

Year 2 Pure Chapter 9: Differentiation – Parametric Equations
Exam Questions (Answers)

Q1.

Question Number	Scheme	Marks
(a)	$x = 4 \sin \left(t + \frac{\pi}{6} \right), \quad y = 3 \cos 2t, \quad 0 \leq t < 2\pi$ $\frac{dx}{dt} = 4 \cos \left(t + \frac{\pi}{6} \right), \quad \frac{dy}{dt} = -6 \sin 2t$ So, $\frac{dy}{dx} = \frac{-6 \sin 2t}{4 \cos \left(t + \frac{\pi}{6} \right)}$	B1 B1 B1 $\sqrt{\quad}$ oe [3]
(b)	$\left\{ \frac{dy}{dx} = 0 \Rightarrow \right\} -6 \sin 2t = 0$ @ $t = 0$, $x = 4 \sin \left(\frac{\pi}{6} \right) = 2$, $y = 3 \cos 0 = 3 \rightarrow (2, 3)$ @ $t = \frac{\pi}{2}$, $x = 4 \sin \left(\frac{2\pi}{3} \right) = \frac{4\sqrt{3}}{2}$, $y = 3 \cos \pi = -3 \rightarrow (2\sqrt{3}, -3)$ @ $t = \pi$, $x = 4 \sin \left(\frac{7\pi}{6} \right) = -2$, $y = 3 \cos 2\pi = 3 \rightarrow (-2, 3)$ @ $t = \frac{3\pi}{2}$, $x = 4 \sin \left(\frac{5\pi}{3} \right) = \frac{4(-\sqrt{3})}{2}$, $y = 3 \cos 3\pi = -3 \rightarrow (-2\sqrt{3}, -3)$	M1 oe M1 A1A1A1 [5]
		8

Q2.

Question Number	Scheme	Notes	Marks
	$x = 3t - 4, \quad y = 5 - \frac{6}{t}, \quad t > 0$		
(a)	$\frac{dx}{dt} = 3, \quad \frac{dy}{dt} = 6t^{-2}$		
	$\frac{dy}{dx} = \frac{6t^{-2}}{3} \left\{ = \frac{6}{3t^2} = 2t^{-2} = \frac{2}{t^2} \right\}$	their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ to give $\frac{dy}{dx}$ in terms of t or their $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$ to give $\frac{dy}{dx}$ in terms of t	M1
		$\frac{6t^{-2}}{3}$, simplified or un-simplified, in terms of t . See note.	A1 isw
	Award Special Case 1 st M1 if both $\frac{dx}{dt}$ and $\frac{dy}{dt}$ are stated correctly and explicitly.		[2]
	Note: You can recover the work for part (a) in part (b).		
(a) Way 2	$y = 5 - \frac{18}{x+4} \Rightarrow \frac{dy}{dx} = \frac{18}{(x+4)^2} = \frac{18}{(3t)^2}$	Writes $\frac{dy}{dx}$ in the form $\frac{\pm \lambda}{(x+4)^2}$, and writes $\frac{dy}{dx}$ as a function of t .	M1
		Correct un-simplified or simplified answer, in terms of t . See note.	A1 isw

(b)	$\left\{ t = \frac{1}{2} \Rightarrow \right\} P\left(-\frac{5}{2}, -7\right)$	$x = -\frac{5}{2}, y = -7$ or $P\left(-\frac{5}{2}, -7\right)$ seen or implied.	B1
	$\frac{dy}{dx} = \frac{2}{\left(\frac{1}{2}\right)^2}$ and either • $y - "-7" = "8"(x - "-\frac{5}{2}")$ • $"-7" = ("8")("-\frac{5}{2}") + c$ So, $y = (\text{their } m_t)x + "c"$	Some attempt to substitute $t = 0.5$ into their $\frac{dy}{dx}$ which contains t in order to find m_t and either applies $y - (\text{their } y_p) = (\text{their } m_t)(x - \text{their } x_p)$ or finds c from $(\text{their } y_p) = (\text{their } m_t)(\text{their } x_p) + c$ and uses their numerical c in $y = (\text{their } m_t)x + c$	M1
	T: $y = 8x + 13$	$y = 8x + 13$ or $y = 13 + 8x$	A1 cso
	Note: their x_p , their y_p and their m_t must be numerical values in order to award M1		[3]

Q3.

Question Number	Scheme	Notes	Marks
	$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$		
(a) Way 1	$\frac{dx}{dt} = 4 \sec^2 t, \quad \frac{dy}{dt} = 10\sqrt{3} \cos 2t$ $\Rightarrow \frac{dy}{dx} = \frac{10\sqrt{3} \cos 2t}{4 \sec^2 t} \left\{ = \frac{5}{2} \sqrt{3} \cos 2t \cos^2 t \right\}$	Either both x and y are differentiated correctly with respect to t or their $\frac{dy}{dt}$ divided by their $\frac{dx}{dt}$ or applies $\frac{dy}{dt}$ multiplied by their $\frac{dt}{dx}$	M1
		Correct $\frac{dy}{dx}$ (Can be implied)	A1 oe
	$\left\{ \text{At } P\left(4\sqrt{3}, \frac{15}{2}\right), t = \frac{\pi}{3} \right\}$		
	$\frac{dy}{dx} = \frac{10\sqrt{3} \cos\left(\frac{2\pi}{3}\right)}{4 \sec^2\left(\frac{\pi}{3}\right)}$	dependent on the previous M mark Some evidence of substituting $t = \frac{\pi}{3}$ or $t = 60^\circ$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ from a correct solution only	A1 cso
[4]			

