

A level Pure Maths: Practice Paper A mark scheme

1	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to substitute $k = 1, k = 2$ and $k = 4$ into $a_k = 2^k + 1, k \geq 1$	M1	1.1b	5th Understand disproof by counter example.
	Shows that $a_1 = 3, a_2 = 5$ and $a_4 = 17$ and these are prime numbers.	A1	1.1b	
			(2)	
(b)	Substitutes a value of k that does not yield a prime number. For example, $a_3 = 9$ or $a_5 = 33$	A1	1.1b	5th Understand disproof by counter example.
	Concludes that their number is not prime. For example, states that $9 = 3 \times 3$, so 9 is not prime.	B1	2.4	
			(2)	
(4 marks)				
Notes				

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2	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Finds $ a = \sqrt{(4)^2 + (-1)^2 + (3)^2} = \sqrt{26}$	M1	1.1b	5th Find the magnitude of a vector in 3 dimensions.
	States $\cos \theta_y = -\frac{1}{\sqrt{26}}$	M1	1.1b	
	Solves to find $\theta_y = 101.309...\circ$. Accept awrt 101.3°	A1	1.1b	
		(3)		
(3 marks)				
Notes				

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3	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Deduces from $3\sin\left(\frac{x}{6}\right)^3 - \frac{1}{10}x - 1 = 0$ that $3\sin\left(\frac{x}{6}\right)^3 = \frac{1}{10}x + 1$	M1	1.1b	5th Understand the concept of roots of equations.
	States $\left(\frac{x}{6}\right)^3 = \arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)$	M1	1.1b	
	Multiplies by 6^3 and then takes the cube root: $x = 6\left(\sqrt[3]{\arcsin\left(\frac{1}{3} + \frac{1}{30}x\right)}\right)$	A1	1.1b	
		(3)		
(b)	Attempts to use iterative procedure to find subsequent values.	M1	1.1b	6th Solve equations approximately using the method of iteration.
	Correctly finds: $x_1 = 4.716$ $x_2 = 4.802$ $x_3 = 4.812$ $x_4 = 4.814$	A1	1.1b	
		(2)		
(5 marks)				
Notes (b) Award M1 if finds at least one correct answer.				

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4	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Recognises that two subsequent values will divide to give an equal ratio and sets up an appropriate equation. $\frac{2k^2}{4k} = \frac{4k}{k+2}$	M1	2.2a	4th Understand simple geometric sequences.
	Makes an attempt to solve the equation. For example, $2k^3 + 4k^2 = 16k^2$ or $2k^3 - 12k^2 = 0$	M1	1.1b	
	Factorises to get $2k^2(k-6)=0$	M1	1.1b	
	States the correct solution: $k = 6$. $k \neq 0$ or $k = 0$ is trivial may also be seen, but is not required.	A1	1.1b	
(4 marks)				
Notes				

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5	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Makes an attempt to set up a long division. For example: $x^2 - 2x - 15 \overline{)x^4 + 2x^3 - 29x^2 - 48x + 90}$ is seen.	M1	2.2a	6th Decompose algebraic fractions into partial fractions – three linear factors.
	Award 1 accuracy mark for each of the following: x^2 seen, $4x$ seen, -6 seen. $\begin{array}{r} x^2 + 4x - 6 \\ x^2 - 2x - 15 \overline{)x^4 + 2x^3 - 29x^2 - 47x + 77} \\ \underline{x^4 - 2x^3 - 15x^2} \\ 4x^3 - 14x^2 - 47x \\ \underline{4x^3 - 8x^2 - 60x} \\ -6x^2 + 13x + 77 \\ \underline{-6x^2 + 12x + 90} \\ x - 13 \end{array}$	A3	1.1b	
	Equates the various terms to obtain the equation: $x - 13 = V(x - 5) + W(x + 3)$ Equating the coefficients of x : $V + W = 1$ Equating constant terms: $-5V + 3W = -13$	M1	2.2a	
	Multiplies one or both of the equations in an effort to equate one of the two variables.	M1	1.1b	
	Finds $W = -1$ and $V = 2$.	A1	1.1b	
(7 marks)				
Notes				

