

Name: _____

Level 2 Further Maths



Trigonometric Equations Corbettmaths

Ensure you have: Pencil or pen

Guidance

1. Read each question carefully before you begin answering it.
2. Check your answers seem right.
3. Always show your workings

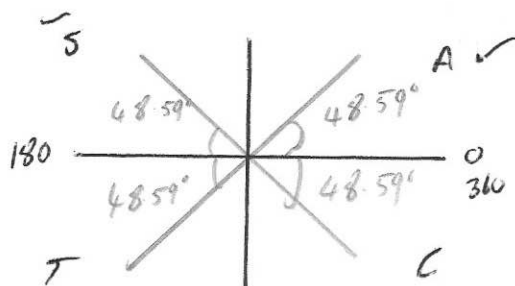
Revision for this topic

www.corbettmaths.com/more/further-maths/



1. Solve $\sin \theta = 0.75$ for $0^\circ \leq \theta \leq 180^\circ$

$$\sin^{-1} 0.75 = 48.59^\circ$$

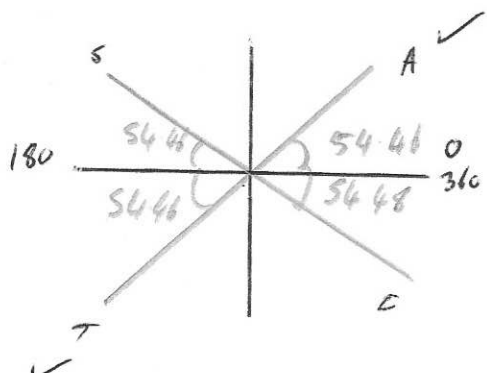


$$48.59^\circ, 131.41^\circ$$

(2)

2. Solve $\tan \theta = 1.4$ for $0^\circ \leq \theta \leq 360^\circ$

$$\tan^{-1} 1.4 = 54.46^\circ$$

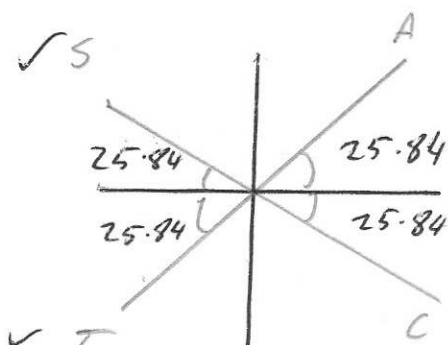


$$54.46^\circ, 234.46^\circ$$

(2)

3. Solve $\cos \theta = -0.9$ for $0^\circ \leq \theta \leq 360^\circ$

$$\cos^{-1} 0.9 = 25.84^\circ$$



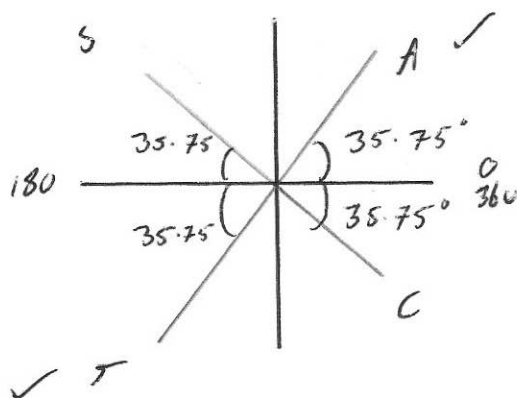
$$154.16^\circ, 205.84^\circ$$

(2)

4. Solve $2\tan\theta = 1.44$ for $0^\circ \leq \theta \leq 360^\circ$

$$\tan\theta = 0.72$$

$$\theta = 35.75^\circ$$



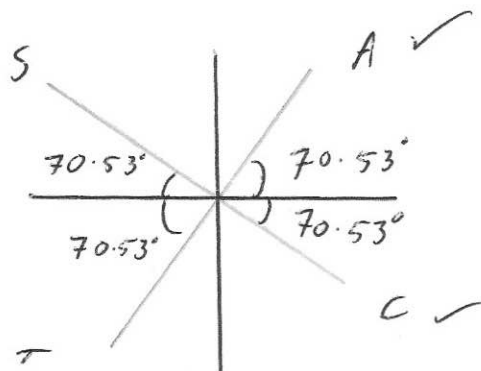
$$35.75^\circ, 215.75^\circ$$

(3)