Question Number	Scheme	Notes	Marks
	$x = 4 \tan t, \quad y = 5\sqrt{3} \sin 2t, \quad 0 \leq t < \frac{\pi}{2}$		
(a) Way 2	$\tan t = \frac{x}{4} \Rightarrow \sin t = \frac{x}{\sqrt{x^2+16}}, \quad \cos t = \frac{4}{\sqrt{x^2+16}} \Rightarrow y = \frac{40\sqrt{3}x}{x^2+16}$		
	$\left\{ \begin{array}{ll} u = 40\sqrt{3}x & v = x^2 + 16 \\ \frac{du}{dx} = 40\sqrt{3} & \frac{dv}{dx} = 2x \end{array} \right\}$		
	$\frac{dy}{dx} = \frac{40\sqrt{3}(x^2+16) - 2x(40\sqrt{3}x)}{(x^2+16)^2} \left\{ = \frac{40\sqrt{3}(16-x^2)}{(x^2+16)^2} \right\}$	$\frac{\pm A(x^2+16) \pm Bx^2}{(x^2+16)^2}$	M1
		Correct $\frac{dy}{dx}$; simplified or un-simplified	A1
	$\frac{dy}{dx} = \frac{40\sqrt{3}(48+16) - 80\sqrt{3}(48)}{(48+16)^2}$	dependent on the previous M mark Some evidence of substituting $x = 4\sqrt{3}$ into their $\frac{dy}{dx}$	dM1
	$\frac{dy}{dx} = -\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$	$-\frac{5}{16}\sqrt{3}$ or $-\frac{15}{16\sqrt{3}}$ from a correct solution only	A1 cso
[4]			

(b)	$\left\{10\sqrt{3}\cos 2t = 0 \Rightarrow t = \frac{\pi}{4}\right\}$		
	So $x = 4\tan\left(\frac{\pi}{4}\right), y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$	At least one of either $x = 4\tan\left(\frac{\pi}{4}\right)$ or $y = 5\sqrt{3}\sin\left(2\left(\frac{\pi}{4}\right)\right)$ or $x = 4$ or $y = 5\sqrt{3}$ or $y = \text{awrt } 8.7$	M1
	Coordinates are $(4, 5\sqrt{3})$	$(4, 5\sqrt{3})$ or $x = 4, y = 5\sqrt{3}$	A1
			[2]
			6

Q4.

Question	Scheme	Marks	AOs
(a)	Attempts $\frac{dy}{dx} = \frac{dy/dt}{dx/dt}$	M1	1.1b
	$\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t} \quad (= 2\sqrt{3}\cos t)$	A1	1.1b
		(2)	
(b)	Substitutes $t = \frac{2\pi}{3}$ in $\frac{dy}{dx} = \frac{\sqrt{3}\sin 2t}{\sin t} = (-\sqrt{3})$	M1	2.1
	Uses gradient of normal $= -\frac{1}{dy/dx} = \left(\frac{1}{\sqrt{3}}\right)$	M1	2.1
	Coordinates of $P = \left(-1, -\frac{\sqrt{3}}{2}\right)$	B1	1.1b
	Correct form of normal $y + \frac{\sqrt{3}}{2} = \frac{1}{\sqrt{3}}(x+1)$	M1	2.1
	Completes proof $\Rightarrow 2x - 2\sqrt{3}y - 1 = 0$ *	A1*	1.1b
		(5)	
(c)	Substitutes $x = 2\cos t$ and $y = \sqrt{3}\cos 2t$ into $2x - 2\sqrt{3}y - 1 = 0$	M1	3.1a
	Uses the identity $\cos 2t = 2\cos^2 t - 1$ to produce a quadratic in $\cos t$	M1	3.1a
	$\Rightarrow 12\cos^2 t - 4\cos t - 5 = 0$	A1	1.1b
	Finds $\cos t = \frac{5}{6}, \frac{7}{8}$	M1	2.4
	Substitutes their $\cos t = \frac{5}{6}$ into $x = 2\cos t, y = \sqrt{3}\cos 2t,$	M1	1.1b
	$Q = \left(\frac{5}{3}, \frac{7}{18}\sqrt{3}\right)$	A1	1.1b
		(6)	
(13 marks)			

Q5.

Question Number	Scheme	Marks
	(a) $\frac{dx}{dt} = 2 \sin t \cos t, \frac{dy}{dt} = 2 \sec^2 t$ $\frac{dy}{dx} = \frac{\sec^2 t}{\sin t \cos t} \left(= \frac{1}{\sin t \cos^3 t} \right)$ or equivalent (b) At $t = \frac{\pi}{3}, x = \frac{3}{4}, y = 2\sqrt{3}$ $\frac{dy}{dx} = \frac{\sec^2 \frac{\pi}{3}}{\sin \frac{\pi}{3} \cos \frac{\pi}{3}} = \frac{16}{\sqrt{3}}$ $y - 2\sqrt{3} = \frac{16}{\sqrt{3}} \left(x - \frac{3}{4} \right)$ $y = 0 \Rightarrow x = \frac{3}{8}$	B1 B1 M1 A1 (4) B1 M1 A1 M1 M1 A1 (6) [10]