(a)	$V = x^3 \Rightarrow \dfrac{\mathrm{d}V}{\mathrm{d}x} = 3x^2$ ✱ cso	B1
		(1)
8(b)	$\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{\mathrm{d}x}{\mathrm{d}V} \times \dfrac{\mathrm{d}V}{\mathrm{d}t} = \dfrac{0.048}{3x^2}$	M1
	At $x = 8$, $\dfrac{\mathrm{d}x}{\mathrm{d}t} = \dfrac{0.048}{3\left(8^2\right)} = 0.00025 \quad \left(\mathrm{cm\,s^{-1}}\right)$ 2.5×10^{-4}	A1
		(2)
8(c)	$S = 6x^2 \Rightarrow \dfrac{\mathrm{d}S}{\mathrm{d}x} = 12x$	B1
	$\dfrac{\mathrm{d}S}{\mathrm{d}t} = \dfrac{\mathrm{d}S}{\mathrm{d}x} \times \dfrac{\mathrm{d}x}{\mathrm{d}t} = 12x\left(\dfrac{0.048}{3x^2}\right)$	M1
	At $x = 8$ $\dfrac{\mathrm{d}S}{\mathrm{d}t} = 0.024 \quad \left(\mathrm{cm^2\,s^{-1}}\right)$	A1
		(3)
		(6 marks)
9(a)	$\dfrac{\mathrm{d}V}{\mathrm{d}h} = \dfrac{1}{2}\pi h - \pi h^2$ or equivalent	M1 A1
	At $h = 0.1$, $\dfrac{\mathrm{d}V}{\mathrm{d}h} = \dfrac{1}{2}\pi\left(0.1\right) - \pi\left(0.1\right)^2 = 0.04\pi$ $\dfrac{\pi}{25}$	M1 A1
		(4)
9(b)	$\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{\mathrm{d}V}{\mathrm{d}t} \div \dfrac{\mathrm{d}V}{\mathrm{d}h} = \dfrac{\pi}{800} \times \dfrac{1}{\frac{1}{2}\pi h - \pi h^2}$ or $\dfrac{\pi}{800} \div$ their (a)	M1
	At $h = 0.1$, $\dfrac{\mathrm{d}h}{\mathrm{d}t} = \dfrac{\pi}{800} \times \dfrac{25}{\pi} = \dfrac{1}{32}$ awrt 0.031	A1
		(2)
		(6 marks)

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	Source paper	Question number	New spec references	Question description	New AOs
1	C2 2014	10	5.7	Differentiation	1.1b, 2.1, 2.2a, 2.4, 3.1b, 3.4
2	C2 2016	9	5.1, 7.1, 7.2 and 7.3	Trigonometry, Differentiation	1.1b, 2.1, 2.2a, 2.4, 3.1b, 3.4
3	C2 2011	8	7.1, 7.2 and 7.3	Differentiation	1.1b, 2.1
4	C4 June 2014	4	7.4	Differentiation	1.1b, 3.1b
5	C3 June 2014	8	6.1, 6.2, 6.3, 6.7, 7.4	Exponential growth, Differentiation of exponential function	1.1b, 3.1a, 3.4, 3.5b
6	C3 2017	8	6.1, 6.2, 6.3, 6.5, 6.7, 7.4	Real life exponential problem, quotient rule	1.1b, 2.1, 3.1a
7	C4 Jan 2011	2	7.2, 6.4	Differentiation	1.1b, 2.1
8	C4 2012	2	7.4	Differentiation	1.1b, 3.1a
9	C4 2011	3	7.4	Differentiation	1.1b