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6	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Use Pythagoras' theorem to show that the length of $OB = 2\sqrt{3}$ or $OD = 2\sqrt{3}$ or states $BD = 4\sqrt{3}$	M1	2.2a	6th Solve problems involving arc length and sector area in context.
	Makes an attempt to find $\angle DAB$ or $\angle DCB$. For example, $\cos \angle DAO = \frac{2}{4}$ is seen.	M1	2.2a	
	Correctly states that $\angle DAB = \frac{2\pi}{3}$ or $\angle DCB = \frac{2\pi}{3}$	A1	1.1b	
	Makes an attempt to find the area of the sector with a radius of 4 and a subtended angle of $\frac{2\pi}{3}$ For example, $A = \frac{1}{2} \times 4^2 \times \frac{2\pi}{3}$ is shown.	M1	2.2a	
	Correctly states that the area of the sector is $\frac{16\pi}{3}$	A1	1.1b	
	Recognises the need to subtract the sector area from the area of the rhombus in an attempt to find the shaded area. For example, $\frac{16\pi}{3} - 8\sqrt{3}$ is seen.	M1	3.2a	
	Recognises that to find the total shaded area this number will need to be multiplied by 2. For example, $2 \times \left(\frac{16\pi}{3} - 8\sqrt{3} \right)$	M1	3.2a	
	Using clear algebra, correctly manipulates the expression and gives a clear final answer of $\frac{2}{3}(16\pi - 24\sqrt{3})$	A1	1.1b	
(8 marks)				
Notes				

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7	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Makes an attempt to rearrange $x = \frac{1+4t}{1-t}$ to make t the subject. For example, $x - xt = 1 + 4t$ is seen.	M1	2.2a	5th Convert between parametric equations and cartesian forms using substitution.
	Correctly states $t = \frac{x-1}{4+x}$	A1	1.1b	
	Makes an attempt to substitute $t = \frac{x-1}{4+x}$ into $y = \frac{2+bt}{1-t}$ For example, $y = \frac{2 + \frac{bx-b}{x+4}}{1 - \frac{x-1}{x+4}} = \frac{2x+8+bx-b}{x+4-x+1}$ is seen.	M1	2.2a	
	Simplifies the expression showing all steps. For example, $y = \frac{2x+8+bx-b}{5} = \left(\frac{2+b}{5}\right)x + \left(\frac{8-b}{5}\right)$	A1	1.1b	
		(4)		
(b)	Interprets the gradient of line being -1 as $\frac{2+b}{5} = -1$ and finds $b = -7$	M1	2.2a	5th Convert between parametric equations and cartesian forms using substitution.
	Substitutes $t = -1$ to find $x = -\frac{3}{2}$ and $y = \frac{9}{2}$ And substitutes $t = 0$ to find $x = 1$ and $y = 2$	M1	1.1b	
	Makes an attempt to use Pythagoras' Theorem to find the length of the line: $\sqrt{\left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2}$	M1	1.1b	
	Correctly finds the length of the line segment, $\frac{5\sqrt{2}}{2}$ or states $a = \frac{5}{2}$	A1	1.1b	
		(4)		
				(8 marks)
Notes				

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8	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Differentiates $u = 4t^{\frac{2}{3}}$ obtaining $\frac{du}{dt} = \frac{8}{3}t^{-\frac{1}{3}}$ and differentiates $v = t^2 + 1$ obtaining $\frac{dv}{dt} = 2t$	M1	1.1b	6th Differentiate using the product rule.
	Makes an attempt to substitute the above values into the product rule formula: $\frac{dH}{dt} = v \frac{du}{dt} - u \frac{dv}{dt}$	M1	2.2a	
	Finds $\frac{dH}{dt} = \frac{\frac{8}{3}t^{\frac{5}{3}} + \frac{8}{3}t^{-\frac{1}{3}} - 8t^{\frac{5}{3}}}{(t^2 + 1)^2}$	M1	1.1b	
	Fully simplifies using correct algebra to obtain $\frac{dH}{dt} = \frac{8(1 - 2t^2)}{3\sqrt[3]{t}(t^2 + 1)^2}$	A1	2.4	
		(4)		
(b)	Makes an attempt to substitute $t = 2$ into $\frac{dH}{dt} = \frac{8(1 - 2t^2)}{3\sqrt[3]{t}(t^2 + 1)^2} = \frac{8(1 - 2(2)^2)}{3\sqrt[3]{2}(2^2 + 1)^2}$	M1 ft	1.1b	6th Differentiate using the product rule.
	Correctly finds $\frac{dH}{dt} = -0.592...$ and concludes that as $\frac{dH}{dt} < 0$ the toy soldier was decreasing in height after 2 seconds.	B1 ft*	3.5a	
		(2)		