5. Solve $3\cos\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$

$$\cos\theta = \frac{1}{3}$$

$$\theta = 70.53^\circ$$



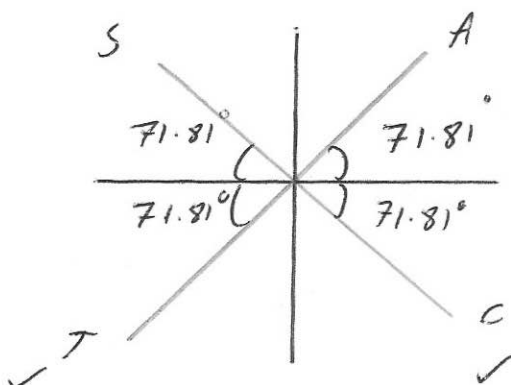
$$70.53^\circ, 289.47^\circ$$

(3)

6. Solve $2 + \sin\theta = 1.05$ for $0^\circ \leq \theta \leq 360^\circ$

$$\sin\theta = -0.95$$

$$\theta = 71.81^\circ$$



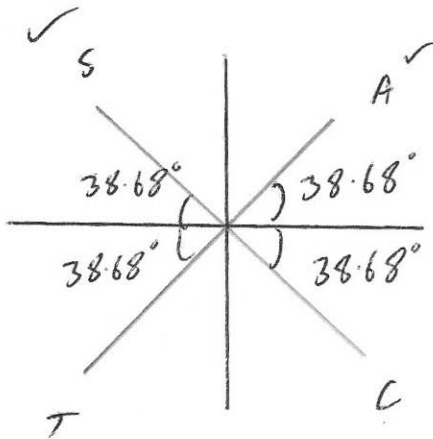
$$251.81^\circ, 288.19^\circ$$

(3)

7. Solve $8\sin\theta - 5 = 0$ for $0^\circ \leq \theta \leq 360^\circ$

$$\sin\theta = \frac{5}{8}$$

$$\theta = 38.68^\circ$$

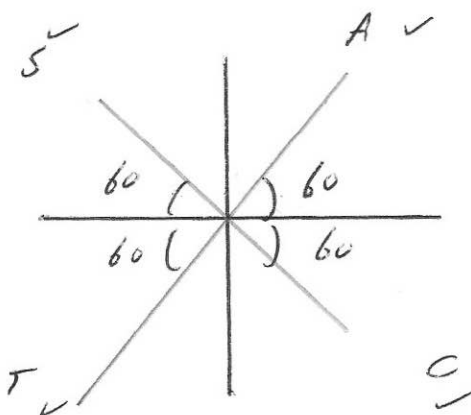


$$38.68^\circ, 141.32^\circ$$

(3)

8. Solve $\cos^2 x = \frac{1}{4}$ for $0^\circ \leq x \leq 360^\circ$

$$\cos x = \pm \frac{1}{2}$$



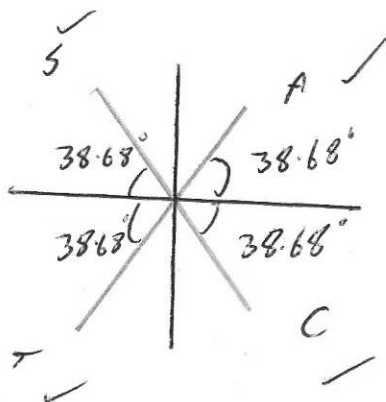
$$60^\circ, 120^\circ, 240^\circ, 300^\circ$$

(3)

9. Solve $64\sin^2 x = 25$ for $0^\circ \leq x \leq 360^\circ$

$$\sin x = \pm \frac{5}{8}$$

$$x = 38.68^\circ$$



$$38.68, 141.32, 218.68, 321.32$$

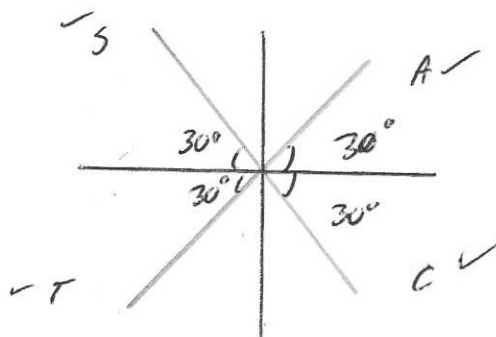
(4)

