Year 1 Pure Chapter 13: Integration with Limits - Exam Questions (Total Marks 55)

1. Use calculus to find the value of

$$\int_{1}^{4} (2x + 3\sqrt{x})\,\mathrm{d}x.$$

(Total 5 marks)

2. $$\mathrm{f}(x) = x^3 + 3x^2 + 5.$$

Find

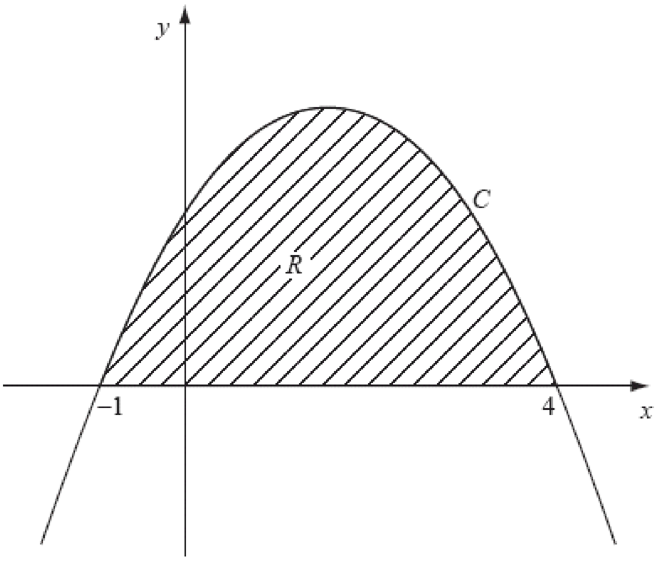
(a) $\mathrm{f}''(x)$,

(3)

(b) $\int_1^2 \mathrm{f}(x)\,\mathrm{d}x$.

(4)
(Total 7 marks)

3.



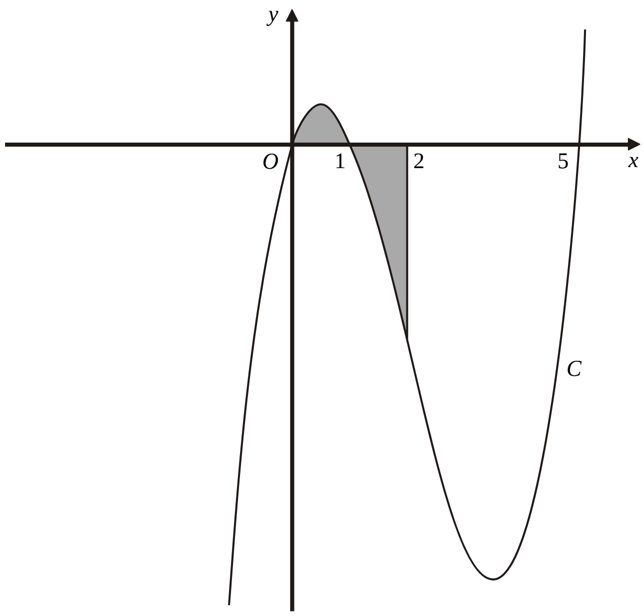
The diagram above shows part of the curve C with equation $y = (1 + x)(4 - x)$.

The curve intersects the x-axis at $x = –1$ and $x = 4$. The region R, shown shaded in the diagram, is bounded by C and the x-axis.

Use calculus to find the exact area of R.

(Total 5 marks)

4.



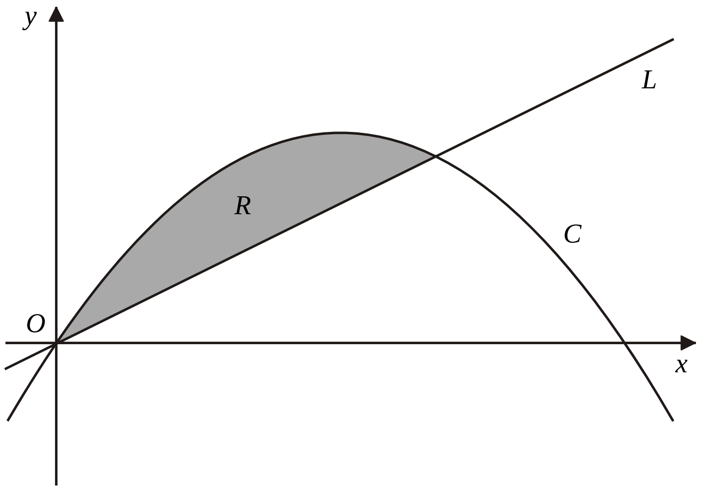
The diagram above shows a sketch of part of the curve C with equation

$$y = x(x - 1)(x - 5).$$

Use calculus to find the total area of the finite region, shown shaded in the diagram, that is between $x = 0$ and $x = 2$ and is bounded by C, the x-axis and the line $x = 2$.