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(c)	$\frac{dH}{dt} = \frac{8(1-2t^2)}{3\sqrt[3]{t(t^2+1)^2}} = 0$ or $8-16t^2 = 0$ at a turning point.	M1 ft	1.1b	6th Differentiate using the product rule.
	Solves $8-16t^2 = 0$ to find $t = \frac{1}{\sqrt{2}}$ Can also state $t \neq -\frac{1}{\sqrt{2}}$	A1 ft	1.1b	
			(2)	
(8 marks)				
Notes				
(b) Award ft marks for a correct answer using an incorrect answer from part a.				
B1: Can also state $\frac{dH}{dt} < 0$ as the numerator of $\frac{dH}{dt}$ is negative and the denominator is positive.				
Award ft marks for a correct answer using an incorrect answer from part a.				

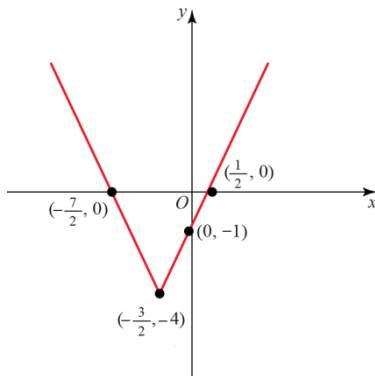
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9	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Recognises the need to write $\tan^4 x \equiv \tan^2 x \tan^2 x$	M1	2.2a	6th Integrate using trigonometric identities.
	Recognises the need to write $\tan^2 x \tan^2 x \equiv (\sec^2 x - 1) \tan^2 x$	M1	2.2a	
	Multiplies out the bracket and makes a further substitution $(\sec^2 x - 1) \tan^2 x$ $\equiv \sec^2 x \tan^2 x - \tan^2 x$ $\equiv \sec^2 x \tan^2 x - (\sec^2 x - 1)$	M1	2.2a	
	States the fully correct final answer $\sec^2 x \tan^2 x + 1 - \sec^2 x$	A1	1.1b	
		(4)		
(b)	States or implies that $\int \sec^2 x \, dx = \tan x$	M1	1.1b	6th Integrate using the reverse chain rule.
	States fully correct integral $\int \tan^4 x \, dx = \frac{1}{3} \tan^3 x + x - \tan x + C$	M1	2.2a	
	Makes an attempt to substitute the limits. For example, $\left[\frac{1}{3} \tan^3 x + x - \tan x \right]_0^{\frac{\pi}{4}} = \left(\frac{1}{3} \left(\tan \frac{\pi}{4} \right)^3 + \frac{\pi}{4} - \tan \frac{\pi}{4} \right) - (0)$ is seen.	M1 ft	1.1b	
	Begins to simplify the expression $\frac{1}{3} + \frac{\pi}{4} - 1$	M1 ft	1.1b	
	States the correct final answer $\frac{3\pi - 8}{12}$	A1 ft	1.1b	
		(5)		
(9 marks)				
Notes				
(b) Student does not need to state '+C' to be awarded the second method mark.				
(b) Award ft marks for a correct answer using an incorrect initial answer.				

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10	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Begins the proof by assuming the opposite is true. ‘Assumption: given a rational number a and an irrational number b , assume that $a - b$ is rational.’	B1	3.1	7th Complete proofs using proof by contradiction.
	Sets up the proof by defining the different rational and irrational numbers. The choice of variables does not matter. Let $a = \frac{m}{n}$ As we are assuming $a - b$ is rational, let $a - b = \frac{p}{q}$ So $a - b = \frac{p}{q} \Rightarrow \frac{m}{n} - b = \frac{p}{q}$	M1	2.2a	
	Solves $\frac{m}{n} - b = \frac{p}{q}$ to make b the subject and rewrites the resulting expression as a single fraction: $\frac{m}{n} - b = \frac{p}{q} \Rightarrow b = \frac{m}{n} - \frac{p}{q} = \frac{mq - pn}{nq}$	M1	1.1b	
	Makes a valid conclusion. $b = \frac{mq - pn}{nq}$, which is rational, contradicts the assumption b is an irrational number. Therefore the difference of a rational number and an irrational number is irrational.	B1	2.4	
(4 marks)				
Notes				