10. Solve $3\tan^2 \theta - 1 = 0$ for $0^\circ \leq \theta \leq 180^\circ$

$$\tan^2 \theta = \frac{1}{3}$$

$$\tan \theta = \pm 0.577$$

$$\theta = 30^\circ$$



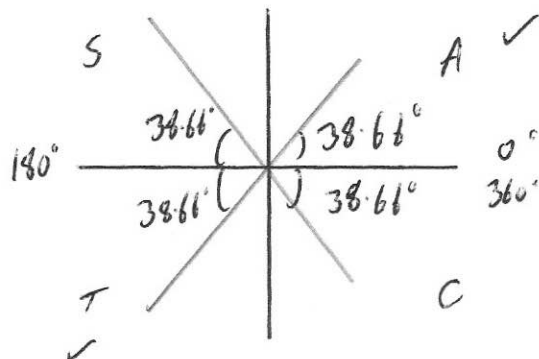
$$30^\circ, 150^\circ$$

(4)

11. Solve $5\sin x = 4\cos x$ for $0^\circ \leq x \leq 360^\circ$

$$\tan x = 0.8$$

$$x = 38.66^\circ$$



$$38.66^\circ, 218.66^\circ$$

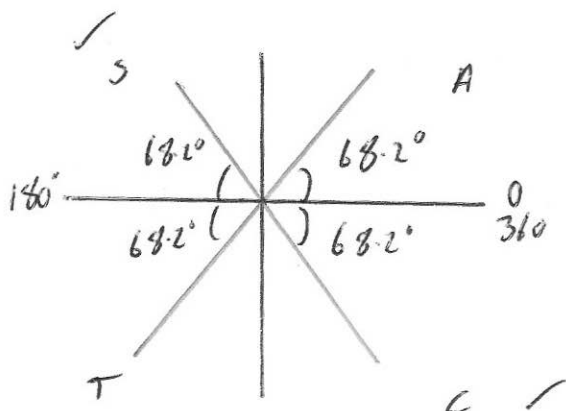
(4)

12. Solve $10\cos\theta + 4\sin\theta = 0$ for $90^\circ \leq \theta \leq 360^\circ$

$$10\cos\theta = -4\sin\theta$$

$$10 = -4\tan\theta$$

$$\tan\theta = -2.5$$

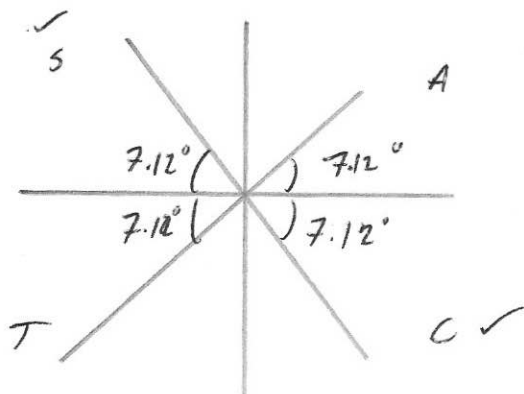


$$111.8^\circ, 291.8^\circ$$

(4)

13. Solve $-\cos x = 8 \sin x$ for $0^\circ \leq x \leq 360^\circ$

$$-\frac{1}{8} = \tan x$$



$$172.88^\circ, 352.88^\circ$$

(4)

14. Solve $\tan^2 x - 2 \tan x = 0$ for $0^\circ \leq x \leq 360^\circ$

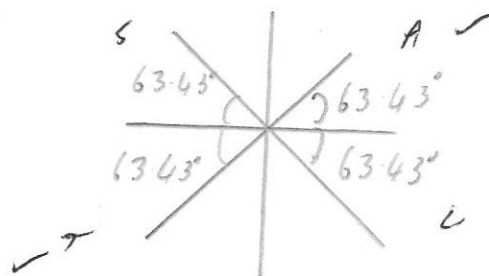
$$\tan x (\tan x - 2) = 0$$

$$\tan x = 0$$

$$x = 0^\circ, 180^\circ, 360^\circ$$

$$\tan x = 2$$

$$x = 63.43^\circ$$



$$0^\circ, 63.4^\circ, 180^\circ, 243.4^\circ, 360^\circ$$

(5)

15. Solve $4\cos^2\theta - 3\cos\theta = 0$ for $0^\circ \leq \theta \leq 360^\circ$

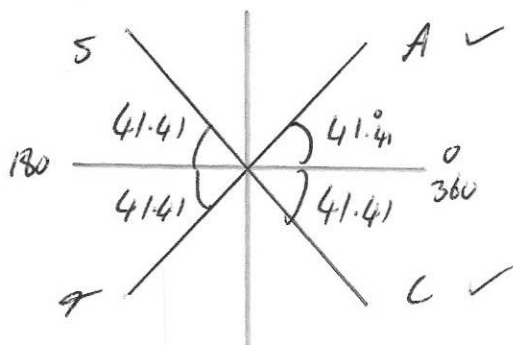
$$\cos\theta (4\cos\theta - 3) = 0$$

$$\cos\theta = 0$$

$$\theta = 90^\circ, 270^\circ$$

$$\cos\theta = \frac{3}{4}$$

$$\theta = 41.41^\circ$$



$$\underline{41.41^\circ, 90^\circ, 270^\circ, 318.59^\circ}$$

(4)

