Year 2 Pure Chapter 10: Numerical Methods Exam Questions (Answers)

1. (a)

$x_1 = 2.32$ or 2.320 M1

$x_1 = 2.32$

$x_2 =$ awrt 2.372 A1

$x_3 =$ awrt 2.356

$x_4 =$ awrt 2.360 or 2.36 A1 cso

(b) Let $f(x) = -x^3 + 2x^2 + 2 = 0$

Choose suitable interval for x, M1

$f(2.3585) = 0.00583577$...e.g. $[2.3585, 2.3595]$ or tighter

$f(2.3595) = -0.00142286$...any one value awrt 1 sf

Sign change and $f(x)$ is continuous dM1

α is such that $\alpha \in (2.3585, 2.3595)$

$\alpha = 2.359$ (3 dp)

both A1

values correct, sign change

and conclusion change of sign, hence root".

2. (a) $f(2) = 0.38 \ldots$

$f(3) = -0.39 \ldots$ M1

Change of sign (and continuity) $\Rightarrow$ root in $(2, 3)$ cso A1

(b) $x_1 = \ln 4.5 + 1 \approx 2.50408$ M1

$x_2 \approx 2.50498$ A1

$x_3 \approx 2.50518$ A1

(c) Selecting $[2.5045, 2.5055]$, or appropriate tighter range,

and

evaluating at both ends.

$f(2.5045) \approx 6 \times 10^{-4}$

$f(2.5055) \approx -2 \times 10^{-4}$

Change of sign

root $\in (2.5045, 2.5055)$

root $= 2.505$ to 3 dp A1

3. (a) $x^2(3 - x) - 1 = 0$ M1

$x = \sqrt{\dfrac{1}{3 - x}}$ A1cso

(b) $x_2 = 0.6455$,

$x_3 = 0.6517$,

$x_4 = 0.6526$ B1; B1

(c) Choose values in interval (0.6525, 0.6535) or tighter M1

f(0.6525) = –0.0005 (372... f(0.6535) = 0.002 (101...

At least one correct "up to bracket", i.e. –0.0005 or 0.002A1

Change of sign, x = 0.653 is a root (correct) to 3 d.p. A1

Requires both correct "up to bracket" and conclusion as above

Alt (i) Continued iterations at least as far as x_6

M1

$x_5 = 0.65268$,

$x_6 = 0.6527$,

$x_7 = \ldots$ two correct to al least 4 s.f. A1

Conclusion: Two values correct to 4 d.p.,

so 0.653 is root to 3 d.p. A1

4.	(a)	f(1) = –2, f(2) = 5 ½	M1
		Change of sign (and continuity) $\Rightarrow$ root $\in (1, 2)$	A1
	(b)	$x_1 = 1.38672\ldots$, $x_2 = 1.39609$ awrt 4dp	B1, B1
		$x_3 = 1.39527\ldots$, $x_4 = 1.39534\ldots$ same to 3dp	M1
		Root is 1.395 (to 3dp) cao	A1
	(c)	Choosing a suitable interval, (1.3945, 1.3955) or tighter.	M1
		f(1.3945) ≈ – 0.005, f(1.3955) ≈ + 0.001	
		Change of sign (and continuity) $\Rightarrow$ root $\in$ (1.3945, 1.3955)	
		root is 1.395 correct to 3dp	A1

5. **(a)** $f(x) = x^3 + 2x^2 - 3x - 11$

$x^3 + 2x^2 - 3x - 11 = 0$

$x^2(x + 2) - 3x - 11 = 0$ M1

$x^2(x + 2) - 3x - 11$

$$x^2 = \frac{3x + 11}{x + 2}$$

$$x = \sqrt{\left(\frac{3x + 11}{x + 2}\right)}$$ A1

(b) Iterative formula:

$$x_{n+1} = \sqrt{\left(\frac{3x_n + 11}{x_n + 2}\right)},\ x_1 = 0$$

$$x_2 = \sqrt{\left(\frac{3(0) + 11}{(0) + 2}\right)}$$

$x_2 = 2.34520788...$	$x_2 =$ awrt 2.345	A1
$x_3 = 2.037324945...$	$x_3 =$ awrt 2.037	
$x_4 = 2.058748112...$	$x_4 =$ awrt 2.059	A1

(c) Let $f(x) = x^3 + 2x^2 - 3x - 11 = 0$

$f(2.0565) = -0.013781637...$

$f(2.0575) = 0.0041401094...$

Sign change and $f(x)$ is continuous

a root α is such that $\alpha \in (2.0565, 2.0575)$

$\alpha = 2.057$ (3 dp)

Choose suitable interval for x, e.g. [2.0565, 2.0575] or tighter M1

any one value awrt 1 sf dM1

both values correct awrt 1sf, sign change and conclusion A1

states "change of sign, hence root"

6. **(a)** $x(2x^2 - 1) = 4$ M1

$2x^2 - 1 = \frac{4}{x}$

$2x^2 = \frac{4+x}{x}$ M1

$x^2 = \frac{4+x}{2x}$

$x = \sqrt{\frac{2}{x} + \frac{1}{2}}$ A1

(b) 1.41, 1.39, 1.39 B1,B1,B1

(1.40765, 1.38593, 1.393941)

(c) $f(1.3915) = -3 \times 10^{-3}$ M1

$f(1.3925) = 7 \times 10^{-3}$ A1

change in sign means root between
1.3915 & 1.3925

B1

7. **(a)** f(1.4) = –0.568 … < 0

f(1.45) = 0.245 … > 0 M1

Change of sign (and continuity) α (1.4, 1.45) A1

(b) $3x^3 = 2x + 6$

$$x^3 = \frac{2x}{3} + 2$$

$$x^2 = \frac{2}{3} + \frac{2}{x}$$

M1A1

$$x = \sqrt{\left(\frac{2}{x} + \frac{2}{3}\right)}$$ cso A1

(c) $x_1 = 1.4371$ B1

$x_2 = 1.4347$ B1

$x_3 = 1.4355$ B1

(d) Choosing the interval (1.4345,1.4355) or tighter interval. M1

f (1.4345) = –0.01 ...

f(1.4355) = 0.003 … M1

Change of sign (and continuity) $\Rightarrow \alpha \in$ (1.4345,1.4355)

α = 1.435, correct to 3 decimal places A1