Introduction to Differentiation – Section Test (Answers)

1.

$$y = 3x^2 - 4$$

$$\frac{dy}{dx} = 6x$$

When x = 2, $\frac{dy}{dx} = 12$

The gradient of the curve at the point (2, 8) is 12.

2.

$$y = x^3 - 2x^2 + x - 3$$

$$\frac{dy}{dx} = 3x^2 - 4x + 1$$

When x = -1, rate of change = $\frac{dy}{dx} = 3(-1)^2 - 4 \times -1 + 1 = 3 + 4 + 1 = 8$

3.

$$y = 2x^2 - 3x + 1$$

$$\frac{dy}{dx} = 4x - 3$$

When $\frac{dy}{dx} = 2$, $4x - 3 = 2$

$4x = 5$

$x = \frac{5}{4}$

4.

$$y = (2x - 1)(x^2 - 3)$$

$$= 2x^3 - x^2 - 6x + 3$$

$$\frac{dy}{dx} = 6x^2 - 2x - 6$$

The derivative is $6x^2 - 2x - 6$

5.

$$y = x^3 - x + 3$$

$$\frac{dy}{dx} = 3x^2 - 1$$

When $x = 1$, $\frac{dy}{dx} = 3 - 1 = 2$

At $(1, 3)$, $y - 3 = 2(x - 1)$

$$y - 3 = 2x - 2$$

$$y = 2x + 1$$

6.

From question 5, gradient of tangent at $(1, 3) = 2$

Gradient of normal at $(1, 3) = -\frac{1}{2}$

Equation of normal is $y - 3 = -\frac{1}{2}(x - 1)$

$$2(y - 3) = -(x - 1)$$

$$2y - 6 = -x + 1$$

$$2y + x = 7$$

7.

$$y = 2 - x^3$$

$$\frac{dy}{dx} = -3x^2$$

Gradient of normal $= 3 \Rightarrow$ gradient of tangent $= -\frac{1}{3}$

$$-3x^2 = -\frac{1}{3}$$

$$x^2 = \frac{1}{9}$$

$$x = \pm\frac{1}{3}$$

8.

$$y = x^3 - 2x^2$$

$$\frac{dy}{dx} = 3x^2 - 4x$$

$$3x^2 - 4x = -1$$

$$3x^2 - 4x + 1 = 0$$

$$(3x - 1)(x - 1) = 0$$

$$x = \tfrac{1}{3} \text{ or } 1$$

When $x = \frac{1}{3}$, $y = \frac{1}{27} - \frac{2}{9} = -\frac{5}{27}$

When $x = 1$, $y = 1 - 2 = -1$

The coordinates of the points with gradient -1 are $\left(\frac{1}{3}, -\frac{5}{27}\right)$ and (1, -1)

9.

$y = x^3 - 2x^2 + 4$

$\frac{dy}{dx} = 3x^2 - 4x$

When $x = 2$, $\frac{dy}{dx} = 3 \times 2^2 - 4 \times 2 = 12 - 8 = 4$

$y - 4 = 4(x - 2)$

$y - 4 = 4x - 8$

Equation of tangent is $y = 4x - 4$

When $y = 0$, $4x - 4 = 0 \quad \Rightarrow x = 1$, so X is (1, 0)

When $x = 0$, $y = -4$, so Y = (0, -4)

Area of triangle $= \frac{1}{2} \times 1 \times 4 = 2$

10.

$y = 2x^2 - x + 3$

$\frac{dy}{dx} = 4x - 1$

When $x = 1$, $\frac{dy}{dx} = 4 - 1 = 3$

Gradient of normal at P $= -\frac{1}{3}$

Equation of normal at P is $y - 4 = -\frac{1}{3}(x - 1)$

$3(y - 4) = -(x - 1)$

$3y - 12 = -x + 1$

$3y + x = 13$

To find where normal meets curve, substitute $y = 2x^2 - x + 3$ into equation of normal:

$3(2x^2 - x + 3) + x = 13$

$6x^2 - 3x + 9 + x = 13$

$6x^2 - 2x - 4 = 0$

$3x^2 - x - 2 = 0$

$(x - 1)(3x + 2) = 0$

$x = 1$ or $x = -\frac{2}{3}$

$x = 1$ is the point P, so the x-coordinate of Q is $-\frac{2}{3}$