16. Solve $2\cos^2\theta - \cos\theta - 1 = 0$ for $0^\circ \leq \theta \leq 360^\circ$

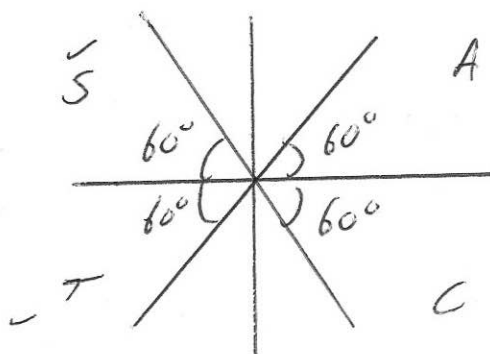
$$(\cos\theta - 1)(2\cos\theta + 1) = 0$$

$$\cos\theta = 1$$

$$\theta = 0^\circ, 360^\circ$$

$$\cos\theta = -\frac{1}{2}$$

$$\theta = 120^\circ$$



$$\underline{0^\circ, 120^\circ, 240^\circ, 360^\circ}$$

(4)

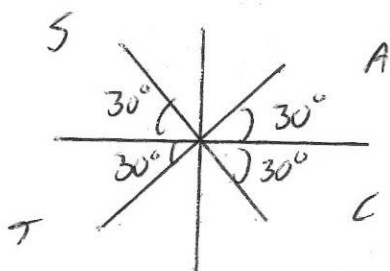
17. Solve $4\sin^2\theta + 4\sin\theta - 3 = 0$ for $0^\circ \leq \theta \leq 360^\circ$

$$x = \sin\theta$$

$$(2x - 1)(2x + 3) = 0$$

$$\sin\theta = \frac{1}{2} \quad \text{or} \quad \sin\theta = -\frac{3}{2}$$

X



30°, 150°

(4)

18. Solve $5\tan^2\theta - 6\tan\theta = -1$ for $0^\circ \leq \theta \leq 360^\circ$

$$5\tan^2\theta - 6\tan\theta + 1 = 0$$

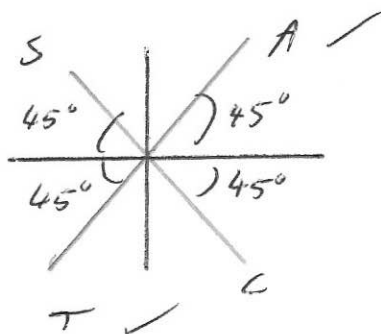
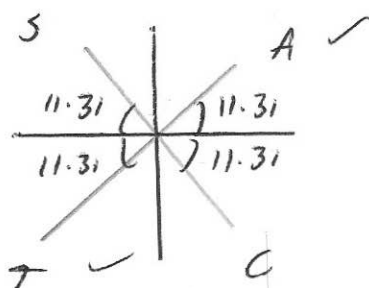
$$(5\tan\theta - 1)(\tan\theta - 1) = 0$$

$$\tan\theta = \frac{1}{5}$$

$$\theta = 11.31^\circ$$

$$\tan\theta = 1$$

$$45^\circ$$



11.31°, 45°, 191.31°, 225°

(5)

19. (a) Prove $\cos^2 x - 2\sin^2 x \equiv 3\cos^2 x - 2$

$$\cos^2 x - 2(1 - \cos^2 x)$$

$$\cos^2 x - 2 + 2\cos^2 x$$

$$3\cos^2 x - 2$$

QED

(2)

- (b) Hence, work out the values of x between 0° and 360° for which

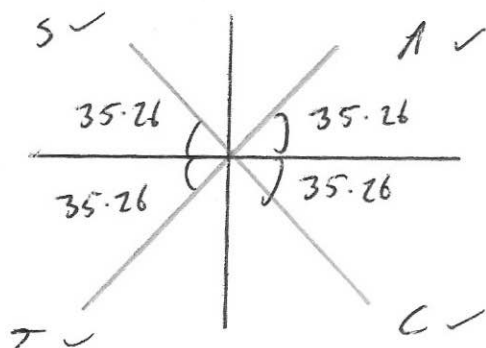
$$\cos^2 x - 2\sin^2 x = 0$$

$$3\cos^2 x - 2 = 0$$

$$\cos^2 x = \frac{2}{3}$$

$$\cos x = \pm \sqrt{\frac{2}{3}}$$

$$x = 35.26$$



$$35.26, 144.74, 215.26, 324.74^\circ$$

.....