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11	Scheme		Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Figure 1 	Graph has a distinct V-shape.	M1	2.2a	5th Sketch the graph of the modulus function of a linear function.
		Labels vertex $\left(-\frac{3}{2}, -4\right)$	A1	2.2a	
		Finds intercept with the y-axis.	M1	1.1b	
		Makes attempt to find x-intercept, for example states that $ 2x + 3 - 4 = 0$	M1	2.2a	
		Successfully finds both x-intercepts.	A1	1.1b	
			(5)		
(b)	Recognises that there are two solutions. For example, writing $2x + 3 = -\frac{1}{4}x + 2$ and $-(2x + 3) = -\frac{1}{4}x + 2$		M1	2.2a	5th Solve equations involving the modulus function.
	Makes an attempt to solve both questions for x, by manipulating the algebra.		M1	1.1b	
	Correctly states $x = -\frac{4}{9}$ or $x = -\frac{20}{7}$. Must state both answers.		A1	1.1b	
	Makes an attempt to substitute to find y.		M1	1.1b	
	Correctly finds y and states both sets of coordinates correctly $\left(-\frac{4}{9}, -\frac{17}{9}\right)$ and $\left(-\frac{20}{7}, -\frac{9}{7}\right)$		A1	1.1b	
			(5)		
(10 marks)					
Notes					

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12	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Writes $(\sin 3\theta + \cos 3\theta)^2 \equiv (\sin 3\theta + \cos 3\theta)(\sin 3\theta + \cos 3\theta)$ $\equiv \sin^2 3\theta + 2\sin 3\theta \cos 3\theta + \cos^2 3\theta$	M1	1.1b	7th Use addition formulae and/or double-angle formulae to solve equations.
	Uses $\sin^2 3q + \cos^2 3q = 1$ and $2\sin 3q \cos 3q = \sin 6q$ to write: $(\sin 3q + \cos 3q)^2 = 1 + \sin 6q$ Award one mark for each correct use of a trigonometric identity.	A2	2.2a	
		(3)		
(b)	States that: $1 + \sin 6\theta = \frac{2 + \sqrt{2}}{2}$	B1	2.2a	7th Use addition formulae and/or double-angle formulae to solve equations.
	Simplifies this to write: $\sin 6\theta = \frac{\sqrt{2}}{2}$	M1	1.1b	
	Correctly finds $6\theta = \frac{\pi}{4}, \frac{3\pi}{4}, \frac{9\pi}{4}, \frac{11\pi}{4}$ Additional answers might be seen, but not necessary in order to award the mark.	M1	1.1b	
	States $q = \frac{p}{24}, \frac{3p}{24}$ Note that $q \neq \frac{9p}{24}, \frac{11p}{24}$. For these values 3θ lies in the third quadrant, therefore $\sin 3\theta$ and $\cos 3\theta$ are both negative and cannot be equal to a positive surd.	A1	1.1b	
		(4)		
(7 marks)				
Notes				
6b				
Award all 4 marks if correct final answer is seen, even if some of the 6θ angles are missing in the preceding step.				

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13	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Correctly writes $6(2+3x)^{-1}$ as: $6\left(2^{-1}\left(1+\frac{3}{2}x\right)^{-1}\right)$ or $3\left(1+\frac{3}{2}x\right)^{-1}$	M1	2.2a	6th Understand the binomial theorem for rational n.
	Completes the binomial expansion: $3\left(1+\frac{3}{2}x\right)^{-1} = 3\left(1+(-1)\left(\frac{3}{2}\right)x + \frac{(-1)(-2)\left(\frac{3}{2}\right)^2 x^2}{2} + \dots\right)$	M1	2.2a	
	Simplifies to obtain $3 - \frac{9}{2}x + \frac{27}{4}x^2 + \dots$	A1	1.1b	
	Correctly writes $4(3-5x)^{-1}$ as: $4\left(3^{-1}\left(1-\frac{5}{3}x\right)^{-1}\right)$ or $\frac{4}{3}\left(1-\frac{5}{3}x\right)^{-1}$	M1	2.2a	
	Completes the binomial expansion: $\frac{4}{3}\left(1-\frac{5}{3}x\right)^{-1} = \frac{4}{3}\left(1+(-1)\left(-\frac{5}{3}\right)x + \frac{(-1)(-2)\left(-\frac{5}{3}\right)^2 x^2}{2} + \dots\right)$	M1	2.2a	
	Simplifies to obtain $\frac{4}{3} + \frac{20}{9}x + \frac{100}{27}x^2 + \dots$	A1	1.1b	
	Simplifies by subtracting to obtain $\frac{5}{3} - \frac{121}{18}x + \frac{329}{108}x^2 + \dots$ Reference to the need to subtract, or the subtracting shown, must be seen in order to award the mark.	A1	1.1b	
		(7)		