(Total 9 marks)

5.



In the diagram above the curve C has equation $y = 6x - x^2$ and the line L has equation $y = 2x$.

(a) Show that the curve C intersects the x-axis at $x = 0$ and $x = 6$.

(1)

(b) Show that the line L intersects the curve C at the points $(0, 0)$ and $(4, 8)$.

(3)

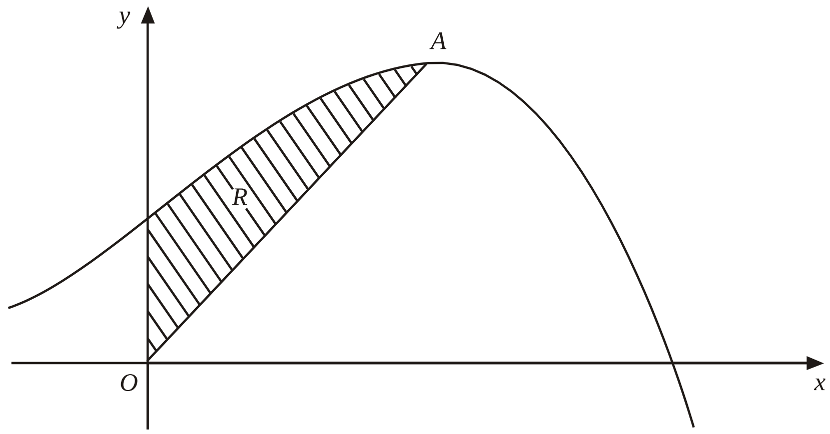
The region R, bounded by the curve C and the line L, is shown shaded in the diagram above.

(c) Use calculus to find the area of R.

(6)

(Total 10 marks)

6.



The diagram above shows a sketch of part of the curve with equation $y = 10 + 8x + x^2 - x^3$.

The curve has a maximum turning point A.

(a) Using calculus, show that the x-coordinate of A is 2.

(3)

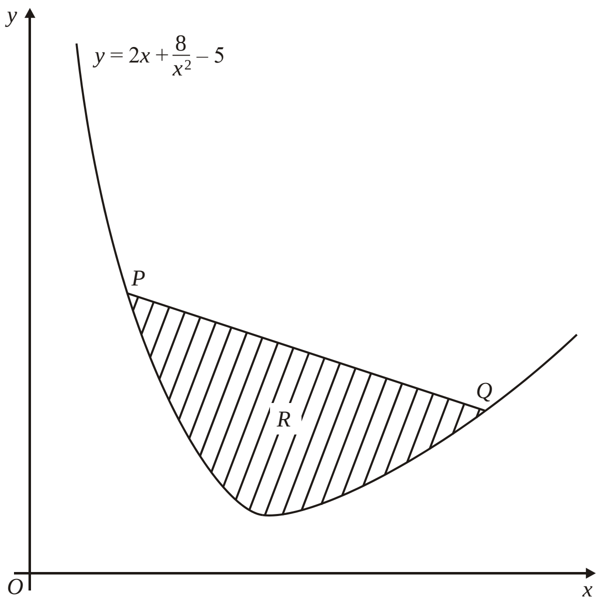
The region R, shown shaded in the diagram, is bounded by the curve, the y-axis and the line from O to A, where O is the origin.

(b) Using calculus, find the exact area of R.

(8)

(Total 11 marks)

7.



This figure shows part of a curve C with equation $y = 2x + \dfrac{8}{x^2} - 5$, $x > 0$.

The points P and Q lie on C and have x-coordinates 1 and 4 respectively. The region R, shaded in the diagram, is bounded by C and the straight line joining P and Q.

Find the exact area of R.

(8)
(Total 8 marks)