(4)

20. (a) Show that $2\cos^2\theta \equiv 2 - 2\sin^2\theta$

$$2(1 - \sin^2\theta)$$

$$= 2 - 2\sin^2\theta$$

QED

(2)

- (b) Hence solve $2\cos^2\theta - \sin\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$

$$2 - 2\sin^2\theta - \sin\theta = 1$$

$$0 = 2\sin^2\theta + \sin\theta - 1$$

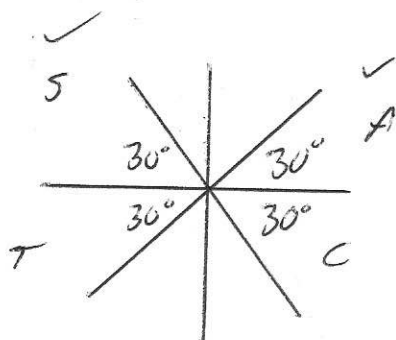
$$(2\sin\theta - 1)(\sin\theta + 1) = 0$$

$$\sin\theta = \frac{1}{2}$$

$$\theta = 30^\circ$$

$$\sin\theta = -1$$

$$\theta = 270^\circ$$



30°, 150°, 270°

(4)

21. Show that $3\cos^2\theta \equiv 3 - 3\sin^2\theta$

$$3(1 - \sin^2\theta)$$

$$3 - 3\sin^2\theta$$

QED

(2)

(b) Hence solve $3\cos^2\theta - 5\sin\theta = 1$ for $0^\circ \leq \theta \leq 360^\circ$

$$3 - 3\sin^2\theta - 5\sin\theta = 1$$

$$0 = 3\sin^2\theta + 5\sin\theta - 2$$

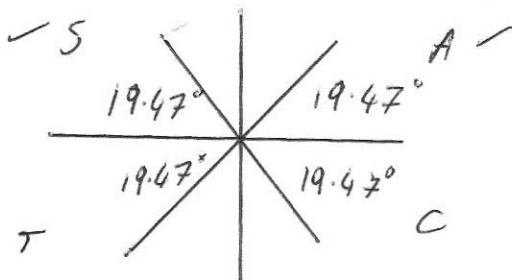
$$(3\sin\theta - 1)(\sin\theta + 2) = 0$$

$$\sin\theta = \frac{1}{3}$$

$$\theta = 19.47^\circ$$

$$\sin\theta = -2$$

x



$$19.47^\circ, 180.53^\circ$$

(4)

22. Solve $6\sin^2\theta + 7\cos\theta - 8 = 0$ for $0^\circ \leq \theta \leq 360^\circ$

$$6(1 - \cos^2\theta) + 7\cos\theta - 8 = 0$$

$$6 - 6\cos^2\theta + 7\cos\theta - 8 = 0$$

$$-6\cos^2\theta + 7\cos\theta - 2 = 0$$

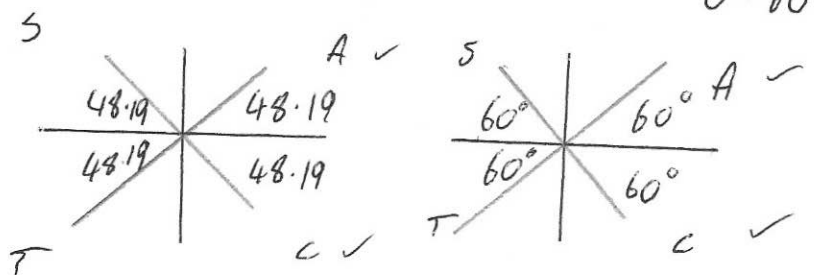
$$0 = 6\cos^2\theta - 7\cos\theta + 2$$

$$(3\cos\theta - 2)(2\cos\theta - 1) = 0$$

$$\cos\theta = \frac{2}{3}$$

$$\cos\theta = \frac{1}{2}$$

$$\theta = 60^\circ$$



$$\underline{48.19^\circ, 60^\circ, 300^\circ, 311.81^\circ}$$

(7)