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(b)	Makes an attempt to substitute $x = 0.01$ into $f(x)$. For example, $\frac{6}{2+3(0.01)} - \frac{4}{3-5(0.01)}$ is seen.	M1	1.1b	6th Understand the binomial theorem for rational n.
	States the answer 1.5997328	A1	1.1b	
		(2)		
(c)	Makes an attempt to substitute $x = 0.01$ into $\frac{5}{3} - \frac{121}{18}x - \frac{329}{108}x^2 + \dots$ For example $\frac{5}{3} - \frac{121}{18}(0.01) + \frac{329}{108}(0.01)^2 + \dots$ is seen.	M1 ft	1.1b	6th Understand the binomial theorem for rational n.
	States the answer 1.59974907... Accept awrt 1.60.	M1 ft	1.1b	
	Finds the percentage error: 0.0010%	A1 ft	1.1b	
		(3)		
(12 marks)				
Notes (a) If one expansion is correct and one is incorrect, or both are incorrect, award the final accuracy mark if they are subtracted correctly. (c) Award all 3 marks for a correct answer using their incorrect answer from part (a).				

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14	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
(a)	Uses $a_n = a + (n - 1)d$ substituting $a = 5$ and $d = 3$ to get $a_n = 5 + (n - 1)3$	M1	3.1b	5th Use arithmetic sequences and series in context.
	Simplifies to state $a_n = 3n + 2$	A1	1.1b	
			(2)	
(b)	Use the sum of an arithmetic series to state $\frac{k}{2}[10 + (k - 1)3] = 948$	M1	3.1b	5th Use arithmetic sequences and series in context.
	States correct final answer $3k^2 + 7k - 1896 = 0$	A1	1.1b	
			(2)	
(4 marks)				
Notes				

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15	Scheme	Marks	AOs	Pearson Progression Step and Progress descriptor
	Understands that integration is required to solve the problem. For example, writes $\int_{\frac{\pi}{2}}^{\pi} (x \sin^2 x) dx$	M1	3.1a	6th Use definite integration to find areas between curves.
	Uses the trigonometric identity $\cos 2x \equiv 1 - 2\sin^2 x$ to rewrite $\int_{\frac{\pi}{2}}^{\pi} x \sin^2 x dx$ as $\int_{\frac{\pi}{2}}^{\pi} \left(\frac{1}{2}x - \frac{1}{2}x \cos 2x \right) dx$ o.e.	M1	2.2a	
	Shows $\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}x dx = \left[\frac{1}{4}x^2 \right]_{\frac{\pi}{2}}^{\pi}$	A1	1.1b	
	Demonstrates an understanding of the need to find $\int_{\frac{\pi}{2}}^{\pi} \frac{1}{2}x \cos 2x dx$ using integration by parts. For example, $u = x, \frac{du}{dx} = 1$ $\frac{dv}{dx} = \cos 2x, v = \frac{1}{2} \sin 2x$ o.e. is seen.	M1	2.2a	
	States fully correct integral $\int_{\frac{\pi}{2}}^{\pi} \left(\frac{1}{2}x - \frac{1}{2}x \cos 2x \right) dx = \left[\frac{1}{4}x^2 - \frac{1}{4}x \sin 2x - \frac{1}{8} \cos 2x \right]_{\frac{\pi}{2}}^{\pi}$	A1	1.1b	
	Makes an attempt to substitute the limits $\left(\frac{\pi^2}{4} - \frac{1}{4}(0) - \frac{1}{8}(1) \right) - \left(\frac{\pi^2}{16} - \frac{1}{4}(0) - \frac{1}{8}(-1) \right)$	M1	2.2a	
	States fully correct answer: either $\frac{3\pi^2}{16} - \frac{1}{4}$ or $\frac{3\pi^2 - 4}{16}$ o.e.	A1	1.1b	
(7 marks)				
Notes				
Integration shown without the limits is acceptable for earlier method and accuracy marks. Must correctly substitute limits at step 6				